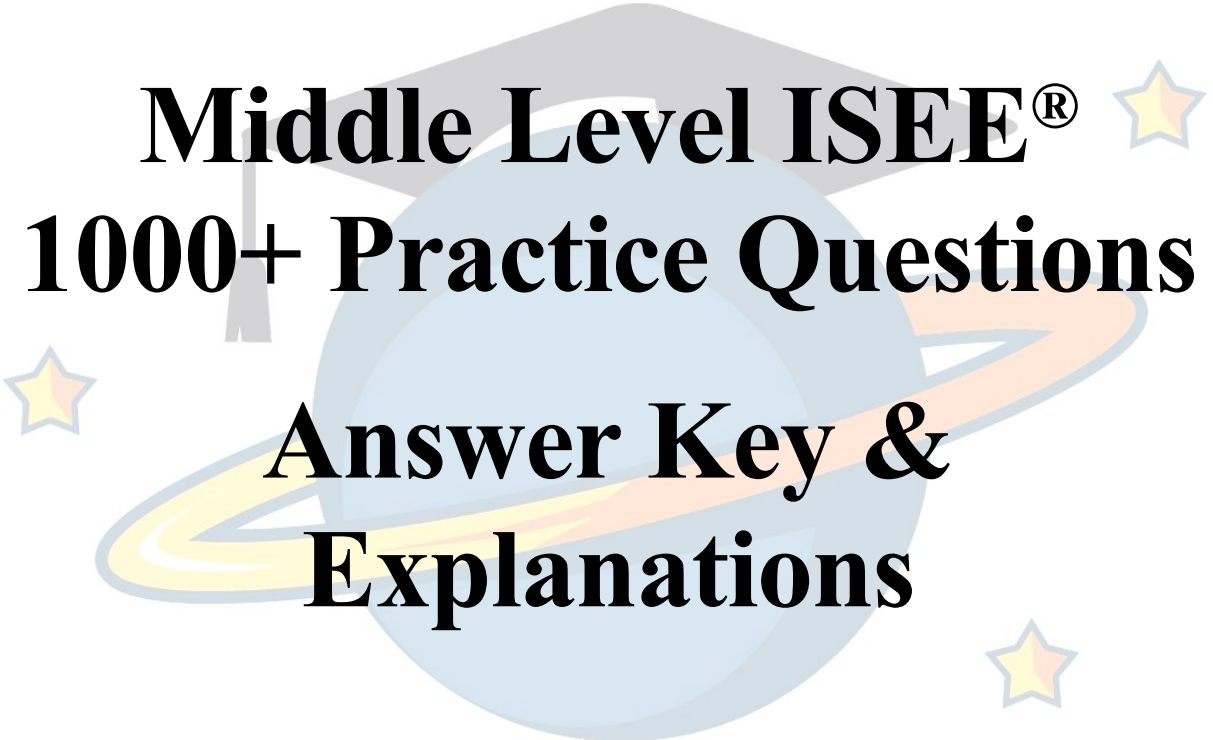


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Note: This answer key and the accompanying explanations apply to the 2026 edition of the workbook. If you are looking for answer key and explanations for the 2025 edition, please visit us at [www.thetutorverse.com/books](http://www.thetutorverse.com/books) to submit a [help ticket request](#). Please note that we are not able to support answer keys and explanations for earlier editions of the workbook.

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## Diagnostic Practice Test (Form A)

### Verbal Reasoning

1. B. “Primary” is similar to the word “prime,” which means the main or most important thing. “The primary goal of the presidency is to serve the American people.”
2. B. “Lofty” has a positive connotation, much like “grand” and “independent” (“heavy” and “basic” are more neutral or negative in connotation). “The lofty goal was difficult to achieve, but worth pursuing.” We can see that the word that best replaces “lofty” is “grand.” “Lofty” borrows from the word “loft,” which we know is a room that is high up in a building. A lofty idea is high, and worthy.
3. C. “Solitary” reminds of us the word “solitude,” which is the state of being alone. When one is alone, one could be said to be isolated. This doesn’t mean “weak,” necessarily, or “enjoyable.” “The solitary mountain peak was majestic to behold.”
4. C. “Accumulate” includes the prefix “ac-,” which means “toward” or “in addition to,” as well as the word “cum,” which means “with.” Together, we see a word that means to bring together more than just one thing, or gather. “Jordan’s accumulated homework was massive, the result of his procrastinating for a week.”
5. B. “Condemn” has a strong negative connotation, and is used to describe the action of expressing disapproval for something. This is most similar to “denounce.” “The principal condemned the actions of those who sprayed graffiti on the wall.”
6. B. “Adequate” combines the prefix “ad-,” which means “toward,” with a word that looks like “equal.” If we move toward the state of being equal, or in balance, we can say that what we have is adequate, enough, or sufficient. “
7. C. “Tentative” has a somewhat negative connotation, unlike “committed,” “confident,” and “reliable.” If something is tentative, it is temporary, or experimental. “The guests made a tentative reservation, since they were unsure whether or not they could commit to the date.”
8. D. “Necessity” is the noun form of the word “necessary.” Something that is necessary is a basic requirement – something that must be had. “The necessity of water to life is obvious.”
9. D. “Consult” is similar to the word “consultation,” which is a word used to describe a meeting where people discuss different topics. A clue as to the word’s meaning is the root word “con,” which means “with” or “together.” Typically, discussions are between more than one being. “He had to consult with a professional, since he was new to the game.”
10. A. “Prediction” contains two root words: “pre-,” which means “before,” and “dict,” which means “speak.” Therefore, “prediction” means something spoken before having full knowledge of something. It’s not just a speech or the way something is spoken – it is an estimate or a guess (since a speech could be full of guesses or full of truths).
11. B. “Vie” is usually used in context of describing people who compete with, or feud, over a certain thing. “The contest allowed designers to vie for the title of World’s Best Costume Dresser.”
12. B. “Plausible” has a positive connotation, which means that something is possible, or capable of being a reality; in other words, realistic. “The scientist’s plans were finally ruled to be plausible, despite seeming outlandish and unrealistic at first.”
13. C. “Dispute” is made up of two root words: “dis-,” which means “opposite of,” and “pute,” which means “think” (i.e. “computer”). To “think” the “opposite of” would be to disagree with someone, or quarrel with that person. “Davey was a contrarian, and would always dispute what someone else said.”
14. B. “Speculate” contains the root word “spec,” which means “look” and “see,” as well as the suffix “-ate,” which means “to cause to be.” So, we know that the word “speculate” has something to do with being able to see or look. If we guess about something, we are trying to “see” or “look” into the future. “There is no use in speculating, since nobody can know the future.”

15. B. “Impede” has two root words: “im-,” which mean “not,” and “ped,” which means “foot.” We can take this to mean that impede has something to do with hindering or preventing someone’s advancement. “The barriers are designed to impede movement.”
16. C. “Flagrant” has a strong negative connotation, which is used to describe something as being obviously wrong or bad. “The bully had a flagrant disregard for the school’s rules.” We can immediately rule out the positive and neutral words “aromatic,” and “conventional.” If we replace “flagrant” with “vagrant,” we get a nonsensical sentence, since “vagrant” is a noun, and “flagrant” describes the degree of disregard the bully had.
17. C. “Intricate” is used to refer to something that is very detailed, intricate, or ornate. “The ornament was covered with fine, intricate details depicting various scenes from the past.” We can see that the ornament wasn’t bare or necessarily elite, but was ornate. The person who made the ornament was proficient in doing so, but the ornament itself isn’t proficient.
18. B. “Languish” has a strong negative connotation, which is used to describe something that is declining or getting weaker. The words “decipher” and “prosper” have neutral or positive connotations. If we use the word in a sentence, we can see that “grieve” is not a proper replacement. Consider: “The plant, given no light or water, was allowed to languish until it dried up complete.”
19. B. “Exuberant” has a strong positive connotation, so we can rule out all of the other answer choices but “enthusiastic,” which is the only other word to have such a positive connotation.
20. A. “Renounce,” like the word “announce,” has something to do with making a statement. We can guess that this is a negative statement, since it has a negative connotation. “The traitor renounced his citizenship and loyalties.” The word “abandon” best describes the traitor’s actions. While the traitor may proclaim or shriek the actual words, the action itself is to abandon his loyalties.
21. A. The sentence describes a difference, and an action that is unable to be taken in respect to that difference. People don’t usually “trace” a difference or “refuse” it (whatever that means). However, people can “disregard” or “detect” it. In this case, we know that the sentence is showing how the difference is difficult to detect even for athletes.
22. D. An ice cube is used to do something to the itching. We can tell that the sentence describes what will happen if the thing is not done. So, we know that the ice cube is used to soothe (rather than intensify or maintain or take advantage of) the itch.
23. A. Abby invited guests over, but she didn’t have enough of something for everyone. This means she wasn’t prepared for the number of guests, so she didn’t “anticipate” having all of the people that she ended up having. Whether she remembered after the fact doesn’t matter. We don’t know that Abby knew that she didn’t have enough utensils and decided to invite people anyway.
24. B. The store tried to correct their mistake, but failed, since the customer was more angry afterwards. This means the attempt was not very good or effective, and the only word that describes something like that is “feeble,” which means that it was a half-hearted attempt.
25. A. Earthquakes are dangerous and deadly, and we know that this was a big one that hit a rural area where it likely didn’t do much damage. If it was in a more populous area, it may have resulted in catastrophe, or disaster. There would likely not be much controversy (disagreement) about it or disgrace as a result of it. A siege is a prolonged attack, and not a good description of a sudden earthquake.
26. B. Quincy brought things that weren’t allowed into the theater, so the usher threw it away. First, though, the usher took it from Quincy, or confiscated it. He would not allow (authorize) Quincy to keep the candy and then throw it away. “Extract” has a meaning similar to “confiscate,” but it is slightly different. Both words mean to take by force, but confiscate is a better fit since it means to take something away that isn’t allowed.
27. B. The sentence warns that animals may be dangerous when provoked. Another word for “provoked” is “antagonized.” Admiring the animals will not likely provoke them. People don’t gratify or raise animals on safaris, as they go just to watch them.
28. C. Lois wants her daughter to give up the lollipop because it’s dinner time. Another word for this is “relinquish.” It doesn’t make sense that she would try to convince her daughter to covet (yearn



for) her lollipop (since she has it already) or reveal it (we don't know that it's hidden).  
"Compromise" is generally not used to describe possession of a thing.

29. A. The sentence tells us that it is important to consider some course of action just in case we run into something unexpected. This sounds like a backup plan, or an "alternative route." This would not be a usual route, but one that could be taken if the usual routes were blocked by traffic or under construction. Panoramic routes describe routes that are visually pleasing. The word "balmy" describes the weather.
30. A. Larry shows an interest in golf, which is the definition of someone who is "avid" at something. If he was not committed (noncommittal) or disgruntled (unhappy with), he would not be studying and playing in his spare time. "Dire" is not usually used to describe a golfer – an expert, perhaps, but not a "dire golfer" (which is nonsensical).
31. B. A revolutionary war is fought by people who disagree with ruling powers. If the Americans rebelled against the British, then they did so against their authority, or right to rule. The sentence hints at this with the phrase "governed by a country."
32. B. If Sam found it "very difficult" to stick to his diet, then his strength of will wasn't very strong at all. This means his will would falter, or lose strength. This is the opposite of "teem," which means "be full of." "Swelter" might be confused for "swell," or "grow," but actually means to sweat profusely because of extreme heat (he might sweat, but probably not because of heat caused by pasta). His will would not conspire (plot against him) every time, but it might abandon him.
33. A. If the plan is "unnecessarily risky," then it is said to be "brazen." Just because something is visible, despicable, or historic, doesn't make it something that is risky.
34. B. The sentence contrasts two different states, one before the book deal and one after. Before, we are told that the author is living modestly. After, we can expect the opposite, which would mean the book deal was "lucrative," or very profitable and beneficial in terms of money. If the book deal was horrible (abhorrent) or unsatisfactory, this would probably not be the case. A rudimentary book deal might be very basic, but still very lucrative. Then again, the book deal could have been very complicated (the opposite of rudimentary).
35. C. The sentence describes the music as "the most beautiful" ever heard, so we are looking for a word that is similarly and strongly positive in connotation. The only word is "sublime," which means "extremely beautiful." The music may or may not have been "conservative" or "vigorous," but neither necessarily means that the music was extremely beautiful.
36. D. The sentence compares the room as it looks now (described by the missing word) and the room "if" it was "simply and plainly" decorated. This tells us that it was not. So, the opposite is something very ornate or extravagant. "Desolate" means empty, so this wouldn't make sense. "Abrasive" and "conceivable" decorations don't make too much sense in context of comparing simple decorations with more extravagant decorations.
37. A. Skunks are known to be animals that excrete a very particular, bad smell. The only adjective that describes smells is "characteristic."
38. B. The phrase "except for" tells us that except for the missing word, the rest of the routine was "boring and uninteresting." So the opposite of "boring and uninteresting" is "flourish," which is a special decorative or beautiful action. The gymnast may or may not have been ambitious, or had a talisman. Repetition tends to be boring and uninteresting to people.
39. D. The sentence tells us that before calculators, complex operations were completed "manually" and "took a long time." Be careful not to choose the opposite here, since the sentence isn't comparing the time it takes to perform calculations before and after calculators. Instead, the sentence is expanding on the idea that it took a long time and was very manual. This describes a tedious process, not a quick one. Substituting "influential" and "liberal" into the sentence doesn't make sense, and doesn't clarify the sentence in terms of defining the dependent clause "as such...complete."
40. B. The sentence describes how new saplings sprout around the edge of a field, which means the forest is getting bigger and the field is getting smaller. This means that the forest is encroaching

on the field, not assembling it or rebelling against it. There is no qualitative judgment about whether or not this is good or bad, so it doesn't make sense to say "improves."

### Quantitative Reasoning

- Algebraic Concepts – Functions & Patterns.* A. The pattern shows that two black dots are removed in each element. By the third element, there are 5 dots remaining. By the fourth, there should only be 3. By the fifth, there should be only 1.
- Algebraic Concepts – Ratios, Proportions, & Scale Factors.* D. The scale factor tells us that the replica is smaller than the actual monument by 15 times.  $15 \times 37 = 555$ .
- Geometry – Coordinates.* D. A rhombus is a parallelogram with four sides of equal length. We know that the distance along the  $y$ -axis must be 5 units, as the distance between  $(3,1)$  and  $(3,-4)$  is  $1 - (-4) = 5$ . So, counting 5 units down from point  $(-1,4)$ , we end up at  $(-1,-1)$ .
- Data & Probability – Interpreting Data.* B. The stem represents the 10's digit, and each leaf represents a units digit. Together, each stem and leaf combination represents the score of a student. So, a 7 in the stem column and a 2 in the leaf portion of the chart gives us a score of 72. We see 88 and 93, too, but not 85.
- Measurements – Units.* A. There are 1,000 grams in a kilogram. In order to convert from kilograms to grams, we must multiply the amount of kilograms by 1,000. So  $7.257 \text{ kg} \times 1,000 = 7,257$ .
- Geometry – Geometric Objects.* B. If each square in the grid has an area of  $4 \text{ in}^2$ , then each side of a square is  $\sqrt{4} = 2 \text{ in}$ . Count the number of sides in the shaded region to find a total of 16, for a perimeter of  $16 \times 2 = 32 \text{ in}$ .
- Numbers & Operations – Estimation.* D. We can round 58 to 60 and divide by 3 to get 20. There is no need to estimate inside the parentheses, which simplifies to  $9 - 3 = 6$ . Finally,  $20(6) = 120$ .
- Data & Probability – Interpreting Data.* D. Based on the circle graph, we can see that pigs outnumber all other animals, and goats are the least numerous animal. This immediately rules out choices A, B, and C, since the number of goats are shown to be equal to other animals.
- Geometry – Geometric Objects.* A. Since there are five sides to the two bases of the prism, we know that we are looking for a pentagonal prism. The only such figure is shown in choice A. The other choices show hexagonal polyhedron, a cylinder, and rectangular prism.
- Algebraic Concepts – Solving Algebraic Equations.* C.  $bx$  means that  $b$  is multiplied by  $x$ . To solve for  $b$ , we must isolate it on one side of the equation. This means dividing out the  $x$  that is multiplying it, to leave an "invisible" 1. If we divide  $x$  from both sides of the equation, then we are left with  $b = 7$ .
- Algebraic Concepts – Ratios, Proportions, & Scale Factors.* B. The most reliable way to solve this question is to set up a proportion. To do so, we have to make sure all units are comparable. Here, we know that time is expressed in hours and in minutes. We know that since there are 60 minutes in 1 hour, 30 minutes represents 0.5 hour. When setting up the proportion, ensure that the numerators and denominators of both fractions represent similar things. In this case, we can put distance in the numerator, and time in the denominator. This gives us  $\frac{8 \text{ miles}}{2 \text{ hours}} = \frac{x \text{ miles}}{0.5 \text{ hours}}$ , where  $x$  represents the answer to the question (how many miles run in 30 minutes). Simply cross multiply and solve for  $x$ :  $(0.5)(8) = (2)(x)$  simplifies to  $4 = 2x$ , and  $x = 2$ .
- Geometry – Geometric Objects.* C. Since ABCD is a square, and it touches the circle at exactly four points, BD represents the circle's diameter. The area of a circle is  $\pi r^2$ , where  $r$  is the radius (which is half the length of the circle's diameter). This means that  $8 = 2r$ , and  $r = 4$ . Thus, the area of the circle is  $\pi(4)^2 = 16\pi \text{ mm}^2$ .
- Geometry – Coordinates.* B. Choice A represents a reflection over the  $y$ -axis. Choice C represents a rotation around another point. Choice D represents a translation. Only Choice B represents a reflection over the  $x$ -axis.
- Measurements – Units.* B. We first should convert the Earth's circumference from meters to kilometers. There are 1,000 meters in a kilometer, so we can convert:  $40,000,000 \div 1,000 =$

40,000 km. From this we divide by the rate at which the snail crawls to see how many years it will take:  $40,000 \div 10 = 4,000$  years.

15. *Algebraic Concepts – Ratios, Proportions, & Scale Factors*. D. Since the triangles are similar, we know that each side corresponds to the same side on the other triangle (for example,  $DE$  to  $AB$ , and  $FE$  to  $CB$ ). Since we know that  $FE$  has a length of  $0.5x$ , and we know that  $CB$  has a length of  $x$ , then we know that  $FE$  is half as long as  $CB$  (however long  $CB$  actually is). This means that  $DE$  must also be half as long as  $AB$ . Therefore,  $AB = (2)(8) = 16$ .
16. *Algebraic Concepts – Slope*. B. The car with the greatest hourly increase in temperature is shown by the line with the greatest slope. Even though the white car starts at a higher temperature, it doesn't increase by as much as the other cars. We can see that the steepest line is that of the black car, which means over a certain period of time, its temperature increases the most.
17. *Numbers & Operations – Estimation*. B. Since we don't know what the square root of 85 is (there is no integer that, when multiplied by itself, gives us 85), we must estimate. We know that  $9 \times 9 = 81$ , and  $10 \times 10 = 100$ . Since 85 is between 81 and 100, the square root of 85 must be between 9 and 10.
18. *Data & Probability – Interpreting Data*. B. In this graph, the  $y$ -axis represents elevation, not speed. Therefore, upward slopes indicate an increasing elevation, while downward slopes represent a decreasing elevation. Flat lines, with a slope of zero, indicate a constant elevation. The graph doesn't tell us about his distance from the beach. This part of the graph only tells us that his elevation remained the same.
19. *Measurements – Units*. B. In liters, the bucket can hold  $1,500 \div 1,000 = 1.5$  L of water. If the bucket is currently a third full, we can represent this as  $1.5 \div 3 = 0.5$  L. This means to fill the bucket, we need another  $1.5 - 0.5 = 1.0$  L of water.
20. *Numbers & Operations – Estimation*. D. Find the average volume of the dice and multiply that by the total number of dice. We can use the range of surface areas to find an approximate average side length. The median of the surface areas is 600, and since there are 6 sides to a cube, each side has an area of  $600 \div 6 = 100$ . To find the length of each side, take the square root of the area:  $\sqrt{100} = 10$ . From there, we can cube the length of the side to find average volume:  $10 \times 10 \times 10 = 10^3 = 1,000$  cubic millimeters. The total volume that all the dice will take up is  $1,000 \times 50 = 50,000$ .
21. *Numbers & Operations – Integers*. D. Sometimes, the best strategy is to try different possibilities. Perfect squares are the result of multiplying an integer by itself. For example,  $4 \times 4 = 16$ ; 16 is a perfect square. We can try several examples:  $16 - 9 = 7$ , or  $9 - 1 = 8$ , or  $25 - 16 = 9$ . These rule out all of the other answer choices but D. A helpful rule to know is that the difference between perfect squares can never be even, non-multiples of 4.
22. *Geometry – Coordinates*. C. The circumference of a circle is given as  $2r\pi$ , so we must find  $r$ , the radius. We know that the center of the circle is at  $(0,0)$ , and that the radius runs from there to point  $(3,4)$ . If we drew a line three units left, and four units down, we see that a right triangle forms, with the hypotenuse running from  $(0,0)$  to  $(3,4)$ . Recall the Pythagorean Theorem:  $c^2 = a^2 + b^2$ , which gives us the length of the hypotenuse of a right triangle,  $c$ . Since we know the two legs ( $a = 3$  and  $b = 4$ ), we know that  $c^2 = 3^2 + 4^2$ , or  $c^2 = 9 + 16$ . From there,  $c^2 = 25$ , and  $c = 5 = r$ . Thus, the circumference is  $2(5)\pi = 10\pi$ . Note that a 3-4-5 triangle is a special right triangle, where the sides of the triangle will always be in the proportion 3-4-5.
23. *Numbers & Operations – Fractions*. C. In Column A, the smallest positive integer of  $x$  that makes  $\frac{1}{x}$  a repeating decimal is 3 (since  $\frac{1}{1} = 1$ , and  $\frac{1}{2} = 0.5$ ). In Column B, the only values of  $n$  which make  $\frac{1}{n} > 0.25$  are 1, 2, and 3. There are 3 values, which means the values in Column A and Column B are equal. Note that if  $n = 4$ , then  $\frac{1}{n} = 0.25$ , not  $\frac{1}{n} > 0.25$ .



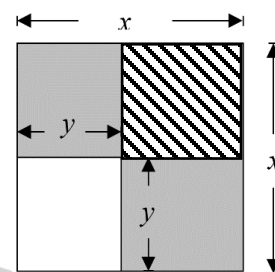
24. *Data & Probability – Mean, Median, Mode, & Range.* D. We do not know the median score because we do not know the individual scores on the test, only that the total sum of scores on all four tests was  $85 \times 4 = 340$ . There could be many combinations of scores that give an average of 90, but which have different medians.
25. *Algebraic Concepts – Ratios, Proportions, & Scale Factors.* A. The value of Column A is  $9 \times 8 = 72$  hours. For Column B, we know that Joe spends 1 hour and 15 minutes working for every 1 hour he spends sleeping. This means in a single day, if Joe spends 8 hours sleeping, he will have spent  $8 \times 1.25 = 10$  hours working (since 15 minutes is 25% of 1 hour). If he does this for one week, that's 7 days, or  $7 \times 10 = 70$  hours.
26. *Numbers & Operations – Integers.* D.  $x$  in Column A is equal to 11 or  $-11$ . In Column B, the radical sign only signifies the positive square root, so the expressions are either equal in one case, or unequal in the other. The answer is D.
27. *Algebraic Concepts – Ratios, Proportions, & Scale Factors.* A. Susan's original stamp has an area of  $2 \times 2 = 4 \text{ cm}^2$ . Each side of the original stamp is first increased by  $\frac{5}{2}$ , then decreased by  $\frac{1}{5}$ . This is the same as the following equation:  $\frac{2}{1} \times \frac{5}{2} \times \frac{1}{5} = \frac{10}{10} = 1$ . This means the final copy of the stamp has an area of  $1 \times 1 = 1 \text{ cm}^2$ .
28. *Geometry – Coordinates.* D. Points in the second quadrant have negative  $x$ -coordinate values, and positive  $y$ -coordinate values. Points in the fourth quadrant have positive  $x$ -coordinate values, and negative  $y$ -coordinate values. If we were to add together the coordinate values of a point in the second or fourth quadrant, we might end up with a negative or a positive number, depending on where the point is. Since this is the case, it's not possible to know which will be greater.
29. *Data & Probability – Interpreting Data.* B. Column A describes the highest point on the scatterplot. This point shows us that a person who is 5.25 ft. tall spent 17 hours per week reading. Column B describes the three points on the 6.5 ft. tall line, which is exactly half way between 6 and 7 ft. tall. There are three data points: 11, 5, 3. The sum of this is 19, which when we divide by 3 to find the average, gives us a number greater than 6. We don't need the exact number – we already know that Column B is greater than Column A.
30. *Data & Probability – Probability.* C. There are 80 bees in total, and  $50 - 2 = 48$  bees sting. So, there is a  $\frac{48}{80} = \frac{6}{10} = \frac{3}{5}$ . Note that  $3 \div 5 = 0.6$ , or 60%.
31. *Algebraic Concepts – Slope.* B. The slope of line  $m$  is negative, since the line is decreasing. The slope of line  $y = \frac{1}{2}x + 3$  is positive. Therefore, the value in Column B is greater.
32. *Numbers & Operations – Factors, Multiples, Primes.* A. Find the GCF using prime factorization.  $27 = 3^3$  and  $32 = 2^5$ . There are no prime factors in common between the two numbers, which means 27 and 32 are relatively prime, meaning their GCF is 1. The smallest prime number is 2, so in any case, Column A is greater.
33. *Data & Probability – Probability.* C. Each suit contains one jack, and there are 4 suits, so there are a total of 4 jacks in a single deck of cards. This means there is a  $\frac{4}{52} = \frac{1}{13}$  chance of getting a single jack. Using this same logic, this is the same probability of getting a single queen. The first card drawn has a  $\frac{1}{13}$  chance of being a jack, just as it has a  $\frac{1}{13}$  chance of being a queen. Since the card is put back before the next card is drawn, the same probability can be applied to the next card drawn. This means the second card still has a  $\frac{1}{13}$  chance of being a jack (or being a queen).



The probability of drawing two jacks or drawing a jack and a queen is  $\frac{1}{13} \times \frac{1}{13}$ , but we don't need to calculate this to determine that the probabilities are the same.

34. *Numbers & Operations – Percents*. C. In Column A, we increase both the number of girls and the total number of students in the class. This gives us  $\frac{12+4}{8+12+4} = \frac{16}{24} = \frac{2}{3}$ . In Column B, we decrease the number of boys in class, as well as the total number of students. This gives us  $\frac{12}{8+12-2} = \frac{12}{18} = \frac{2}{3}$ . Both amounts are equal.

35. *Geometry – Geometric Objects*. A. The area of the large square is  $x^2$ . The area of the small square is  $y^2$ . The area of the shaded region is  $x^2 - y^2$ . We can see that the striped area in the figure at right represents  $(x - y)(x - y)$ , or  $(x - y)^2$ . This is clearly not as large as the area of the shaded region. Alternatively, plug in values to test out whether Column A or Column B is greater. Let's use  $x = 3$  and  $y = 1$ . Column A would give us  $3^2 - 1^2 = 9 - 1 = 8$ . Column B would give us  $(3 - 1)^2 = 2^2 = 4$ . Therefore, Column A is a greater value.



36. *Algebraic Concepts – Solving Algebraic Equations*. C. If we combine like terms, we see that the equation becomes  $14x + 14y = 0$ . We could divide both sides by 14 to arrive at  $\frac{14x + 14y}{14} = \frac{0}{14}$ , which simplifies to  $x + y = 0$ . This means that no matter how many times we have the term  $x + y$  (7 times, or 28 times), the result will always be zero.
37. *Numbers & Operations – Decimals*. A. We know that  $8 \times 8 = 64$ , and  $7 \times 7 = 49$ , so we have  $8 - 7 = 1$  in Column A. We don't know the values of  $\sqrt{6.4}$  or  $\sqrt{4.9}$ , but we know that each of them is somewhere between 2 and 3 (because we know that  $2^2 = 4$ , and  $3^2 = 9$ , and that both  $\sqrt{4.9}$  and  $\sqrt{6.4}$  are between  $\sqrt{4}$  and  $\sqrt{9}$ ). Subtracting a number between 2 and 3 from another number between 2 and 3 will always give us something less than 1. For example, if we try to make the biggest number we can by subtracting a number between 2 and 3 from another number between 2 and 3, we could try  $2.99 - 2.01 = 0.98$ , which is less than 1.

### Reading Comprehension

1. A. *Main Idea*. The tradition described is the school calendar, which is shared by many people across the country. The passage takes the point of view that this tradition should be changed, and gives many reasons to support this idea.
2. D. *Supporting Idea*. The author uses these colloquial phrases as examples of how the school schedule has impacted American culture, since we even have special phrases to describe these periods of time.
3. C. *Vocabulary*. The author uses a series of positive words to describe the programs that parents look for. Only "interesting" fits this list of positive traits.
4. D. *Inference*.
5. B. *Tone/Language/Style*. The passage is written as an editorial where an opinion is shared. There is no personal story or narrative used in the passage, nor are there instructions given. A reference book would be filled with facts, as opposed to opinions.
6. B. *Organization/Logic*. The author provides his opinion on school calendars, which is not so much a theory as an idea or point of view. There isn't one particular event that is described, and there isn't a particular order to the passage in terms of timeline.
7. C. *Main Idea*. Train stations are mentioned as being busy and having more than one function, but the purpose of the passage is to describe the ceiling of the train station as a work of art. In particular, one particular ceiling is described.

8. C. *Supporting Idea*. The passage mentions how many people see and notice the other parts of Grand Central, but don't always look up to look at the ceiling. It never mentions how the ceiling is ruined because of the mistake, or that the color is not right.
9. C. *Organization/Logic*. The paragraph delves into the physical appearance of the ceiling, and doesn't make comparisons with other parts of the ceiling. The mistakes aren't described until the next paragraph.
10. A. *Tone/Language/Style*. The patterns referred to are constellations, which are the subject of the third paragraph. The Milky Way is described as a cluster of stars, not as any particular pattern. The orbit is described as a golden arch.
11. D. *Vocabulary*. The context tells us that transposing most nearly means "copying," since the rest of the sentence describes how the art was replicated from a sketch onto the ceiling.
12. D. *Supporting Idea*. The passage mentions all of these qualities, but implies that Gemini and Taurus' orientation was a mistake.
13. B. *Main Idea*. The passage describes the author's personal experience of realizing that he has more responsibilities along with his freedoms (which is summarized in the second to last paragraph). The author doesn't want to do chores, yet ends up doing so because they are part of his new responsibilities. Part of accepting this is the fact that the author is grocery shopping despite the fact that he is not a parent. The author doesn't mention that a driver's license is more or less important than other things.
14. C. *Tone/Language/Style*. In context, we can tell that the author is referring to the fact that he has more freedoms (like being able to drive to the lake to skip stones) now that he has a driver's license. There's no mention that the author has a new sports car, or that the license was given to him on his 18<sup>th</sup> birthday. The freedoms come with additional responsibilities, not fewer.
15. B. *Supporting Idea*. The author mentions that his mother didn't mention specifically the type of peanut butter to buy, or that she doesn't approve of skipping stones. The passage also doesn't mention anything about his father telling him to listen to his mother. Instead, the passage states that "through experience, I knew that there was no arguing with her" when she speaks a certain way (lines 23-24).
16. A. *Tone/Language/Style*. We can rule out the positive emotions "satisfied" and "excited" because the author uses the word "frustrated" in line 45. This leaves "depressed" and "annoyed." If the author were very sad, we might say he was depressed. But here, the emotion being expressed isn't sadness, but frustration, or annoyance.
17. C. *Vocabulary*. The author describes an idea that just came to mind, or occurred to him. The phrase "dawned on" doesn't mean that something rose up or brightened up (even dawn is a time when the sun rises). The idea doesn't make any demands of the author.
18. C. *Inference*. We don't actually know that the author can drive around whenever or with whomever he wants, especially since he actually has to do chores before he can hang out with his friend Jordan. Though the author follows his mother's instructions, it's not because of this that he feels more like an adult. The last couple of paragraphs tell us that his mother's trust is what is important to him feeling like an adult (lines 55-60).
19. C. *Main Idea*. The passage never mentions that the water cycle is unique to Earth. In fact, the sentence "on Earth, the sun powers the water cycle..." implies that the water cycle exists in other places as well. The passage implies that water is necessary for life on Earth, but that isn't the central point of the passage. Instead, the author spends a great deal of time describing how the water cycle influences the environment, from the weather to geology.
20. A. *Vocabulary*. In context, this phrase tells us that frozen water can stay frozen for as long as it remains cold enough. We don't know how long this is before it melts, but it could be forever. The definition of the word "indefinitely" is basically "until an uncertain time in the future." This doesn't mean ice and water are "purposefully" frozen, or even "stubbornly" so. The water may be frozen permanently, or temporarily – we don't really know.

21. A. *Supporting Idea*. The passage tells us that frozen has an impact on the environment by shaping much of modern-day Earth. The passage describes how various geological features like mountains and valleys are influenced by frozen water (in this case, glaciers).
22. C. *Organization/Logic*. The passage is not written like a story or a novel, instead sharing factual information about the water cycle like a scholarly paper (think research paper or report). The passage doesn't seek to persuade, nor does it share personal information in the form of a letter.
23. B. *Supporting Idea*. The passage tells us in the second-to-last paragraph that temperatures are influenced by evaporation and condensation, not geological features or precipitation.
24. D. *Inference*. The author presents an objective view of the water cycle, not stating that the water cycle is more or less efficient than it was in the past. The passage doesn't mention how the atmosphere hinders the water cycle, only that it influences the water cycle. Frozen water is mentioned as also having an impact on the environment in that it can shape the geology. The author mentions how the process of the water cycle can influence the world.
25. D. *Main Idea*. The author shares a particular experience. She tells us about the orchard and about how the peaches are sometimes red like apples, but doesn't spend most of the time doing this. Instead, the focus is on the experience as a whole – arriving, picking, eating, peaches.
26. C. *Supporting Idea*. The sign never tells us that the trees were endless, and there's no mention that the orchard was mentioned in a newspaper. Instead, we are told that the hills wind up and down, and that the trees go up or around those hills.
27. A. *Inference*. The passage describes an experience at a peach orchard, and we know from line 14 that there are decaying fruits on the ground. It wouldn't make sense for there to be apples decaying on the ground, and we don't know anything about insects from what is written in the passage.
28. C. *Tone/Language/Style*. The phrase is used to describe how the peaches are colored by the light from the sun. This doesn't have to do with the temperature outside or the brightness of the sun, or even getting sunburned. The context tells us that the color of the peaches depend on exposure to sunlight.
29. D. *Supporting Idea*. The passage mentions in lines 43-51 the various criteria people used to pick peaches. This mentions all of the factors except for smell.
30. B. *Vocabulary*. The word "motioned" is used to describe how the peach pickers are gesturing (making motions or signals) to one another to "come to our respective tree." We know this has to do with movement of some kind, but it's not right to substitute the word "raced" for a word that has to do with calling someone to move a certain way.
31. B. *Main Idea*. The author writes that some people make ripples in history, while others make waves. This contradicts the first choice. The author mentions tolerance for other cultures, and also focuses on several criteria, but doesn't actually say who is greater or more important than the other.
32. D. *Vocabulary*. The word "aspire" means to yearn for, hope for, or dream for something. In other words, it means to want something. We can tell this because the sentences after this compares conquering the world to a dream, or an endeavor.
33. C. *Supporting Idea*. The passage never mentions how luck plays into greatness. It mentions how Alexander the Great was very ambitious, and how he displayed great leadership and vision. This was the same as Genghis Khan.
34. C. *Supporting Idea*. The author uses examples of Alexander the Great's legacy to help support the idea that his impact was very great. This includes the fact that cities were named after him (like Alexandria), the fact that Greek culture spread around the world, and that many people studied his methods. This is all highlighted in the third paragraph, but doesn't mention coins.
35. B. *Organization/Logic*. The author never mentions how cities were named after Genghis Khan, nor does it suggest that this tolerance led to wise rule by his descendants, or even the fact that this led to his control over Eurasia. Instead, it suggests in lines 55-60 that his tolerance led to an "exchange of ideas," and gives religions as examples.



36. *C. Inference.* The phrase implies that everyone has an impact on the world, but some people have greater impacts than others. This is the comparison between a ripple (a small disturbance of water) and a wave (a large disturbance of water).

### Mathematics Achievement

- Numbers & Operations – Factors, Multiples, Primes.* D. One can list every factor of 250, but this is time consuming. Instead, use prime factorization, which is the process of breaking a number down into a list of all of its prime factors. We do this by factoring each resulting factor until we reach a prime number. For even numbers, it's easiest to divide the number by 2, which is the smallest prime number. In the factor pair 2 and 125, the factor 125 can be broken down further into 5 and 25, and so on. This results in a prime factor tree that looks like the figure at right.  $250 = 2 \times 5 \times 5 \times 5$ . We can also express this as  $2 \times 5^3$ .
 

$$\begin{array}{r}
 250 \\
 / \quad \backslash \\
 2 \quad 125 \\
 \quad \quad / \quad \backslash \\
 \quad \quad 5 \quad 25 \\
 \quad \quad \quad \quad / \quad \backslash \\
 \quad \quad \quad \quad 5 \quad 5
 \end{array}$$
- Algebraic Concepts – Functions & Patterns.* B. We don't know how many cookies Devon or Mason baked, but we know that however many Devon baked, Mason baked three times as much. So, we can let the number of cookies that Devon baked be represented by a variable, in this case,  $x$ . This means that Mason baked  $3x$  (since he baked 3 times as much). Therefore, the total number of cookies baked was  $x + 3x$  (the amount that Devon and Mason each baked). Since this is 36, we get an equation:  $4x = 36$ .
- Numbers & Operations – Percents.* C. A 25% increase on 24 hotdogs can be expressed as  $1.25 \times 24 = 30$ .
- Numbers & Operations – Decimals.* D. To solve this equation, we can either convert the decimal into a fraction or the fraction into a decimal. If we do the former, we have  $\frac{13}{10} \div \frac{1}{4}$ . Dividing by a fraction is the same as multiplying by its reciprocal:  $\frac{13}{10} \times \frac{4}{1} = \frac{52}{10}$ . This simplifies to 5.2.
- Data & Probability – Mean, Median, Mode, & Range.* A. First find the median. The temperature values are already shown in order from least to greatest. In this case, there are two values in the middle: March (50) and April (60). Therefore, take the average:  $(50 + 60) \div 2 = 55$ . Then, find the mean. The set of numbers is  $\{20, 40, 50, 60, 70, 90\}$ . The sum of this is 330. There are 6 values in this set, so  $330 \div 6 = 55$ . The median and the mean are equal, meaning there is no difference.
- Measurements – Formulas.* A. Substitute known values into the equation:  $144 = 12 \times h$ . If we divide both sides by 12, we find that the height is 12 in.
- Algebraic Concepts – Solving Algebraic Equations.* D. First, add 3 to both sides of the equation, to give us  $\frac{a}{5} = 12$ . Then, recognize that  $\frac{a}{5}$  means  $a \div 5$ . So, we need to multiply both sides by 5 to isolate the variable  $a$ . This leaves us with  $a = 60$ .
- Data & Probability – Probability.* B. The total number of possibilities (in this case, the total number of apples in the barrel) must be a multiple of the number in the denominator of a probability. In this case, there could be 4, 8, or 12 apples in the barrel, because it would be possible to calculate the number of bruised apples in the barrel ( $\frac{3}{4} \times 4 = 3$ ,  $\frac{3}{4} \times 8 = 6$ , etc.). If there were 6 total apples in the barrel, it would not be possible to calculate the number of bruised apples in the barrel ( $\frac{3}{4} \times 6 = \frac{9}{2}$ ).
- Numbers & Operations – Integers.* B. Subtracting a larger number from a smaller number results in a negative number.  $602 - 429 = 173$ . Remember to carry all values and to flip the sign to  $-173$ .
- Numbers & Operations – Fractions.* A. Since ABGF and BCHG are congruent, their sides are the same length. This means that the small triangles in each square are the same size. Square BEGD



is made up of 2 of the small triangles, and there are 8 of them in total in rectangle ACHF. This can be represented as  $\frac{4}{8}$ , which simplifies to  $\frac{1}{4}$ .

11. *Numbers & Operations – Factors, Multiples, Primes*. B. First, find the value of  $x$ . Utilize prime factorization to determine that  $81 = 3^4$ . This means that  $x = 4$ , and  $3(4) = 12$ .
12. *Data & Probability – Mean, Median, Mode, & Range*. B. Since there are 7 total scores, and we know that 89 is the median, it must be the fourth highest and fourth lowest value in the list. If  $x = 88$ , 89 would be the median, since there would be 3 numbers fewer than 89, and 3 numbers greater than 89. However, this is also the case if  $x = 89$ , and the question asks what the highest possible value for  $x$  is.
13. *Measurements – Formulas*. B. Pay close attention to units. We are given all of the measurements to find the volume of a rectangular prism.  $10 \text{ ft} \times 18 \text{ ft} \times 0.5 \text{ ft}$  (or 6 inches). This gives us  $90 \text{ ft}^3$ .
14. *Data & Probability – Probability*. A. The probabilities of rolling a 6 and the spinner landing on green are independent of each other. The chance of rolling a 6 is  $\frac{1}{6}$ , while the chance of the spinner landing on green is  $\frac{1}{4}$ . Therefore, the probability of both things happening is  $\frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$ .
15. *Numbers & Operations – Decimals*. B. If  $x = 16$ , then the number of free throws made on Tuesday is  $1.25x = 1.25(16) = 20$ . The number of free throws made on Wednesday is  $1.75(20) = 35$ . In total, Jack made  $16 + 20 + 35 = 71$  free throws.
16. *Numbers & Operations – Percents*. B. The width of 10 ft. is increased by 10%, so  $10 \times 1.1 = 11$ . The length of 10 ft. is decreased by 10%, so  $10 \times 0.9 = 9$ . The new area is  $11 \times 9 = 99$ .
17. *Numbers & Operations – Fractions*. C. Create improper fractions first by converting the mixed fractions into improper fractions.  $2\frac{1}{6} \div 4\frac{2}{5} = \frac{13}{6} \div \frac{22}{5}$ . Then, multiply the dividend  $\left(\frac{13}{6}\right)$  by the reciprocal of the divisor  $\left(\frac{22}{5}\right)$ . Thus:  $\frac{13}{6} \times \frac{5}{22} = \frac{65}{132}$ .
18. *Algebraic Concepts – Slope*. A. If the amount of water is represented by  $y$ , and the amount of time that passes is represented by  $x$ , then we know we must look for an equation with a negative slope. This is because a negative slope decreases the value of  $y$  as the value of  $x$  increases. Only choices A and B have negative slopes. Next, we must look at the  $y$ -intercepts, which are 500 and  $-500$ . The pool can't start with less than 0 amount of water, so the only choice that makes sense is A, having the pool start at a positive amount of water. This is because the  $y$ -intercept tells us the amount of water in the pool when  $x = 0$  (i.e. when time is 0, before any draining has been performed).
19. *Algebraic Concepts – Solving Algebraic Equations*. C. To isolate  $y$ , first divide both sides by 12. This leaves us with  $101 = y + 1$ . Then, subtract 1 from both sides, giving us  $100 = y$ .
20. *Numbers & Operations – Integers*. D. We need only take the difference between the starting temperature (when Frank went to sleep) and the ending temperature (when Frank woke up) to find the increase. This would be  $12 - (-15) = 27$ . Subtracting a negative is the same as adding a positive.
21. *Numbers & Operations – Factors, Multiples, Primes*. A. We only need to find the number of prime factors of 120, not the number of total factors. We can substitute 5 for  $x$  and get  $5 \times 4 \times 3 \times 2$ . We don't need to multiply and find the product – the expression already breaks the product down into its factors. 5, 3, and 2 are distinct primes.
22. *Measurements – Formulas*. B. The surface area of a cube is given by the formula  $SA = 6(s^2)$ , since  $s^2$  is the area of each face of the cube and there are 6 faces on a cube. Therefore,  $150 = 6s^2$ . Solving for  $s$ , we find that  $25 = s^2$  and  $s = 5$ . The volume of a cube is simply  $s^3$ . In this case,  $5^3 = 125 \text{ cm}^3$ .

23. *Numbers & Operations – Fractions.* A. Erica spent  $\frac{2}{3} \times 300 = 200$  on rent. She spent  $\frac{1}{4} \times 300 = 75$  on food. This means she spent a total of  $200 + 75 = 275$  on rent and food. Of 300, this means \$25 remains.
24. *Data & Probability – Interpreting Data.* A. Since a circle has  $360^\circ$ , then  $90^\circ$  represents 25% of the total. Since apples represents 25% of the total number of students, and there are 25 apples, then there must be a total of  $25 \div 0.25 = 100$  students. The sum of the number of students who voted for all other fruit besides orange is  $25 + 20 + 40 + 10 = 95$ . This means there are 5 students who voted for oranges.
25. *Algebraic Concepts – Functions & Patterns.* B. If Moore has  $x$  nickels, then he has  $x + 4$  quarters. He therefore has  $0.05x + 0.25(x + 4) = 3.10$ , or  $0.3x + 1 = 3.1$
26. *Algebraic Concepts – Slope.* C. To determine the slope, represent the equation in the format  $y = mx + b$  by isolating  $y$ . The equation becomes  $5y = 2x - 12$ . We need to divide both sides by 5 to isolate  $y$ , so the equation becomes  $y = \frac{2x - 12}{5}$ , or  $y = \frac{2x}{5} - \frac{12}{5}$ . Note that we don't need to simplify further – the slope is simply  $\frac{2}{5}$ .
27. *Numbers & Operations – Percents.* D. If there were 25% fewer boys born in Hospital B, then  $20 - 25\%(20) = 15$ . If there were 50% more girls born in hospital B, then  $12 + 50\%(12) = 18$ . Alternatively, recognize that we can represent “25% fewer boys” as  $20 \times 0.75 = 15$ , since  $0.75 = 1 - 0.25$ . Similarly, we can represent “50% more girls” as  $12 \times 1.5 = 18$ , since  $1.5 = 1 + 0.5$ . Thus,  $20 + 15 + 12 + 18 = 65$ .
28. *Geometry – Geometric Objects.* C. There are two sets of opposite sides in a quadrilateral. If each set is parallel, then it is a parallelogram, and we can rule out a trapezoid right away. A rhombus and square both have sides that are equally long, but we don't know this is the case for this particular quadrilateral. Therefore, we only know that the shape is a rectangle, since it has 4 right angles and is a parallelogram.
29. *Numbers & Operations – Decimals.* B. One way to solve this question is to perform long division ( $9.000 \div 37.000$ ) to find that the result is  $0.243\dots$ . Another way to solve this question is to recognize that the fraction is slightly less than  $0.25 = \frac{1}{4} = \frac{9}{36}$ . Therefore, the fraction is closer to 0.2 than it is to 0.3.
30. *Measurements – Formulas.* C. Use the Pythagorean theorem on right triangles:  $c^2 = a^2 + b^2$ , where  $c$  is the hypotenuse, and  $a$  and  $b$  are the legs of the triangle. We can use the values given and plug them into the formula:  $10^2 = 8^2 + b^2$ , which simplifies to  $100 = 64 + b^2$ . This becomes  $36 = b^2$ , and finally  $6 = b$  when we take the square root of both sides.
31. *Algebraic Concepts – Functions & Patterns.* C. We can see that the number of participants is halved when moving from one round to the next. In the 4<sup>th</sup> round, there will be  $40 \div 2 = 20$  participants left. In the 5<sup>th</sup> round, there will be  $20 \div 2 = 10$  participants left.
32. *Numbers & Operations – Fractions.* A. Since Gary washes a car in 1 hour, his Barry can wash 2 cars in 1 hour, since it takes Barry half as long as Gary to wash a car. We could say that it takes Barry only 30 minutes to wash a car, where it takes Gary 60 minutes to wash a car. Since they're washing a single car together, and Barry can wash a car by himself in 30 minutes, the only choice that makes sense is choice A, since the two brothers working together could wash a car faster than Barry could by himself.
33. *Algebraic Concepts – Ratios, Proportions, & Scale Factors.* D. We can manipulate values as long as we preserve the ratio of those values. For instance, we are told that 2 fruit snacks are worth 12 graham crackers. From this, we know that 4 fruit snacks would be worth 24 graham crackers (twice as many fruit snacks for twice as many graham crackers). We also know that 4 fruit snacks are worth 9 pretzels. So, we know that 9 pretzels are worth 24 graham crackers. Next, set up a

proportion:  $\frac{9 \text{ pretzels}}{24 \text{ graham crackers}} = \frac{12 \text{ pretzels}}{x \text{ graham crackers}}$ . Cross multiply and solve for  $x$ :  $(12)(24) =$

$9x$  simplifies to  $(4)(24) = 3x$ , and then  $(4)(8) = x$  (because we can see that we are able to divide both sides by 3 two times. We are left with  $x = 32$ ).

34. *Data & Probability – Probability*. C. The square would have a side length of 8, since the radius of the circle would be half the side of the square. This means the square has a total area of  $8^2 = 64$ . Meanwhile, the circle has an area of  $\pi r^2$ . With a radius of 4, this gives us  $4^2 = 16$ , and an area of  $16\pi$ . Since we want to know the probability of finding a point inside the circle, we take the ratio  $\frac{16\pi}{64}$  or  $\frac{\pi}{4}$ .
35. *Data & Probability – Mean, Median, Mode, & Range*. C. The range of temperatures is the highest value minus the lowest value. In this case, 29 is the highest value, and 25 is the lowest, so the range is  $29 - 25 = 4$ . The median, out of 30 days, would be the average of the 15<sup>th</sup> and 16<sup>th</sup> highest temperatures. Working from 25 degrees (which appears 2 times) and 26 (which appears 4 times), we keep counting days until we find the 15<sup>th</sup> and 16<sup>th</sup> days. Both of these are 28, so the average of these two days is 28. The mode is also 28, since there were 10 days with highs of 28, more than for any other temperature. Therefore, the median and mode are equal. Choice C is correct.
36. *Data & Probability – Interpreting Data*. A. The column on the left gives us the number of packages per order. If there are 5 packages of cookies in an order, then the cookies themselves cost  $5 \times 5 = 25$ , and tax represents an additional 10% of that, which is  $0.1 \times 25 = 2.5$ . This means the total cost of the cookies plus tax is  $25 + 2.5 = 27.5$ . But we are told in the second column that the total cost of the order is 32.5. The difference must be the shipping charge, which is  $32.5 - 27.5 = 5$ . We are told that this shipping fee is charged based on the number of packages per order. Since there are 5 packages in the order, and the shipping cost is \$5, then we know that the cost is \$1 per package. Verify this using another row (for example, 20 packages in an order).
37. *Algebraic Concepts – Ratios, Proportions, & Scale Factors*. A. Since the red-colored cube has a volume of  $27 \text{ cm}^3$ , we know that its sides must have sides of  $3 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm}$ . The scale factor applies to each of these dimensions, not to the volume as a whole. So,  $3 \times \frac{2}{3} = 2 \text{ cm}$ . This means each side of the blue-colored cube is 2 cm, making the volume  $2^3 = 8 \text{ cm}^3$ .
38. *Algebraic Concepts – Solving Algebraic Equations*. D. Note that none of the answer choices have  $y$  as a denominator, so we know we must cancel that out. The equation can be rewritten as  $x = 3\left(\frac{1}{y}\right) + y$ . The question asks us to manipulate the equation. If we multiply both sides by  $y$ , we multiply each term by  $y$ . In this case,  $x$  becomes  $xy$ , and  $3\left(\frac{1}{y}\right)(y)$  becomes 3 (since  $y \div y = 1$ ). Then,  $y \times y = y^2$ . This leaves us with  $xy = 3 + y^2$ . Remember that what is done to one side of an equation must be done to the other.
39. *Numbers & Operations – Decimals*. B. Since there are 128 marbles in the box on Monday morning, and 0.25 times that number are lost by the end of Monday, then there remains  $128 - 0.25(128) = 96$  marbles. This is because  $128 \times 0.25 = 32$ , which is the number of marbles lost. Therefore,  $128 - 32 = 96$  marbles. We repeat this for Tuesday. Since there are 96 marbles at the beginning of Tuesday, then there remains  $96(0.75) = 72$  marbles by the end of Tuesday, which is the same number that starts on Wednesday morning.
40. *Algebraic Concepts – Solving Algebraic Equations*. C. First, simplify the equation by distributing the 4. This gives us  $x = 4y + 16$ . Since we're looking for  $x - 2$ , we need only subtract 2 from both sides of the equation:  $x - 2 = 4y + 16 - 2$ . This simplifies to  $x - 2 = 4y + 14$

41. *Data & Probability – Probability*. C. Of the different numbers on the wheel (1, 2, and 3), only 2 and 3 are prime (remember: 1 is not prime!). There are a total of four 2's and four 3's, for a total of 8 prime numbers on the wheel, out of 12 total numbers. Therefore, the probability is  $\frac{8}{12} = \frac{2}{3}$ .
42. *Measurements – Formulas*. B. From the formula, we can plug in the given values, leaving  $\pi$  in the equation, since the answer choices all have  $\pi$  in them.  $SA = (2)(\pi)(3^2) + (2)(\pi)(3)(8)$ . This simplifies to  $18\pi + 48\pi = 66\pi$  in<sup>2</sup>.
43. *Numbers & Operations - Percents*. A. At Dunwood Prep, 72 out of 180 days were rainy. This can be represented as  $\frac{72}{180} = \frac{2}{5} = 0.4$ , or 40%. At Donovan Prep, 72 out of 160 days were rainy. This can be represented as  $\frac{72}{160} = \frac{9}{20} = 0.45$ , or 45%. Thus,  $45\% - 40\% = 5\%$ .
44. *Algebraic Concepts – Ratios, Proportions, & Scale Factors*. A. Set up a proportion to solve this question reliably.  $\frac{6 \text{ minutes}}{20 \text{ newspapers}} = \frac{x \text{ minutes}}{70 \text{ newspapers}}$ . Cross multiply and solve for  $x$ :  $(6)(70) = 20x$  simplifies to  $(6)(7) = 2x$ , and then  $42 = 2x$ . When we divide both sides by 2, we are left with  $x = 21$ .
45. *Data & Probability – Mean, Median, Mode, & Range*. B. If a set of 6 numbers has a mean of 20, then let  $x$  represent the sum of the numbers in that set:  $\frac{x}{6} = 20$ . This means  $x = 120$ . If we change the set of numbers by adding one number, we increase the average from 20 to 24. This can be represented algebraically as  $\frac{120+x}{7} = 24$ . Solving for  $x$ , we simplify to  $120 + x = 168$ , and  $168 - 120 = 48$ . The number that must be added is 48.
46. *Measurements – Units*. C. The area of the rectangular patio is  $6 \times 8 = 48$  ft<sup>2</sup>. Each square tile measures 6 inches, or  $\frac{1}{2}$  ft. long. This means Abigail can fit  $6 \times 2 = 12$  tiles along the length and  $8 \times 2 = 16$  tiles along the width. The area in square tiles is  $12 \times 16 = 192$ .
47. *Algebraic Concepts – Solving Algebraic Equations*. D. One way to solve this question is to substitute any value for  $x$  into the expression and the answer choices. An easy-to-work with value like  $x = 1$  shows us that only D is equal to the expression shown. We can also recognize that the radical and exponent cancel each other out, which doesn't leave us with choice A, B, or C.

## Verbal Reasoning – Synonyms

### Introductory

- B. "Anxious" has a negative connotation, and comes from the same Latin word as "anxiety." To be anxious is to experience nervousness, uncertainty, or unease. "John was anxious about the test he had not studied for."
- D. "Obnoxious" has a negative connotation, as does the word "noxious." Something that is obnoxious is unpleasant in some way. "The 24-hour construction outside my window was obnoxious."
- D. "Prior" has a neutral connotation, and is itself a Latin word that describes when something happened. If some event (let's call it Event A) happened prior to another event (let's call it Event B), then Event A happened before Event B. It doesn't mean necessarily that Event A is an original.
- A. "Devour" has a neutral to negative connotation, and includes the Latin root "de-," which means "down" (think "descend"). When we devour something, consume it, and send it down into our stomachs. "Look at that hungry bear devour the fish!"



5. B. “Evaluate” has a neutral connotation, which means to assess or estimate. The word “value” is part of “evaluate,” which gives us a clue that the word means to figure out what the value of something is. “The test was designed to evaluate how much the students remembered.”
6. B. “Debate” has a neutral to negative connotation which shares a common Latin root word with “battle.” To debate is to discuss or argue. “The politicians will debate the proposed law tomorrow night.”
7. A. “Benefit” has a positive connotation because of the prefix “bene-” in front of it, which means “good/well” in Latin. Whenever this prefix is included in front of a word, we know that the word has something to do with something good. In this case, this excludes “obstacle,” which is negative. A benefit is something that is advantageous to someone or something. “Having a balanced diet was a great benefit to the athlete, who won the gold medal.”
8. D. “Accomplish” has a positive connotation and shares a Latin root word with “complete.” This is how we know that to accomplish something is to complete it. For example, “Hard work enabled Janet to accomplish her goal.” Don’t be fooled by the fact that accommodate and accustom both have the same prefix.
9. A. “Acknowledge” has a positive connotation and is a verb that builds upon the noun “knowledge.” Since we know that knowledge has something to do with an idea, fact, or concept, we know that the synonym should have something to do with this. To acknowledge an idea is to accept it, or recognize the truth of it. This is different from contemplating an idea, which is only to think about an idea. “After reading the facts, the oil baron acknowledge the existence of global warming.”
10. D. “Abolish” contains the prefix “ab-,” which means “away” (for example, a student who is absent is away from school). To abolish is to “put it away forever” – to formally end something. “The 13<sup>th</sup> Amendment to the Constitution abolished slavery, ending the practice forever.”
11. A. “Context” has a neutral connotation, and contains the prefix “con-,” which means “with” or “together.” Think of context as all of the things that surround something that together give meaning – the background of a story or situation, for example. When people use “context clues” to figure out the meaning of a new word, they are using all of the other words and ideas around the new word to help. “The jury heard testimony that provided them with the necessary context for making a decision.”
12. D. “Compose” has a neutral connotation, and actually contains two roots: “com-,” which means “together,” and “pos,” which means “place/put.” Together, the two roots mean “put together. When we write something, we put ideas and words together. “You must compose a cover letter to send to your employer.”
13. C. “Optimistic” is the adjective used to describe someone who is full of “optimism,” a noun which comes from the Latin word “optimum.” Something that is “the optimum” is in its perfect or best state or form. This means “optimistic” has a positive connotation, which only choices A and C share. To choose between the two, remember that we are talking about a person who is full of “optimism” – a person who wants things to be in their perfect or best state or form. This would be a hopeful person, not a friendly person. “No matter what lemons life throws at him, Marcus is always optimistic.”
14. B. “Strategy” is a word that is often used in sentences that talk about how to take some course of action for some specific purpose. For example, “The best strategy for saving money is to clip coupons.” In this case, the course of action is to clip coupons, and the specific purpose is to save money. We could say that clipping coupons is a method of saving money.
15. A. “Adapt” begins with the prefix “ad-,” which means “to” or “toward.” To adapt to something is to become more like it – to change and move toward being like it. “When Josh started his new diet, he found it difficult to adapt to the new lifestyle.”
16. A. “Exhibit” is often used in sentences that talk about sharing something with others. For example, an exhibit at a museum is used to show or display information to visitors. A salesman who puts on an exhibit showcases and displays his wares. Someone who looks at an exhibit would be watching or observing it.

17. D. “Consecutive” is made up of two Latin roots: “con-,” which means “with” or “together,” and “secu-,” which means “follow.” If we put these together, we have a word that means “follow together.” Consecutive actually means that something happens continuously, in an unbroken chain. “Johnny forgot his lunch money for the fourth consecutive day.”
18. D. “Collide” is made up of two Latin roots: “col-,” which means “together,” and “lide,” which means “strike.” We can infer that collide means to strike together. “Rounding the corner quickly, Sophie collided with Eric.”
19. D. “Excel” has a positive connotation which is used to show how something is better than other things. It includes the prefix “ex-,” which means “out of” or “away from.” We can reason that someone who excels at something is so much better than other people at it that she is “away (apart) from” them (in a good way). “It took a lot of practice for Ludy to excel at ballerina.”
20. A. “Encounter” is often used to describe a situation where two parties are meeting. This might involve waiting, but it also might not. “The encounter with the pride of lions left the travelers feeling excited.”
21. D. “Abnormal” contains a prefix “ab-,” which means “away” (for example, a student who is absent is away from school). The word “normal” means usual, plain, or average, so something that is “away” from normal is a departure from it. It would be something unusual, or strange. “Rain in the desert is abnormal.”
22. C. “Remorse” has a strong negative connotation that is often used to show that someone has strong negative feelings. Even if you don’t know that remorse means deep regret or guilt (as in “The judge took pity on the thief, who was filled with remorse for his crimes.”), we know that it is a strongly negative word. The only such word is “grief.”
23. C. “Portable” is made up of two root words: “port,” which means “carry,” and “able,” which means ability. Therefore, “portable” means something that can be carried or moved around. This doesn’t mean that it’s clean or awkward or exposed, but that it’s convenient.
24. D. “Spontaneous” is used in sentences that describe a surprise, or something is unexpected. “The spontaneous eruption of the volcano caused thousands to flee.” This means that the thing that happened was unintended, or unprompted. Several things may happen unexpectedly at the same time, but only one thing could happen unexpectedly, and it would still be spontaneous.
25. B. “Toxic” draws from the root word “tox,” which means “poison.” “Toxic” also has a highly negative connotation, which would rule out “functional” and “splendid” automatically. Something that is poisonous is bad for one’s health, or harmful to it. It may or may not be nauseating.
26. B. “Data” refers to facts and figures, or information about things. “Data from the cafeteria showed that pizza is the most popular menu item.”
27. B. “Overwhelm” contains the root word “over,” which means “above” or “excessive.” If something overwhelms something else, it completely overtakes it, or is too much to be handled. This is not necessarily a punishment. To be able to deflect something would mean not being overwhelmed by it. That something is overwhelmed is not necessarily a punishment.
28. C. “Merely” is used to minimize something. “Despite falling down the stairs, Bernie merely suffered a small scratch.” The only word to minimize something is “only,” which we can substitute directly for “merely.”
29. B. “Dismal” has a strong negative connotation. In fact, the key root word “mal” means “bad.” This helps us to eliminate “beautiful” and “cheery.” If an outlook is “bleak,” then it is not very positive. Something can be ridiculous without being bad. “The dismal weather left everyone feeling sad.”
30. D. “Thrive” has a strong positive connotation which is only matched by “prosper.” Thrive means to prosper, flourish, or grow vigorously. “Years of favorable growing conditions saw the farm thrive.”
31. D. “Ignite” comes from the word “ignis,” which means fire. The only word that relates to this is “spark,” which is a word used to express how something is caused (a fire, or strong emotion). “The eloquent and passionate speech would go on to ignite the hearts and minds of millions.”

32. A. “Omit” has a negative connotation, and is used to show that something has been forgotten or remains undone. This most closely means “ignore,” which can happen intentionally or unintentionally. “If you omit your name from the test, the teacher won’t know whose test it is!”
33. C. “Hoax” has a slightly negative connotation, and is used to refer to some sort of trick or deception, just like the word “prank.” For example, “The students covered a locker with tape, a hoax that the vice-principal did not find amusing.”

### Intermediate

1. B. “Dwell” is used to show that something lives in or at a certain place. It doesn’t mean that the thing sleeps in or improves wherever it is, since the thing that lives in another place could be an inanimate object. “Bravery dwells in the hearts of soldiers.”
2. B. “Appeal” used to express an earnest request. “The lawyer tried to appeal to the judge for mercy.”
3. A. “Reinforce” is made up of two root words: “re-,” which means “again,” and “forc,” which means “strong” or “strength.” To strengthen again would be to boost or increase the strength of something. “Reinforce the levy with extra sandbags.”
4. A. “Inevitable” is used to suggest that something is unavoidable, or that will certainly happen. “After disrupting class, she waited for the inevitable call to the principal’s office.”
5. D. “Chronological” contains the root word “chron,” which means “time.” This means we know that the word has to deal with time, which flows in a certain order. If something is logical, it proceeds in a certain order. In this case, that order has to do with time. “The story is told chronologically, from beginning, to middle, to end.”
6. A. “Barren” has a strong negative connotation, so we can rule out “lush” right away. “Barren” is used to refer to something that is bleak, lifeless, and empty. “Mars is a barren wasteland of rock and dust.” Just because something is far away or isolated does not mean it is barren.
7. D. “Boycott” has a strong negative connotation, so we can rule out all choices except “reject.” “To make a statement, the people boycotted the company by refusing to buy their pasta.”
8. A. “Concise” is made up of two root words: “con-,” which means “with” or “together,” and “cise,” which means “to cut” (think of “incision”). A “concise” statement is one which is cut down to the most important information, and still remains coherent (“together”) as an idea. “With no time to spare, Harvey left a concise message on the pad of paper.”
9. D. “Employ” is used to show how something is used, or utilized, by something else. To be in someone’s employ, for example, is to work for them. A villain could employ a devious ruse to trick a hero. However, this does not mean that “employ” means “to trick.” Rather, the villain “utilized” a devious ruse.
10. D. “Correspond” is the word “respond” with a prefix “cor-” in front of it, which means “with” or “together.” If two people “respond together” or say something together, they are saying the same thing. Those things match. “Sherlock wrote the letter because the handwriting corresponds to that in the book.”
11. A. “Agitate” has a strong negative connotation, which rules out “enjoy” and “relax.” To agitate is to disturb calm, or to cause distress. “Sudden ripples appeared as the insect agitated the water’s surface.”
12. B. “Compel” has a strong negative connotation, which is used to describe how someone or something is forced into doing something. This is stronger than a request, since the thing or person has no choice but to do it. “The threat of timeout compelled the child to eat his broccoli.”
13. C. “Adjacent” is made up of two root words: “ad,” which means “near” and “by,” and “jac/ject,” which means “throw” (think “eject”). Something that is “adjacent” means that it is close enough to throw something there. In other words, it is nearby, or neighboring. “The adjacent buildings share a common wall.”
14. C. “Persist” is made up of two root words: “per,” which means “through” or “intensive” and “sist,” which means to “stand” or “withstand.” Something that persists is about to “withstand” and last “through” difficulty. It is something that can endure. “She persisted in her studies thanks to her parents’ support.”



15. B. “Notorious” has a negative connotation, so we can rule out “ordinary” right away. The word is frequently used to describe notable, famous, or well-known people. This is the opposite of “obscure,” and the people may or may not be famous because of how “deadly” they are. “The politician was notorious for passing off lies as the truth.”
16. B. “Indifferent” is actually made up of the prefix “in-,” meaning “not,” and “different.” In this case, the different doesn’t mean dissimilar, or not alike. Instead, it refers to a difference of some kind. “Indifferent” therefore means “no difference,” or “neutral.” This is the opposite of opinionated, and doesn’t mean that something is similar or not. “She was indifferent to the outcome of the game.”
17. A. “Gruesome” has a strong negative connotation, so we can rule out all other choices but “appalling.” “The haunted house had gruesome decorations in it, so no children were allowed.”
18. A. “Treacherous” has a strong negative connotation, so we can rule out “education” and “loyal.” The word is frequently used to describe something that does not appear to be as it is; in other words, deceptive. “The seas are treacherous because storms often strike with no forewarning.”
19. B. “Pasture” is another word for land, or field. There may be wheat growing in the field, but not necessarily. “The sheep like to graze in the pasture.”
20. D. “Oblivious” has a slightly negative connotation, so we might eliminate “furious,” which is a strongly negative word. The word is used to describe someone or something that is unaware of something else. We might also think of the word “oblivion,” which refers to nothingness. “The grazing gazelle was oblivious to the cheetah hiding in the grass.”
21. D. “Nimble” has a positive connotation, so we might rule out all of the choices but “spry.” The word “nimble” is used to describe something quick and agile. Another word for this quality would be spry. “The nimble mouse was able to escape the cat’s pursuit.”
22. C. “Brawl” has a negative connotation, so we might rule out “accept” and “befriend,” both of which are positive. Of the remaining, we should try to use the word in a sentence. “The exchange of words devolved into an all-out brawl.” So the word doesn’t mean ignore, but “fight” would work.
23. D. “Abrasive” has a negative connotation, so we might rule out “considerate” and “kind.” Let’s try and use the word in a sentence. “The sandpaper was abrasive on my skin.” So we know that “indecisive” doesn’t describe sandpaper, but rough does.
24. A. “Copious” derives from the Latin word “copia,” which means “plenty.” Think of “cornucopia,” from Thanksgiving. Another word for “plenty” and “copious” is “abundant.”
25. B. “Avert” means to turn away from, or prevent something bad from happening. “Tragedy could have been averted if there had been more lifeboats on the Titanic.”
26. D. “Aspire” has a positive connotation, where “criticize” and “descend” do not. Use “aspire” in a sentence. “Ford aspires to compete in the Olympics one day.” We can see that this does not mean “generate.”
27. A. “Benign” begins with the prefix “bene-,” which means “good” or “well.” There is only one word with a positive connotation here, and that is “affectionate.”
28. A. “Bestow” is used to describe something that is given to something else. In this case, the word “award” is used as a verb – to give someone something, for example, a diploma at graduation.
29. A. “Controversy” is used to describe a disagreement of some kind. It has a negative connotation, which eliminates “proof” and “truth.” Using it in a sentence shows us that it does not mean refutation: “Controversy surrounded the school’s decision to remove the football field.”
30. A. “Defect” has two root words: “de-,” which means “opposite” or “reverse” and “fect,” which means “make.” A defect is something that has the opposite effect of “making” something – a problem with the thing that was made, or an error. “The store refunded the customer’s money because there was a defect in the shirt.”
31. D. “Subjective” describes something that a person believes, which may be different from what someone else believes. “Jeremy’s views are subjective, based only on his personal opinions.” This is the opposite of something that is impartial.



32. A. “Belligerent” includes the root word “belli,” which means “war.” In war, there are different sides – different aggressors. To be belligerent is to be “aggressive,” as in war. There is a negative connotation here, unlike “consistent” and “intelligent.”
33. C. “Pinnacle” refers to a high point or place. “The climbers finally reached the pinnacle of the mountain.” We can see that this doesn’t have anything to do with the other words.

### Advanced

1. C. “Perspective” contains two root words: “per-,” which means “through” and “spect,” which means “to look/see.” We can see that the word has something to do with vision, or sight. “From the hill, Jill had a perspective of the town’s layout.” The only word that relates to this is “outlook.”
2. D. “Rebuke” has a strong negative connotation. This allows us to rule out the first three choices, since “admire” and “devote” have positive connotations and “instill” has a neutral connotation. Only “scold” has a negative connotation. “Jack’s mother rebuked him for coloring on the wall.”
3. D. “Cite” comes from root word “cit,” which means “call” (think “incite” or “citation”). We can use the word in a sentence. “The teacher told the students to cite the sources in their research papers.” We can see that to “cite” most nearly means “quote,” which is not quite the same thing as “evoke.”
4. A. “Endeavor” has a positive connotation, and is used to describe how someone tries very hard to accomplish something. We can rule out “fail” and “waver,” since they have negative connotations. To try very hard does not necessarily mean to encourage, though someone might encourage someone to try hard. “She endeavored to get 100% on the test, studying every night for hours.”
5. D. “Articulate” is used to describe something that is expressed. “He was so shocked that he could not articulate his emotions.” From this, we can see that he had trouble vocalizing his emotions.
6. C. “Gullible” is used to describe someone who is too trusting, and has a slightly negative connotation. This is the opposite of “adventurous” and “experienced,” which have slightly positive connotations. “The gullible man believed everything he was told, even that the sun would rise in the West!” This man is easily influenced, or impressionable, not stubborn.
7. B. “Comprehend” begins with the prefix “com-,” which means “complete” or “together.” The word also includes the root word “prehendere,” which means “grasp.” The word therefore has something to do with fully grasping, understanding, or realizing something. “He could not comprehend the sheer size of the universe.”
8. C. “Dejected” has a strong negative connotation, so we can rule out “discovered” and “improved,” since they don’t have quite the same connotation. “Sally was dejected when she found out her favorite band was breaking up.” We could also say that Sally was forlorn, but it doesn’t make sense that Sally was “removed” from anything.
9. B. “Barter” is used to refer to a trade or exchange. “Donald had no money, so he had to barter his watch for a drink of water.” We can see that we can’t really replace “barter” with “banter,” “foster,” or “pioneer.”
10. C. “Component” is made up of two root words: “com-,” which means “complete” or “together,” and “pon,” which means “put.” If we combine these ideas together, we get a word that means to put something – like pieces or parts of something – together to form a complete object. This sounds a lot like “ingredient,” which when combined form a meal. “Computers are complicated machines with many components.”
11. C. “Integrate” means to combine things together, or mingle them together. There are two root words that can help us with this word: “integer,” which means “whole” (think of integers in math, which are whole numbers) and “-ate,” which means “to cause to be.” So, to integrate something is to cause that thing to be whole. If we mingle things together, we’re doing just that.
12. D. “Capricious” has a negative connotation, so we can rule out all of the other choices, which have neutral to positive connotations. “Capricious” is used to describe how something or

someone is subject to sudden and unpredictable changes. “Sailing is dangerous given the capricious nature of the weather.”

13. C. “Fortify” means to strengthen. “We should fortify the door against burglars.” By using the word in a sentence, we can see that none of the other words make much sense as a replacement. We can think also of a fort, which is strong and defensible, to help us think of a synonym for “fortify.”
14. D. “Characterize” is used to refer to the way in which something is represented or depicted. “How would you characterize the actor’s portrayal of the character?” We can see that neither “forge” nor “objectify” really work as a replacement for the word “characterize.” “Compare” might seem like a possibility, but someone’s representation of something doesn’t necessarily have to be in comparison to something else.
15. D. “Sleek” has a neutral to positive connotation, and is used to describe the appearance of something smooth. We can rule out “eccentric” and “lonely” since they have more negative connotations. “The car was polished to a shine and looked more sleek and streamlined.” We can see then that “modest” doesn’t work to replace sleek, but “smooth” does.
16. A. “Calamity” is a strongly negative word, and the only other such word in the list is “catastrophe.” “Nervousness,” “uncertainty,” and “cliché” all have negative connotations, but a calamity is a disaster so great that it has a strong negative connotation, just as the word “catastrophe” does. “The calamity that struck the region destroyed virtually every building.”
17. A. “Anarchy” is made up of several root words that give us a clue about its meaning. “an-” is a prefix that means “without” and “arch” means “rule” (think “monarchy” or “patriarchy”). The suffix “-y” shows that the rest of the word “has” or is “marked by” a certain quality. So, we are looking for a word that describes how there is a state of “being without rule.” The only word that compares to this is “chaos,” the opposite of “control” and “dictatorship,” and even “regime.” The word has a strongly negative connotation, as does chaos. “The election results were widely disputed, and the country fell into anarchy.”
18. B. “Commemorate” is made up of the root words “com,” which means “with” or “together” and “mem,” which means “recall” or “remember.” The suffix “-ate” means “to cause to be.” So, if we put all of this together, we get a word which means something like “to cause to be remembered together.” One of the things that this describes is the word “honor,” where something is remembered and valued. This is a strongly positive connotation, whose only similarly positive connotation is found in “honor.” The other words are neutral to negative.
19. C. “Derive” is used to describe that something was obtained from something else. “Since we know that  $2 + 2 = 4$ , it is possible to derive the fact that  $4 - 2 = 2$ .” We can see that none of the answer choices make sense.
20. A. “Incense” has a strongly negative connotation, which is often used to describe someone who is very upset or angry. We can think of the word “incendiary,” which means flammable, to help us with this word. The other words all have neutral to positive connotations. “The politician was easily incensed, and would shake with rage at even playful jokes.”
21. C. “Proprietor” is very similar to the word “property,” which refers to possessions or objects. However, the suffix “-or” often means that there is a person who is related to the rest of the word. In this case, a person who is in charge of those possessions or objects. This would be an owner. “The proprietor of the restaurant wanted his staff to be friendly to all customers.”
22. B. “Allot” is a verb, which we can confirm if we use it in a sentence. “The teacher could not decide how to allot the treats to all of the students.” The word “allot” is similar to “allocate,” which means “to distribute.” The other words don’t make sense in this context. Notice that “numerous” isn’t even the same part of speech, since it is an adjective.
23. D. “Succumb” is used to show how something gives in to something else. Another word for this action is “yield,” which is the opposite of “resist.” The other words don’t make sense if we use “succumb” in a sentence: “After fighting off the idea of bedrest for days, Eric finally succumbed to his doctor’s advice.”

24. A. “Leeway” is often used to describe wiggle room, or flexibility to do something. “The student asked his teacher for some leeway, since his dog ate his homework.” None of the other words make sense in context of this sentence.
25. A. “Abdicate” is made up of many root words. “ab-” means “away from” or “off,” “dic” means “say” or “speak,” and “cat” means “down.” So, we know that “abdicate” has something to do with a person saying that he or she is going to step down or off from something, perhaps a position of power. This is another way of saying “quit.” In a sentence, this could be: “The king refused to abdicate, even though none of his people liked him.”
26. C. “Prone” has a somewhat negative connotation, which means to be likely to suffer from something. The other words all have positive connotations. “He avoided contact sports, since he was prone to bruising.”
27. A. “Incredulous” is made up of several root words. “In-” means “not,” “cred” means “believe” (think “credible”), and “-ous” means “having the quality of” or “relating to.” This means that the word “incredulous” means that there is something that has the quality of being not believable. It also has a negative connotation, which would rule out “genuine” and “substantial.”
28. A. “Admonish” has strong negative connotations, so we can rule out all other answer choices immediately, which are neutral to positive. In addition, the word consists of two roots: “ad-,” which means “to” and “mon,” which means “warn” or “remind” (think about how a monument reminds us of things). So, “admonish” means “to warn.”
29. B. “Famished” has a strong negative connotation, unlike all of the answer choices except for “hungry.” “After weeks on a strict diet, Martin was famished.”
30. A. “Irate” has a strong negative connotation, as does “furious” and “vindictive.” When used in a sentence, we can see that the word with the closest meaning to irate is “furious,” not “vindictive,” which means “vengeful” or “spiteful.” “The teacher grew irate, and even shouted, when the students would not quiet down for the lecture.”
31. D. “Vocation” comes from two root words: “voc,” which means “call,” and the suffix “-ation,” which refers to an “action” or “resulting state of being.” This simply means that vocation has something to do with someone’s calling – someone’s mission or objective to make what they are “called” to do a reality. “Sally always felt that being a vet was her true vocation.”
32. C. “Obstinate” has a negative connotation, but so do the other choices. We can try to use the word in a sentence: “Henry was obstinate in his refusal to eat peas, even though his parents promised him dessert as a reward.” From this, we could see that Henry might be stubborn in his refusal, not immune or wary (suspicious) in his refusal. The root word “ob,” which means “against” helps give us a clue about this word.
33. A. “Reticent” has a slightly negative connotation, and is used to refer to someone who is reluctant to share his or her feelings. Another word for this is guarded. Note that “plentiful” and “rewarding” both have positive connotations. “Verbose” is a negative word too, but it actually means wordy, the opposite of reticent. “Try as he might, the guidance counselor could not get a word out of Sally, who was known to be very reticent.”

## Verbal Reasoning – Sentence Completion

### Introductory

1. A. First, see what part of speech is missing (a noun, verb, adjective, or adverb). Then, use context clues to see if there’s a word or phrase that somehow relates to the missing part of speech. We can see that a verb is missing (partly because all of the answer choices are verbs), so we should look for another verb in the rest of the sentence. We see a few more: “determine” and “drink.” We know the scientist had a job to do, which was to “determine” if the water was safe for “drinking.” We don’t actually know if scientist is drinking the water, but we know that he is determining something about it. Therefore, we should look for a verb that has to do with making a determination. The word that best fits this is “analyze.”



2. B. Ask yourself “What happens in the sentence?” If we broke down the sentence into smaller parts, it might look like this: someone made a call to take an action; (s)he was told that that action would not be possible for a certain reason. The action that the caller wanted to make was to “renew” the book, which we can infer relates to the lending period of the book. Since the book was overdue (past its lending period), the borrowing period couldn’t be renewed, or extended, anymore.
3. D. The sentence describes that the student was sent to detention. Why? Because of his prank. The best word to use here is “consequence,” since it shows that something happened as a result of (or because of) his prank.
4. D. Sometimes the only way to pick the right word is by trying each word and seeing which sounds most appropriate. In this case, the word “transfer” is commonly used in sentences with busses, traveling, and routes.
5. C. The word “whereas” gives us a clue that Xena does the opposite of Yardly. Since Yardly avoids conflicts, we can expect that Xena enjoys conflict. So, Xena would not appease (agree with), commend (praise), or skirt (avoid) her enemies, since that would be something that is positive that Yardly would do. Instead, she would confront them, and not back down from her enemies.
6. B. We know that the missing word is an adjective that describes “the fact that he finished” the marathon. Marathons are long and difficult tasks, that usually require training. So, if Greg did something difficult without any training, this would be unexpected, positive, fantastic, or impressive.
7. D. We’re looking for an adjective that describes the word “portion.” In this case, the “portion” refers to the amount of the test that will be about multiplication. Since the teacher had been focusing on this topic, we can expect that a lot, or a significant amount, of the test will be on multiplication. The other words aren’t typically used to describe the amount of something.
8. A. We’re looking for words that describe things that sugar can do. We can eliminate “erupt” and “perish,” since we don’t really think of sugar as doing that. The word “instead” tells us that what actually happened (“sat in clumps at the bottom”) was the opposite of what should have happened. So, what was the opposite of “sat in clumps at the bottom”? Only “dissolve” makes sense.
9. B. What happens when someone continues “to blow into [a] balloon”? It gets bigger and bigger until it explodes. What word could we use instead of “bigger and bigger”? Only “expand” makes sense, since “contract” means to get smaller, and “plummet” and “withdraw” are not used to describe things that get bigger or smaller.
10. A. We know that people believed the invention was “useless.” This is a negative idea, which is used along with another negative idea, since it would not make sense for someone to say something both negative and positive about the invention. The only word that fits is “absurd.” It doesn’t make sense that something is useless and beneficial, or that it is useless and practical.
11. C. What Darren did was “unlike” him. Darren is “usually very proactive” and “early.” The opposite of “proactive” and “early” is “late” or “delayed,” not “hurry.” “Commit” and “approach” don’t have anything to do with when something happens.
12. C. We are looking for an adjective that describes how the guards “remain,” or stand. We know that the guards are “still as statues,” and the only word that relates to this is “stationary,” which means “unmoving” and “motionless.” The guards may be marvelous, and standing still may be very tiring, but in context, the only word that makes sense is for the guards to be unmoving despite distraction.
13. A. We can see that the other choices are synonyms of something getting worse. Russel was trying to make his headache go away (by resting, water, and medication), not get worse. The only word that makes sense is “abate,” which means “lessen.”
14. B. Another way to read this sentence would be to say that the pigeons were resting peacefully until Harriet did something. To harass is to bother or disturb, resulting in the opposite of peaceful



- rest. So, we can infer that Harriet was harassing the pigeon, causing her mother to scold her and the pigeons to no longer be resting. Harriet could have fed the pigeons without harassing them.
15. D. The adjective that describes the need to get out of the way of an ambulance is “urgent,” especially since we are told that there is an emergency. This is the opposite of “urgent” or “futile” (useless).
16. B. If the father is usually calm and patient, and everyone was surprised by his reaction to the waiter’s mistake, then the reaction must have been the opposite of how he normally reacts. The word that best fits this is “infuriated,” as “relaxed” would be similar to how he usually is, and “distant” would be more neutral or removed than calm or not calm.
17. D. The wildlife preserve is described as being land that is “set aside” and that is “protected.” We are looking for a noun that carries a similar meaning to this. Only “sanctuary” makes sense, as “myth” and “illusion” refer to things that are not real, where wildlife preserves are very real. They are not drawbacks for animals, but are instead benefits.
18. A. Florence did something to let her father know that she wanted to stay up late. She did this “with pleading eyes.” A similar word to “plead” is “beg.” The other choices don’t make sense. Florence would probably not have pleading eyes as she fights – perhaps instead having anger in her eyes.
19. D. We are told that Victor hiked for six hours and it took all of his strength. This would leave him feeling tired, or weary. The word “fragile,” when used to refer to a person, usually means ill, weak, or unwell, which is not quite the same as feeling tired.
20. D. The televisions allow viewers to watch multiple shows “at the same time.” To be able to watch two or three shows “at the same time” would be to watch them “simultaneously.” “Consecutively” means one after the other, which is not the same thing as “at the same time.”
21. B. Edward had “so much going” which made it difficult to do a certain thing with his time and energy. When someone is very busy, they need to organize their time and energy – to prioritize what is important. Edward wouldn’t need to predict his time or sift through it; he knows that he has homework, basketball, and newspaper.
22. A. There could be many words to use to describe the amount that a harvest provided. We are told that they had “enough food” for a long time. This would mean that the amount the harvest provided was a lot. Another way of saying this is to say that the harvest was bountiful. Something that is prevalent is high profile, which might make sense for a successful harvest, but does not really make sense in context of this sentence.
23. C. The sentence is comparing two different types of geysers – ones that are regular and predictable like Old Faithful, and ones that are not. The opposite of regular and predictable would be something like irregular and unpredictable. Something that is regular and predictable could also be coordinate and unsurprising, so we can rule out those words, leaving us with “hesitant” and “irregular.” Something could be hesitant but still regular, but something that is irregular cannot be (and is the opposite of) regular.
24. A. Xavier is working on a “problem” that he believes is “unsolvable.” An enigma is just such a problem. Notice also that the other words don’t make sense when plugged into the sentence.
25. C. Since the bank was robbed and was “powerless to stop thieves,” it wasn’t very safe or secure. The word “though” is a clue that what seemed to be the case actually wasn’t the case. Since what actually happened was that the bank was robbed, the bank must have seemed secure and safe. Notice that “vulnerable” describes what the bank was actually like, not how it seemed.
26. A. The word “though” is important, since it tell us that what Jenny wanted to think was different from what she actually thought. The second half of the sentence tells us that Jenny “knew that winning” was “due to” luck, not skill. The opposite of this should be stated in the first half of the sentence. Another way of saying “due to” is to say “credit” or “attribute.”
27. C. Maurice made a mistake in permanent pen. We’re told that because of this, he was unable to do something to his response on the questionnaire. What would someone want to do to change a response on the questionnaire? The only word that makes sense is “modify,” since “highlight” would emphasize the mistake, not help change it.

28. D. We know that the fire was fierce and that the fact that there was relief when the firefighters emerged. What could happen to leave people feeling relieved? The firefighters were all safe, uninjured, or unscathed.
29. C. The key phrase here is “reassembled later” meaning that the table is currently assembled. If the table is assembled now, and needs to be reassembled later, then it must be first taken apart, or dismantled. “Compile” means to put together. And while the family is moving the table, “export” isn’t quite the right word to use to describe this action, let alone to describe what needs to be done before the table can be later reassembled.
30. B. Evelyn was certain about the fact that she did poorly, but then saw that she had done very well. What could Evelyn’s reaction be? She was likely surprised, or bewildered, not upset over doing well.
31. A. There is something slightly wrong with Kristen’s desk, leading to feel uneasy or “off” about it. The word most similar to this is “amiss,” which means something is “not quite right.” Given the negative feelings expressed in the sentence, “cheerful” doesn’t make much sense. And we don’t know that the desk was clean or dirty.
32. C. If the son “could not be quieted,” then he was not tired or senseless. Nor was he “legendary” (like a hero). He was simply the opposite of quieted, calmed, and soothed. The only word that fits this tone is “perturbed,” which means bothered or upset.
33. C. Something that leads to “serious medical conditions” is very bad. We can rule out “routine” and “advantageous,” since these have neutral and positive connotations. Something that is detrimental is damaging to that thing. In this case, lack of activity is damaging to health. Contagious has to do with health, but is used to describe the spread of something to another, which doesn’t make sense here.

### Intermediate

1. B. We can infer that people no longer use public payphones because they have “turned to more convenient options.” This means that payphones are no longer used, or useful. Another way to describe this is to say that they have become more obsolete, not popular, difficult to think about, or particularly troublesome.
2. A. If something is used to the point where nothing remains, then it is said to be depleted. To preserve something is to keep it safe and use it sparingly – something we do to things that are valuable to us. To supplement means to add things back, which would help to prevent something from being depleted.
3. C. Something good “recess” was cancelled. This must mean that something bad must have happened in the cafeteria. “Abundance” and “opportunity” have positive connotations, so we can rule it out. “Clamber” is actually a verb that describes movement (not to be confused for clamor). “Incident” refers to some sort of event.
4. B. Larry likes to do something to his sister’s anger, which he does by running around and yelling. We can infer that doing this is likely to make someone more angry, not less angry. The only word that means to increase is “provoke.”
5. A. Someone who does not want to be recognized widely or publicly is someone who likes to be anonymous, or unnamed. This has nothing to do with whether the person is very plainly spoken, patient, or reliable.
6. C. The sentence deals with the amount of work that people do. People work to make or do things, which is measured by how productive they are. Working and being elegant or suspicious are unrelated. To be languid is to be lazy and relaxed, which is the opposite of how one might feel after working more than 40 hours in a week.
7. D. We can infer that the politician isn’t going to forgive the “vicious claims” since he is going to reply with his own “hurtful claims.” Another way of saying this is to say that he will “retaliate” with his own claims. “Celebrate” and “collaborate” have positive connotations, while “retaliate” has strong negative connotations.
8. B. If something can “disguise itself,” then it can be said to camouflage itself. When something is camouflaged, it is similar to – or mimics – its surroundings. For something to mimic something

else, it doesn't necessarily need to understand it or pick it apart. And it certainly doesn't limit it in anyway.

9. C. The owner denied the loud group admittance, which means he refused to let them into his restaurant – something he “did not like” to do. The act of refusing the group entry is the same as excluding them from the restaurant, and not accommodating them. To abandon something, one must first have it (the group was not kicked out of the restaurant, but excluded from the beginning). The owner didn't necessarily intimidate them.
10. C. We are told that Polaris is “unchanging” in its location. Another word for “unchanging” is “constant.” There is no judgement made about its location or size, so “appropriate” and “massive” don't make sense. We're told that the location is more exact than approximate.
11. B. The thing that charities want people to do is to make donations. Another word for “make” is “contribute,” not “compete,” “offend,” or “reject.”
12. B. We can tell that Fran tried to convince her mother, since we are told that “though” she tried her best, her mother could not be. Another word for “convince” is “persuade.” The other words don't make sense in context of Fran trying to convince her mother.
13. A. If there is no difference between day and night, then it was either light all the time or dark all the time. We can infer that it was dark all the time, because of the clouds covering the sun. To do this, the clouds must have been very thick, dark, and “dense.”
14. B. If people supported increasing the speed limit, then they were not very worried about the danger of speeding. We know that the government had the opposite view because of the word “though.” So, they were worried that higher speeds would put people in danger, or endanger them. This would not enhance the public safety, or appease the people.
15. A. The buyers thought about “many qualities,” but there was one that was more important than the rest (the location). Another word for “quality” is “factor.” “Genre” refers to categories of books. The verdict itself was to buy the house, which was based on many qualities/factors.
16. A. The word “although” tells us that how Milton is generally regarded was different from how his managers felt about when he couldn't work with the new equipment. Someone who demonstrates understanding and ability is said to be competent. His managers felt that his failure made him look inept, so the opposite of that is competent. Milton could have been believable or composed, but he might not have been; these are irrelevant to the measure of his abilities as a worker.
17. D. We know that the person is concerned about becoming more popular, or to increase her social position or status. We aren't given any hints about whether or not she has any particular social policies or that she needs a refuge. She wouldn't want to increase her social risk.
18. A. We're told that the stones are “strange” but despite this (“though”), they are also beautiful. So, we are looking for a word that might be the opposite of beautiful. None of the words except “bizarre” work in this context, since something might be drastic, extensive, or repetitive and still beautiful.
19. D. The professor had an idea that Jordan could not argue with. Another way of saying “idea” is to call it a “viewpoint.” We don't have a clue that Jordan is having trouble disputing the professor's academic abilities. It doesn't make sense that Jordan would be unable to argue with a mistake, and less so that the mistake made “logical sense.”
20. D. Because of the word “whereas,” we know that the old manager is being compared with the new one. The new manager is focused and detail-oriented, so we know the old one must have been the opposite. Indeed, he made mistakes, and was also negligent (careless). This is the opposite of “attentive.” We don't know that the manager was blunt or rude, only that he didn't do his job well.
21. B. The investigator was able to do something that many of “her peers could not.” This means she can do things that the average investigator can't, which makes her “more than” her peers, in a positive way. Based on this, we can rule out the other answer choices, which are neutral to negative in connotation. To be “astute” is to be “smart” or “perceptive.”



22. C. The sentence tells us that “different ways” of thinking is valued by schools and businesses. Another way of saying “different ways” is to say to say “diverse ways.” This is the opposite of conformity (sameness), and has nothing to do with honesty or appearance.
23. B. We know that instead of supporting ideas with facts, Dana does so with exclamation marks. Both things are done to support or emphasize her ideas, not to show that they necessarily relate to anything else, or to paraphrase them. Dana might not be nurturing of her ideas if she doesn’t support them with anything else but exclamation marks, which usually follow strong statements.
24. C. We are told that “though” the athlete experienced negative things, she “nevertheless” carried on. This means that she pursued the medal despite these bad things. The only positive word here is “resolute,” as the other words have negative connotations.
25. C. To satisfy thirst, we quench it ( just as we would satisfy an itch by scratching it). We know that nothing could satisfy, or quench, her thirst for understanding. She was not trying to preserve her thirst, or to understand it.
26. B. Based on the way the sentence is structured, we know that the first part will have something to do with the second part. The sentence talks about a response to an insult, the definition of which is a retort. One generally doesn’t make a promise or vow when “responding negatively to an insult.” A whim is an idea that comes suddenly. While a retort may be a whim, this sentence requires a more specific word.
27. B. The party experience was positive, and the guests were happy, saying the host was gracious. So, we should look for another positive word, and find it in “hospitable.” The other words all have negative connotations.
28. A. Referees are supposed to be impartial judges. If they were siding with a particular team by accepting bribes, then they were not doing what they were supposed to. One could say that they were corrupt, or that their purpose was corrupted. There’s no suggestion that they didn’t know what was going on (oblivious). Whether or not the referees were basic is irrelevant (and nonsensical).
29. B. If a word is hurtful to groups of people, then that word is derogatory, not customary. Predatory, though also having a negative connotation, means to prey on or take advantage of something. This child is young and “ignorant,” so wasn’t likely preying on anyone when she used a hurtful word.
30. B. A flying pig is impossible, or at least improbable. Torval was most likely surprised, or flabbergasted, by this. This circumstance doesn’t make Torval ignorant (naïve) or irritated (peevish). Torval was never accused of anything, so he couldn’t be found innocent of it (exonerated).
31. C. The sentence tells us that some handbags can be identified as something negative but still sold as “legitimate and original” products. This gives us a clue that the negative quality might have something to do with whether or not it is real. There are two words that deal with whether or not something is real: “authentic” and “counterfeit.” If the handbag is real, then they wouldn’t escape detection. If they were fake – counterfeit – then they would need to fool someone into thinking they were real.
32. B. The word “epidemic” is the only one usually associated with illness, and refers to a disease that has spread far and wide.
33. A. The teacher is comparing Zephyr with Andreas. We know this because of the word “whereas.” If Zephyr had a desire to learn, then Andreas had the opposite. The opposite of interest and desire is apathy, not curiosity or great ability.

### Advanced

1. B. Based on the way the sentence is written, we can tell that the missing word is described by the clause “who controlled...power.” We also know that the sentence is describing a person, and that person has “royal” jewels. So, we can infer that the person is someone with power, and likely royalty. This fits the definition of a monarch, which is a king, queen, or other similar type of ruler. Even an actor playing a monarch would not be able to actually control the country.



2. D. We know that the politicians “rarely agreed on anything,” so we know that the unsurprising thing would be if they were divided or not in agreement about a decision. However, we are told that the decision made was surprising, which means that it was unusual, or the opposite of what normally happens. If the politicians are normally divided and do not agree on the same thing, then they were in this case in agreement, or unanimous, about their decision. We don’t know if the politicians are tired or inexperienced.
3. A. If something is “unbroken,” it is continuous in nature. If we substitute answer choices into the sentence, we see that none of the choices make sense except for “unbroken.” “Disjointed” has the opposite meaning.
4. C. We know that the woman being described is powerful and holds many “highly visible positions.” This is almost exactly the definition of the word “prominent.” “Inconspicuous” is the opposite of this, meaning something difficult to notice. Someone who is “reserved” could be said to be shy or not outgoing who would not likely be satisfied in “visible” positions. “Nonchalant” describes a carefree and easy manner in which something is done.
5. D. Kolya is described as someone who has a “difficult time” understanding jokes and sarcasm, which are part of the American language. This would mean that he does not understand that sometimes, words and phrases are not supposed to be taken literally. If Kolya had a figurative understanding of American words and phrases, he would likely not have a difficult time understanding jokes and sarcasm. This is the case also if he had an admirable or complex understanding of the same.
6. D. We check to see which word is typically used with “the price of” something. We don’t generally assemble or eclipse the price of things, but we might document or manipulate the price of things. However, the context of the passage tells us that the fact that a small group of people controls all the oil doesn’t have much to do with their ability to document prices – anyone could do that. But only people with control could change – or manipulate – the price of something.
7. B. The fact that the cat “later found the experience to be tolerable” tells us that the cat did not at first find this to be the case. So, the opposite of “tolerable” would be something terrible, or intolerable. One would not be eager, yearning, or resolved to do something terrible. One would instead be reluctant to do so.
8. D. A journal or diary is used to record things that happened during the day. So, we are looking for a word that has a similar meaning. The only word that comes close to this is “recount,” which means to relive or retell some experience.
9. B. We can tell that Ben wanted to ask the professor a question, but that the professor had already moved on. One asks questions when one is unclear about something, or needs more information about something. Another word for this is to ask for someone to elaborate, or expand, on that thing.
10. A. Since Ms. Smith wanted to avoid confusion about her expectations, she established them quickly. To exaggerate the rules or pursue them doesn’t make sense. To “reflect” rules might mean to live by them, but this is something that happens over time, and would not help to prevent confusion about the rules in advance.
11. B. Things that are displayed on flags and banners – especially symbols, like animals – are referred to as emblems. If we know what the other choices mean (a requirement, an excited statement, or something terrible that happened), and we substitute those words into the sentence, we can see that they don’t make any sense. So, even if we don’t know the definition of “emblem,” we know that the other choices don’t make much sense.
12. C. A grudge is a lasting feeling of ill-will. “Perception” is a point of view or ability to see something. A warrant is a document that gives someone permission to do something else. “Pseudonym” is made up of two roots, “pseudo,” which means “fake,” and “-nym,” which means name. So, a famous person who wants to maintain his or her privacy because their names are well-known, might want a fake name. This is the only word that makes sense.
13. A. We know that the words used to describe Gerry’s abilities are positive, since the sentence says that she would represent interests “well.” There is only one word with a positive connotation, and

that is “advocate,” which means to champion or to fight for something. In this case, Gerry would fight for (advocate) their needs.

14. C. We know that Fabian has a positive outlook on artists who draw, since he compliments their form of art. So, we know that he has a positive respect for those artists. The only positive phrase created by the answer choices is “profound respect,” which means “a deep respect.” A “disdainful” respect is nonsensical, since someone who disdains something hates it very much. “Superficial” is the opposite meaning of “profound,” indicating that Fabian doesn’t have much respect at all for the artists. To be blasé about something is to be unimpressed, which we know is not the case based on the sentence.
15. A. If land produces a lot of food, it is said to be fertile, or productive. Land could be pungent (intensely smelly), or it could be formidable (daunting or scary), and it could even be resilient (coming back again and again), but in context of being productive, “fertile” best describes the land.
16. A. When we contrast one thing against another, we are comparing those two things. So, it doesn’t make sense that Lara was trying to deploy (make happen), evolve (change over time), or fend (defend or repel) different possibilities. It only makes sense that she was trying to compare them to make a decision.
17. A. Since Yanez had “no hope” of getting done what he wanted, we know that the support of powerful friends was very important, or “crucial,” to the plan’s success. If the friends’ help was marginal, or inconsequential (think at the edges, or margins, of something), not having it wouldn’t have influenced his plan at all. Whether the friends’ help was repeating is irrelevant to the meaning of the sentence.
18. C. We know that Will is going to do the opposite of keeping the error a secret, since the sentence begins with “rather.” This means that he will do the opposite of keeping a secret, which is to say clearly and openly, or overtly, that he made a mistake. A disingenuous admission would be one that is not heartfelt. A tired (haggard) or sedative (tiring) admission would not do much to help him move past his mistake.
19. D. The sentence compares Lisa’s “casual” mention of the number of shoes she owns with Sharon’s response. The phrase “only the most expensive shoes” tells us that Sharon takes pride in her shoe collection, and thinks that she is better than Lisa accordingly. The word “haughtily” describes an action that is done arrogantly, or disdainfully. If Sharon cared about Lisa’s feelings, she may have said as much gingerly, but would not have used such language to show that she had many times Lisa’s shoes, all of which were “very expensive.”
20. C. The sentence describes “endless” waves. Another way to express the idea of an unending body of water would be to call it “infinite.” There is no context to support the fact that the ocean is angry (belligerent) or proud, or even that it is exotic (unusual).
21. B. Since Irene had no other choices; she must admit defeat. To admit defeat is to give up, or forfeit. To chastise is to punish, to inaugurate is to select, and to operate is to take action. None of these choices makes sense except to forfeit.
22. C. When part of a sentence can stand alone as a sentence, it is called an independent clause. We know that the clause “By the time...already -----” is an independent clause because it can stand alone as a sentence. So, we know we are looking for an adjective to describe historical sites. The clause after this (“having...artifacts”) is called a dependent clause because it describes a part of the independent clause. In this case, that is the historical sites. So, the dependent clause is giving us a clue what the missing word is. If something has been “robbed of important and valuable artifacts,” then it has been plundered, not preserved (pristine). That the site is “coherent” doesn’t make much sense, nor does the fact that it is “deceptive.”
23. B. When two words are used together with “and,” we know that they should be related in idea. The idea of “peace” is the opposite of “distractions and cares.” A complementary word for “peace” is “serenity,” not suspense. Evidence is a noun that describes the support for something, and “transition describes the moving between two different things. Neither of these makes much sense.

24. D. We know that Gina was angry and upset and would not “return to the store” because of the “unfair” treatment. So, we are looking for a negative word. We know that “acclaim” (praise) and “candor” (familiarity) are positive, as is “mirth” (cheerfulness or amusement). Only “umbrage,” which means “offense,” has a negative connotation.
25. B. We know that the fact that “lightning can never strike the same place twice” is a “mistaken belief.” Another word for this is a “fallacy.” A decree is an order that must be obeyed, which is not quite the same thing as a widely-held belief. It’s not quite right to call a belief a procedure. Nor is a belief necessarily a belief about the future, as a prophecy is.
26. C. We are told that the company generates something from Bob, and that Bob is associated with “income.” Another word for “income” is “revenue,” not “expense” or any of the other choices.
27. C. The sentence is describing periods of time. Only “epoch” and “semester” are measurements of periods of time. An arena is a place of competition or combat, and a burden is a heavy responsibility. A semester is a short period of time, whereas an epoch is a long period of time, or an era. The only choice that makes sense in describing the period of time where people are using electricity is to describe it as an era, or an epoch.
28. C. The attack on Pearl Harbor can be many things, but in this sentence, it is described as a moment in history that “helped spur” the country to war. To spur something on is to help make something happen. In this case, it is to change the United States from being neutral to being part of the war. This is a pivot, or a change from one point of view to another. This has nothing to do with whether the moment was virtuous (highly moral), carefree, or insightful (educational).
29. A. We can infer that Oscar’s surroundings were overwhelming – that there were many sensory inputs (Sights, smells, etc.) that he was trying to process. His senses would be “assailed,” or “barraged,” by these sensations. These were not likely comforting, since they were new surroundings. His senses were not “elapsed” (over) or “instituted” (put into place), since these verbs don’t make sense when describing what happens to senses.
30. A. If a group recruits new people to join them, then they are hoping to expand the number of people in the group. The only word that means to grow or increase is to “augment.” This is the opposite of “minimize,” or reduce numbers. The other words – caption and encompass – don’t make sense to use in context of the number of people in a group.
31. D. An abandoned house that looks haunted is probably not an inviting place, so we would not expect to use words with positive connotations to describe that place. Such a place would inspire fear, or foreboding, in a person, causing that person to avoid the place. The person would not be filled with eagerness (anticipation), or necessarily hatred (contempt). He would certainly not be determined to visit the place.
32. D. If something is “revolutionary,” it is not necessarily impractical. However, if it was, then it would not be able to “change the lives of billions.” Another way of saying “revolutionary” is to say “inventive.” “Hackneyed” is a word with a strong negative connotation, that would not be used to describe something positive happening (it means that something is uninventive).
33. B. Perry is described positively, so we can rule out “greedy” and “mutinous” as adjectives that describe his laugh. We might not know what “wry” means, but it doesn’t sound particularly positive (in fact, when used to describe a smile, it is often used to mean a smile that is disingenuous, mocking, or sarcastic). A “hearty” smile, on the other hand, is very positive, and is in line with “genuine.”

## Reading Comprehension

### Passage 1

1. B. *Main Idea.* The passage does list many different types of holiday decorations, as well as the weather when the decorations are put up, but it does so to describe the overall tradition at Sycamore Lane. If the passage was primarily concerned with listing decorations or describing the weather, it would not bother spending time speaking to the festival – Mrs. Jones, “Jingle Bells,” etc.
2. B. *Supporting Idea.* The passage mentions many complicated decorations, which many people help set up. This might mean the decorations are exhausting, but they could be fairly straight



forward, and not too tiring at all. The passage doesn't mention how the music is chosen. The fact that many people who don't even live on Sycamore Lane attend the festival means that many people are aware of it, and that it is famous.

3. D. *Tone/Language/Style*. The author compares the strings to the cocoons not to describe a pest problem, but to show how, like butterflies, the lights become unwrapped and show their beauty. There's no rule that's mentioned saying that the trees can only be used every eleven months.
4. D. *Vocabulary*. The word "envelope" has many meanings. As a noun, it refers to a paper product that needs to be opened to reveal a letter. As a verb, it means to surround. It could also mean to contain, but in this case, it doesn't make sense that a festive mood would contain the lane.
5. D. *Organization/Logic*. The passage indicates that Mrs. Jones gives a speech to the gathered spectators, meaning that the people must have gathered first before listening to a speech. The music comes after the speech ends.
6. D. *Inference*. The passage describes how the decorations are synchronized and that the exhibition isn't just one house or a few houses. Instead, there is a coordinated effort to light up lawns, trees, and homes, all set to music. There is no mention of this being the most important (or any other) holiday tradition, or indeed even an advertisement.

### Passage 2

1. D. *Main Idea*. The author doesn't take a particular stance on any topic in this passage. Instead, he shows that the formation of coal is a complicated process with many different factors. He never mentions that coal was the only way people could cook, that people kill plants for coal, or that coal should be replaced with gas.
2. D. *Supporting Idea*. The first paragraph describes how coal can be used by people, while the "story of coal begins" in the third paragraph. The rest of the paragraphs describe parts of the coal formation process happening later on.
3. A. *Supporting Idea*. The author lists the various factors that turn lignite into bituminous coal in the 5<sup>th</sup> paragraph. Acid is not mentioned here, but is mentioned as one of the forces that help to delay the decay of plant matter. This is a step that happens earlier than the transformation of lignite into bituminous coal.
4. A. *Vocabulary*. The word "precarious" doesn't mean dangerous or deadly, but something that is very sensitive to various factors and conditions. This makes it vulnerable to interruption, not confusing. Precarious can describe something unbalanced, but in this case, the word more appropriately refers to the fact that something is easily disrupted.
5. C. *Inference*. The passage describes how one unit of coal starts with as much as five times as much plant matter, which means the formation process requires a lot of plant matter, not a little. The passage states the coal started forming 300-400 million years ago, but doesn't say that coal only forms every 300-400 million years. Instead, the process is ongoing, since all that is required is for lots of plant matter to accumulate and to become buried and subjected to time, pressure, and heat.
6. A. *Tone/Language/Style*. The author's asides (lines 51-52) and exclamations (lines 7-8) are evidence of his admiration for the coal formation process. The author never criticizes anything, or seems saddened by anything mentioned in the passage. Neither is the author questioning. Instead, he is sharing information that appears to fascinate him.

### Passage 3

1. C. *Main Idea*. The author compares his mornings now, which he dreads, with the eagerness of waking up to read comics. The author enjoys comics, and would probably agree that as a child, they were the most important part of the newspaper. However, the main idea of passage has to do with the contrast between mornings now and mornings as a child, with something to look forward to.
2. A. *Vocabulary*. "Dreary" has a negative connotation, which we see also mirrors the word "unpleasant" used in line 3. The only word that carries a similar connotation is "cheerless." Something uplifting is enjoyable or inspiring.



3. *C. Inference.* The passage describes the author's memory of being in middle school. Therefore, the author must be older than middle-school aged. He may or may not be an adult – he might be in high school, for example. The author doesn't mention coffee.
4. *D. Tone/Language/Style.* The fact that the author skips the bathroom, developed a method of swinging downstairs, and runs across the lawn demonstrates how excited and eager he is. There is no information to suggest that the author is hungry, late, or being scolded. Instead, this reinforces the fact that the author is very focused on the comics.
5. *C. Tone/Language/Style.* The Sunday treasures can be inferred to be the newspaper comics, the thing that helps him get out of bed and that excites him enough to run around in the morning. The author doesn't mention drinking coffee (only that he is up before his parents start making coffee). The mailbox itself is what keeps the treasures dry; it isn't the treasure itself.
6. *A. Organization/Logic.* The passage describes how he would still be wet because of running through dew on the lawn. The author doesn't drink coffee, and skips the bathroom entirely.

## Passage 4

1. *B. Main Idea.* The passage describes a few easy-to-miss details of a famous landmark, the Eiffel Tower. These include latticework, and the names of French scientists, as well as the color of the tower. Though it is true that the Eiffel Tower is recognizable, and that the color has changed, but these by themselves are not the primary concern of the passage. The passage doesn't take a stance on which characteristic is most distinguishing, nor does it say that visitors must know about little details to appreciate the tower.
2. *B. Vocabulary.* In this case, the word "tribute" doesn't mean tax, sacrifice, or contribution (though sometimes, it could). Instead, it is talking about how the latticework is designed to commemorate, or memorialize the famous scientists.
3. *C. Supporting Idea.* There's no mention of a vote over the color of the Tower. There's no mention that the designer was friends with the 72 people, nor that they all worked on designing the Tower. Instead, we are told that they were famous. The author describes the scientists as "great thinkers" (line 30), which we can infer means that they had a great impact on their fields.
4. *C. Organization/Logic.* The author indicates that the tower is painted "every several years to prevent it from rusting" (lines 35-36). The color hasn't changed much in recent history. The names of the scientists are engraved, not painted.
5. *A. Supporting Idea.* The author mentions latticework, French scientists' names, and shades of color in the second and third paragraphs, which deal with easy-to-miss details. The first paragraph, where curved archways are described, deals with the general appearance of the Tower, which is described as widely known.
6. *B. Inference.* The author doesn't claim that brown is better than red or yellow. The Eiffel Tower is iconic, but the author doesn't claim that it is the most iconic, since he doesn't compare it to other landmarks. Instead, the author seems primarily concerned with understanding little-known facts of the Tower.

## Passage 5

1. *A. Main Idea.* The author describes how clothing can be a tool that separates people or helps bring them together. She describes how clothing can make a big difference in schools, and students should think about how clothing affects the people they're friends with.
2. *D. Vocabulary.* The paragraph discusses how clothing can bring people together and build community. This is similar to the idea of "unity" as opposed to "individuality." The idea of "formality" is something that might be inferred from wearing uniforms, but in this case, the context calls for a word that means "togetherness."
3. *B. Inference.* Clothing is described as the distracting source of difference, which we can glean from the rest of the sentences in the paragraph. It's not likely that the uniforms, which are all the same, would be sources of differences. The author doesn't mention how the opinions and personality are the sources of differences. Rather, these are things that are covered up by different clothing styles.

4. D. *Supporting Idea*. Rather than explain the importance of individuality, the last paragraph describes how uniforms can be beneficial to students. It doesn't specify exactly what needs to be done in terms of a uniform requirement. The benefits are tangible, not intangible.
5. C. *Tone/Language/Style*. The phrase refers to how uniforms can eliminate a lot of differences and help students see each other for more important qualities. It doesn't refer to the fact that there are similar customs, nor does it refer specifically to uniforms worn by soldiers or athletes, or even to fashion trends. Instead, the phrase describes the benefits to students.
6. C. *Organization/Logic*. The passage never goes into detail with respect to the types of uniforms. The author supports her claim with various reasons and examples, but doesn't actually use facts or figures to refute a theory. The passage doesn't uphold a belief, but rather overturns it (the idea that many students dislike uniforms).

## Passage 6

1. C. *Main Idea*. The passage describes an otherwise normal day that suddenly turns extraordinary. All of the other choices are mentioned, but only focus on one particular part of the story.
2. D. *Inference*. The author never tells us her name, but we find out that Mr. Bernard was going to call the author's name next and didn't manage to finish saying "Skyler" before things take a turn.
3. C. *Vocabulary*. Something that is very loud can cause one to go deaf, or the state of being unable to hear. The word relates to sound (the sentence also says how everyone covered their ears), so we know that it doesn't have anything to do with sight or taste.
4. D. *Organization/Logic*. The children ran to the window after rollcall began. There's no mention of recess, or a disrespect for Mr. Bernard. Because this happens after the loud crash, then we can infer that it was curiosity that drove the students to the window.
5. D. *Supporting Idea*. There is speculation among the students about whether a comet or alien had crashed into the playground. We're told later in line 46 that a meteorite had landed in the playground. There was no mention of recess.
6. B. *Tone/Language/Style*. The author's use of exclamation points and the casual language used (contractions, etc.) let us know that the author isn't serious, but excited and energetic. There's no indication that the author was annoyed by the events or angered by them. Throughout the passage, the author describes curiosity and excitement at the events happening outside the window.

## Passage 7

1. D. *Main Idea*. The passage doesn't really delve into why estuaries are important – it really just tells us about how we might think about different types of estuaries. However, it doesn't talk about which estuaries are the best. In addition, there are no such things as fresh or salt water estuaries. The passage defines an estuary as a body of brackish water – that is, one that has both salt and fresh water in it.
2. B. *Supporting Idea*. The passage describes how one way to classify estuaries is by how "fresh and salt water interacts" (line 13). This is another way of referring to water circulation, or how water moves around, but not to the number of salt water layers. A salt wedge is a type of estuary, and marine life isn't mentioned in the passage.
3. A. *Supporting Idea*. If the salt and fresh water is poorly mixed, then it is a salt wedge, where the fresh water sits on top of the salt water. If the water is thoroughly mixed, it might be a homogenous estuary. The passage never describes a "permanently mixed" estuary, or an estuary where salt and fresh water never mix.
4. A. *Supporting Idea*. The passage tells us in lines 18, 28, and 35 that tidal force is important to the mixing of fresh and salt water. It never mentions boat traffic, temperature, or deposits. It does mention melting snow or heavy rainfall, but these choices are not available.
5. B. *Organization/Logic*. The passage describes different examples of an estuary. The author leaves his opinion out of the paper, not arguing for or against anything. The passage doesn't describe any specific event, and it doesn't "reject" any sort of concept. The passage simply provides us with information.

6. C. *Vocabulary*. The last paragraph describes an estuary that is sometimes there, but sometimes not, depending on whether or not there is fresh water. So, we wouldn't call this a constant estuary. Nor do we know if it is apparent (without the fresh water) or valuable. It would be considered irregular, or "temporary" or "seasonal," as we are told in the last sentence.

## Passage 8

1. A. *Main Idea*. The passage doesn't try to convince the reader of an opinion, but instead tells a personal story or experience. The passage tells us of a beach, but only in context of the overall experience of escaping heat and having to look for parking. The author never shares technical terms or facts, just the experience.
2. B. *Tone/Language/Style*. The author uses negative adjectives and adverbs throughout the passage, like "aimlessly" (line 20), "frustrated" and "impatient" (line 38). So, we know that the passage is generally about a negative experience. The best choice is "hopeless," especially since in the following paragraphs we are told that the author wanted to give up (line 30).
3. A. *Vocabulary*. Something that is vacant is empty (think of a hotel's vacancies, or a blank/vacant stare).
4. A. *Tone/Language/Style*. This is a colloquial phrase that describes an experience where people are very hot and uncomfortable. It doesn't literally mean that a person is sweating a bucketful of sweat.
5. C. *Supporting Idea*. The reason why it was difficult to find parking space was described early in the passage (it was hot and there were a lot of people who had just arrived at the beach). The author describes her mood just before finding a parking space.
6. D. *Organization/Logic*. The author mentions that a shout from the back seat scared her, but that it alerted her to a car that was pulling onto the road, out of a parking space. The heat is what caused so many people to go to the beach. The passage doesn't mention a car crash.

## Passage 9

1. D. *Main Idea*. The passage never gives tips on how to mountain climb, nor does it actually explain what a *torii* gate is, or mention that this is the largest *torii* gate in the world. The gate itself remains quite mysterious. Instead, an experience is related.
2. B. *Organization/Logic*. The passage indicates in lines 6-12 that the gate was very large and seemed even bigger because there weren't any large plants to make it look smaller. This, the color of the gate (reddish) is contrasted against the color of the rocks (grey), and the summit of the mountain behind it make the gate seem grand and impressive.
3. A. *Tone/Language/Style*. The author compares the shape of the gate to a letter "n." We are told that there are intricate carvings, which suggests that they are not just like the letter "n." We're told that there are no trees, and rocks are only mentioned in passing, as a contrast between the gate and the ground's colors.
4. A. *Vocabulary*. The word is used as an action, or a verb. In this case, context tells us that the wind and rain did something to the pillars that resulted in the stripping away of paint. This is another way of saying "worn away."
5. C. *Supporting Idea*. The fresh air helped to clear the author's mind, and put him at ease, not give him the impression that the area was sacred. There is no mention of birds or rustling of animals. Instead, the author writes that the inscriptions (line 37) added to the feeling that the place was sacred.
6. A. *Tone/Language/Style*. The author writes with great reverence for his experience. He uses words like "sacred" and "mysterious" and "important" to describe the experience. So, we can eliminate the negative emotions like "confused" and "sarcastic." The author could be sentimental, but there's nothing that strikes us as "forgiving" in the passage.

## Passage 10

1. A. *Main Idea*. The author mentions saving money and time, but doesn't say which is more valuable. he also mentions how gardens produce healthy food, but not that this is the only way to get healthy food. The author mentions technology and science and the way it's helped farmers



and people make cheaper and more plentiful food. However, the point of the passage is to defend an “old” idea of having one’s own garden.

2. B. *Supporting Idea*. The third paragraph describes how science has improved people’s ability to farm more effectively. This is thanks to the ability of people to fight pests and manage droughts. In addition, transportation is mentioned as leading to cheaper and more accessible.
3. D. *Tone/Language/Style*. The passage describes how victory gardens were so-called because they were a way for people to help the war efforts of World War I and II. The government courage people to grow their own food. There’s no mention of athletes in this passage.
4. C. *Inference*. Charles Pack is mentioned in line 38 as the person who founded the War Gardens Commission. We can surmise that he believed that vegetable gardens were more useful than just for personal amusement. He would have supported Eleanor Roosevelt’s garden as an example of a wartime contribution. Pack would likely not have thought about food as health versus money, instead thinking that lots of people growing their own food could contribute to something as big as a war.
5. B. *Vocabulary*. The last paragraph describes how the author is wishful, that she wants to work with her hands, and wants to emulate the resourcefulness of others. Being wishful is similar to being jealous, or wanting something.
6. C. *Inference*. We are told explicitly that the author likes to work outdoors. We don’t know if the author works with her hands frequently or not, or whether she is often engaged in creative activities. We only know the author’s position on gardens, and that she wants to start her own vegetable garden.

#### Passage 11

1. D. *Main Idea*. The author mentions early on how painting requires planning and equipment, which is more than a “little effort.” To do it well, painting should be planned, not done on a whim. It may be true that there are many easy ways to improve a room, but the passage doesn’t really discuss this in detail. Instead, the passage describes the many things to think about before and while painting.
2. A. *Vocabulary*. The word “depth” can have several meanings. In this case, it doesn’t mean distance, or depth. Instead, the word is used to mean depth of atmosphere, ambiance or richness.
3. A. *Inference*. The author emphasizes planning as a way to ensure that the process goes smoothly and has good results. The opposite can be inferred from the author’s position on planning – that not doing so will result in a poor process and poor results. The author indicates that without planning, furniture could get covered in paint, and the process could take much longer.
4. A. *Tone/Language/Style*. The phrase is used to describe parts of a wall that are difficult to reach, which will benefit from painter’s tape. We know this because the preceding sentences describe the different benefits of using painter’s tape.
5. C. *Tone/Language/Style*. The author doesn’t describe the actual steps to paint in an aggressive manner. Neither does he do so in a bored tone (be careful not to confuse your own attitude or feelings with those of the author’s). Instead, the author simply tells us what the next steps are in painting, and does so without any sort of emotional attachment. Therefore, we could call this an objective tone.
6. C. *Supporting Idea*. The author tells us throughout the passage that we need to carefully plan, purchase the necessary equipment, and clean the painting surface. We’re never told that we need experienced painters.

#### Passage 12

1. A. *Main Idea*. The passage describes different ways people consider diamonds to be valuable. However, the passage does so in context of several famous examples of famous diamonds. In fact, the second half of the passage is entirely about a specific diamond.
2. B. *Supporting Idea*. The passage describes the *Millennium Star* in the third paragraph, and mentions how it is valuable for its color and clarity. The *Hope Diamond* was famous for its ownership and newspaper publicity, while the *Cullinan* was famous for its size.



3. C. *Vocabulary*. The sentence compares “mere fame” with becoming “legendary,” suggesting that being legendary is greater than, or more than, just fame. We could say that to be legendary is to go beyond – or, exceed – “just” fame.
4. D. *Supporting Idea*. The passage describes the physical beauty of certain diamonds in the first half of the passage. In the second half, the passage describes the *Hope Diamond*, which was famous because of its history and ownership. The nicknames are certainly creative, but this alone does not make a diamond famous. While diamonds are used in industry, the passage never mentions famous diamonds which were used in industry.
5. B. *Inference*. The passage suggests that, as in the case of the *Cullinan*, large diamonds are cut and recut, but that all of the pieces were saved (over 100 in all), not that they were lost or destroyed. We are given several examples of royalty that collects diamonds. We are told that the *Hope Diamond* is believed to be cursed because of the “misfortune” it brought to its owners (which included Marie Antoinette and Louis XVI, NOT Louis XIV. The passage describes how some standards that make diamonds famous are objective (color, clarity, weight), but that there are also other factors (like notoriety) that make diamonds famous.
6. A. *Inference*. The size of the diamond (measured in carats, which we know from the second paragraph) contributes to a diamond’s worth. There is no supporting evidence to suggest that the smaller the diamond, the better the color or clarity (we are only given a couple of examples of this idea, but not told that this is fact). The example of the *Hope Diamond* shows us that what people believe determines a diamond’s fame, not just the weight or color of the diamond.

## Passage 13

1. A. *Main Idea*. The passage mentions nuclear fusion, but doesn’t really tell us more than what it is, and that it produces a lot of energy. Instead, the passage focuses on the lifecycle of the sun, taking us from its creation to its eventual demise. What happens to the outer planets, and the qualities of the sun, are described to us in context of this journey.
2. D. *Tone/Language/Style*. The author uses words like “incomprehensibly” and “inconceivably” to describe the information he is sharing. There is no evidence of any regret or confusion, or even uncertainty. Instead, the author seems to be describing facts to us with a sense of awe.
3. A. *Supporting Idea*. The author tells us that gravitation collapse was the process that created the sun. The sun shines because of nuclear fusion. There is no information shared about increasing the mass of the sun, and temperature is mentioned as increasing only after the sun runs out of hydrogen.
4. D. *Vocabulary*. The idea of something being engulfed by another is that it is completely consumed, or overtaken. We can infer this because the process describes how the sun is “swelling,” so expanding to overtake the other planets.
5. C. *Supporting Idea*. The sun will expand, and get hotter, and therefore melt ice that was never before melted.
6. D. *Organization/Logic*. There is no opinion or point of view shared. Instead, a theory, or hypothesis, is shared in order from earlier to later (chronologically). There aren’t comparisons between theories made, though several theories are discussed.

## Passage 14

1. D. *Main Idea*. The passage describes a story about how Mary comes up with an idea for a story. The life and career of Mary’s mother are discussed briefly, as is the idea of the power behind life. We don’t know the outcome of the writing contest yet.
2. A. *Supporting Idea*. The author writes in lines 16-19 that Byron was the “most famous writer in the world” and that the other authors stood no chance. There is no indication that Mary’s mother told her she wasn’t good, or that Percy told her not to compete.
3. D. *Vocabulary*. The word “obliged” means that Mary felt like she had to do something – that there was an obligation to do so. This is similar to saying that there was a requirement to do so, not necessarily a desire to do so.
4. A. *Inference*. We know that there is lightning outside, so the rumble is likely caused by thunder. There’s no suggestion that Percy was sharing a room with Mary, or that Mary was particularly

hungry. We also don't know anything about Mary's room, including whether or not there was a fireplace.

5. C. *Inference*. The context of the paragraph tells us that Mary was scared, since she was thinking about a scary story. Mary may have been cold, or the lightning may have been bright, but we can't know that for sure. What we do know is that Mary was frightened, leading her to pull the sheets over her head.
6. A. *Organization/Logic*. Mary is kept awake by the fact that she wanted to think of an idea to enter into the writing contest. Byron's scary story is mentioned only as an introduction to the idea of a writing contest. The other choices are included in the passage as a result of Mary's thought process while trying to come up with an idea.

#### Passage 15

1. D. *Main Idea*. The passage describes various aspects of the Tea Garden, from the Tea House itself to the Pagoda to the Moon Bridge. The passage glosses over a detailed history of the place. It doesn't tell the reader to visit, and just offers information about the Tea Garden.
2. A. *Supporting Idea*. We know from the third paragraph that Hagiwara's ancestral home was Japan because we are told that he imported things from his ancestral home, which is described as Japan. We don't know if Hagiwara became rich serving tea to visitors, or that he made lanterns. We also can't conclude that Hagiwara was a landscape designer just because he designed the Tea Garden.
3. B. *Supporting Idea*. Lines 40-42 describe the different tiers of the pagoda as having to do with natural elements. These are from Japanese philosophy, but not necessarily Buddhism. The tiers don't represent plants, or the number of years the Hagiwaras lived in the garden.
4. A. *Tone/Language/Style*. The author writes that "though" the Moon Bridge spanned a small water feature, its arch was high enough so that the reflection in the water created a full circle. This is a pretty steep arch. The author doesn't suggest that it is dangerous, or unstable. The arch was certainly a particular design choice, but not necessarily "eccentric," strange, or not customary.
5. A. *Tone/Language/Style*. The phrase refers to a visitor's mood, or mindset. The phrase is used to describe the effect of the bridge on slowing visitors down mentally.
6. B. *Organization/Logic*. In lines 19-21, we are told that Hagiwara wanted to make sure that the gardens emulated, or mirrored, the gardens of his home in Japan. This is to say that he wanted to make the Tea Garden authentic. Importing features from Japan would have cost more money, not saved it. While the Tea Garden today attracts many people, we don't know that that was the motivating factor that caused Hagiwara to emulate the gardens of Japan.

#### Passage 16

1. C. *Main Idea*. The passage doesn't seek to convince the readers of anything in particular. Instead, it seems to share a memory of a nice day. The weather is compared only as a way to emphasize how nice this particular day is.
2. A. *Vocabulary*. Something that is rare is scarce, or special. It's not necessarily unbelievable, though it could be pretty incredible. Food that is prepared rare is undercooked, but that doesn't make sense in this context.
3. C. *Tone/Language/Style*. The author uses the word "blemish" figuratively. He describes the sky as a giant sapphire with no flaws, meaning it is perfectly blue with no other colors in it. In context, we can tell that this isn't humidity or wind, since those things can't be seen.
4. D. *Supporting Idea*. We are told in the fourth paragraph that the song birds are "too well-hidden" and can't be seen. The author "can't spot the songbirds." The robins were observed hopping on the ground, the blue jays flying, as are the cardinals.
5. A. *Inference*. In the last paragraph, we're told that the author doesn't "know the next time" he'll have time to enjoy a day like today. From this we can infer that he is very busy. We don't know what the author does for a living, if he is a gardener, student of the weather, or poet.
6. B. *Organization/Logic*. The passage never mentions questions or presents an argument. There is no theory presented either. Instead, the events of a particular day are discussed in order as they happen.

## Passage 17

1. B. *Main Idea*. The author tells us about a problem from the past which he feels is still a going to be a problem in the present. This problem is monoculture, and he says that we will experience a similar problem as the potato famine but this time for wheat. He goes on to say that people need to do something to prevent this from happening again. We can infer that the author believes it is more important to have diverse crops than inexpensive crops.
2. C. *Supporting Idea*. In the second and third paragraphs, the author lists various reasons that contributed to the famine. One of these was the fact that the Irish were kept in poverty because they were not allowed to be educated. This caused them to rely on a single crop, which was ultimately wiped out by disease. If the Irish had instead grown a number of different types of potatoes, the passage suggests that the famine would not have been so bad.
3. D. *Vocabulary*. “Rampant” is used to describe how poverty was everywhere in Ireland. We know this because the preceding paragraph and the following sentences describe just how widespread the poverty was. This is like saying that the poverty was uncontrolled, or wild. It doesn’t make sense to say that poverty was “ruined” in Ireland (which would have been a good thing). “Fortified” makes a little bit more sense, but doesn’t capture the idea that the poverty was everywhere. “Static” describes how something is unchanging, which is not the case, as poverty was spreading throughout Ireland.
4. A. *Inference*. The author draws a comparison between the two crops to show that the problem of monoculture is still here today because both crops can be wiped out by disease. This was bad for Ireland then, and could be bad for the world today. The passage never mentions how either crop tastes.
5. D. *Tone/Language/Style*. The author is clearly not objective, as he strongly believes that people must do something about the problem of monoculture. He does not use humor to help drive home his point, and is far from indifferent (uncaring) for the problem. Instead, he feels passionately about the problem, and uses questions to directly engage the reader.
6. D. *Organization/Logic*. A historical event (the famine in Ireland) is discussed in detail, but this is only half of the passage. Instead, this event is used as support to share a particular point of view about monoculture. The author doesn’t disprove an idea or simply define a term (though he does this in context of advancing an argument).

## Passage 18

1. C. *Main Idea*. The author presents both sides of the issue of reservations, describing both how people who like and dislike the idea of reservations think. This means we should rule out the first two answer choices. The author also describes several different possible solutions, but doesn’t say that protecting restaurants or diners is more important.
2. D. *Supporting Idea*. We are told in the first paragraph that “no-shows” are a problem because they “pose a real problem to a restaurant’s financial success.” This is because (lines 7-10) restaurants hold tables for reservations, and if people don’t show up, the tables are empty and nobody spends money at the restaurant. The passage never mentions publicity or the fact that people will see a restaurant that is empty.
3. D. *Tone/Language/Style*. We can infer that the “opponents” to charging for reservations believe this this is a negative practice, so we are looking for negative words that might replace “flies in the face of.” The only choice here is “undermines and contradicts.” It wouldn’t make sense for an opponent of selling reservations to say that the practice is similar to or represents hospitality.
4. A. *Inference*. The paragraph states that the wealthy people will be the only ones who will have money to pay for reservations, not that the poor will never be permitted to dine at restaurants. They will, but they won’t be able to make reservations. Similarly, restaurants will cook for whomever dines at the restaurant, not only the wealthy, but only the wealthy will get reservations.
5. D. *Vocabulary*. The paragraph is discussing selling tickets for “a specific date and time” (lines 52-53). This is like scheduling a ticket time, or arriving at an appointed time (think “appointment”).



6. D. *Organization/Logic*. The passage discusses several possible solutions, and presents multiple points of view, not just one. There are several opinions expressed, but none that are supported with a personal experience as an example.

## Passage 19

1. B. *Main Idea*. Agnes and Eleanora's relationship, as well as Eleanora and her mother's relationship, are each part of the story, but not all of it. We are viewing life through Eleanora's eyes, including her relationship with her mother and with Agnes. There are many things covered here, including Eleanora's hopes and aspirations (possibly traveling to America, a relationship with Lars, etc.).
2. A. *Inference*. The passage never explicitly says so, but we can see from her reaction to Agnes' letter (lines 15-25) that going to bed so late is very unusual in Skara. We are told also that Eleanora was "surprised to find Mama...even though the church bell had already struck eight." This leads us to believe that a bed time after 10 P.M. is unusual, not necessarily ridiculous or even necessarily bad. It certainly wouldn't feel customary.
3. C. *Supporting Idea*. We are told that she is thinking about what it would "be like to live there" (line 26). In addition, Eleanora thinks to herself that "if only she wasn't waiting for Lars, maybe she could have a life like Agnes." This would mean Agnes' actions aren't irresponsible. We never see that Eleanora shares her feelings about America with her mother, nor do we know if Eleanora has ever been outside of Skara before.
4. A. *Inference*. We know that Eleanora is getting home late (lines 29-34) because there is darkness around, the stars are out, and her mother is up late (past 8 o'clock).
5. B. *Vocabulary*. The fire is close to burning out (line 58), so we can infer that the fire is dying, not strong like an inferno. "Remnants" describes the left over pieces of something (think "remainder"), in this case, the log. This wouldn't be the "smoke of the logs" or the "sparks of the logs."
6. D. *Inference*. We're not told explicitly why Eleanora burns the letter, but we know that she is "waiting for Lars," otherwise she would very much like to "have a life like Agnes" (lines 62-63). This means that she is choosing Lars over life in America. We're never told that Lars writes Eleanora a letter, nor does the passage suggest that Eleanora is angry with Agnes.

## Passage 20

1. A. *Main Idea*. The author does not take a stance in this passage about whether the PRC or ROC should control the island of Taiwan. Neither does the author mention what the United Nations should do regarding Taiwan. Instead, the author provides information to illustrate how complicated the issue of Taiwan is.
2. B. *Vocabulary*. The word "adversaries" is similar to the word "adverse," which has a very strong negative connotation. We know then that people who are adversaries are not colleagues (people who work together). They could be peers (people sharing something in common) or even relatives, but the word "adversaries" is used to describe people who are enemies, or opponents. This perfectly describes the relationship between the communists and nationalists.
3. A. *Inference*. The passage tells us that a peace treaty or armistice represent a "legal end" (lines 49-50). This means that the Chinese Civil War has not legally ended. However, if it *had* been legally ended, then there likely wouldn't have been a conflict over control of Taiwan.
4. C. *Supporting Idea*. The idea that different countries go back and forth over which country should control Taiwan makes control political. This is highlighted in the rest of the paragraph. The passage doesn't discuss if the United States influences other countries' opinions, or state that it is because of the Japanese that the situation is political.
5. D. *Organization/Logic*. The author doesn't express a point of view or a solution. Instead, he only seeks to share information about a problem and the reasons for that problem.
6. A. *Tone/Language/Style*. The author keeps a neutral attitude toward the issue of Taiwan's independence. He doesn't suggest that either the PRC or ROC should be in control.

## Passage 21



1. A. *Main Idea*. The passage is concerned primarily with describing the art and life of Jackson Pollock. The passage doesn't spend a lot of time comparing one style of art with another, nor does it go into detail about a particular work of art. The artist doesn't adopt a point of view that tries to convince the reader to become an artist.
2. C. *Organization/Logic*. While many things led to Pollock's eventual popularity, only his patronage with Peggy Guggenheim can be said to have helped propel Pollock into fame. The *Time* article, and display came after he was famous. The importance of his education is doubtful, since he was "not much of a student." The Works Progress Administration is mentioned as something that helped keep Pollock afloat, not propelling him to fame.
3. A. *Vocabulary*. The word "acclaim" refers to something positive, like praise (think "accolade"). This is something like an award. Don't let the word "critical" distract you from the context, which is that his paintings were also described as "iconic" and "now-famous." This means that lots of people liked his work, even critics, who gave him acclaim (praise).
4. A. *Supporting Idea*. We know from lines 38 to 51 that Pollock first moved to Long Island before experimenting with new techniques, "including his now-famous 'drip' method of painting." We're not told that Pollock came up with the nickname for himself. We know that his works are on display at the Metropolitan Museum of Art, but we don't know that he donated them. His later works didn't sell well because they weren't similar enough to his most famous works (lines 59-61).
5. B. *Inference*. We know that Pollock didn't do well in school, but don't know that he pursued art because of this. We're told that he studied art "despite" his lack of success in an academic setting. The author signals that the artist was influenced heavily by his mentorship under Benton, and that "art historians believe that Pollock's artistic independence is due to" his relationship with Benton. So, we can infer that Pollock didn't realize this, but was nonetheless influenced by Benton. We're not told what Pollock would have done had he not followed his brother. We aren't provided information to know whether or not he would have ended up in New York City anyway.
6. C. *Tone/Language/Style*. The author adopts an objective, or fair, tone throughout the passage. He doesn't use language to suggest that he is critical of Pollock's life or works. Nor does he adopt language which is particularly serious or, on the other hand, humorous. Instead, he paints a fairly objective and unbiased view of Pollock.

## Passage 22

1. C. *Main Idea*. The dangers of the glacier aren't a focus of the passage. The passage does describe the blueness of a glacier, but this is done in context of the broader experience itself. There's no indication that the experience is a dream. From what we know, it actually happened to the writer.
2. A. *Tone/Language/Style*. The first paragraph compares the glacier with a giant sapphire. The author uses figurative language to help convey the color and brilliance of the surrounding ice.
3. D. *Supporting Idea*. The author, in the third paragraph, points to the elevation ("only a thousand feet above sea level"), temperature ("warmer than I had expected"), and landscape ("temperate rainforest") as examples of why the glacier he hiked was unique. The author never mentions that the glacier is unique because of the number of visitors to it.
4. B. *Organization/Logic*. The author never describes being nervous, instead saying that he needed to hold his breath in order to "slide between the two walls" (line 33). This suggests that it is very narrow where he is.
5. A. *Inference*. We can tell that the glacier is very old, but we don't know how old. We don't know why the author was walking the glacier, nor do we know how warm it was. However, we are told that glaciers form when snow becomes "ice under its own weight" (lines 42-43).
6. C. *Vocabulary*. The word "profound" is used in this context to emphasize how the experience was special, and will be "forever burned into" the author's memory. This means that it was a remarkable, or touching experience. The author doesn't seem to have regrets. An "exaggerated experience" doesn't make sense.

## Passage 23

1. B. *Main Idea*. The passage describes a problem and a potential solution. This is that people can help solve the problem of global warming by taking advantage of or helping along existing natural processes. The fact that phytoplankton have regulated the atmosphere for a long time, and the fact that people can fertilize the oceans, are mentioned as facts as part of the passage's broader point.
2. A. *Inference*. The author states that a warming climate leads to "powerful and destructive storms." In addition, she indicates that a warming climate is partly caused by increased greenhouse gases in the atmosphere. Therefore, we can infer that fewer greenhouse gases in the atmosphere would mean less powerful storms. We don't know that reducing greenhouse gases would decrease carbon sinks, increase plankton (in fact, we would expect the opposite), or increase iron in the world's oceans.
3. B. *Supporting Idea*. In the third paragraph, the terms "carbon sink" and "carbon source" are defined. We know that the processes that "absorb more carbon dioxide than is given off" is called a carbon sink (lines 28-30). The opposite of this – things that increase the amount of carbon dioxide in the atmosphere – are called carbon sources (lines 26-28).
4. A. *Vocabulary*. The phytoplankton are numerous – so numerous that they produce between 50-85% of the world's oxygen. The word "ample" means plentiful, the opposite of "scarce." The phytoplankton are important, but in context, the appropriate synonym for "abundant" is "ample."
5. D. *Tone/Language/Style*. The author seems shocked or even fascinated by the fact that phytoplankton have been around for so long. The fact that the author uses an exclamation point at the end of fourth paragraph gives us this clue, as do the words she chooses to use to describe their age. The author doesn't seem angry or in a rush (frantic), nor even particularly grateful. Instead, the author is awestruck and impressed by this fact.
6. C. *Organization/Logic*. The last paragraph tells us that to increase the number of phytoplankton, we need more than just carbon dioxide. This is because the phytoplankton feed also on other nutrients, like iron.

#### Passage 24

1. D. *Main Idea*. The passage describes one person's experience with a strong storm. Different aspects of the storm are discussed, but the point of the story isn't simply to list the fact that storms have lightning and thunder. Nor is it simply to tell the reader how to prepare for a storm. The passage never gets into the causes of the thunderstorm. Instead, the passage is about all of these things, but from the point of view of one person's experience.
2. B. *Supporting Idea*. The passage tells us that it is because of the author's various senses – what he saw, heard, and felt – that he knew that the storm was coming. There's no mention of neighbors, newspapers, or televisions, only the author's own personal observations.
3. B. *Vocabulary*. The word "campaign" refers to some sort of great effort (think of a political campaign, or an advertising campaign). In this case, the word is used figuratively to describe the wind, which we can infer is blowing very hard (hard enough to shake the storm shutters violently).
4. A. *Tone/Language/Style*. The author uses figuratively language here to compare the few drops of rain that quickly became a "deafening deluge." The dependent clause "a million...roof" can't stand alone as a sentence, and is dependent on the first part of the sentence, which describes the rain. This means that the million nails refers to the rain.
5. C. *Organization/Logic*. The author writes that "the branches of a tree – a victim of the wind or lightning – fell to the earth, taking with it the electrical lines that powered my home." This suggests that the power outage was caused not by thunder or rain, or construction (which wasn't mentioned at all), but the wind or lightning. There are no literal giants in the sky, only the sound of thunder which is loud like "a giant's war drums."
6. D. *Inference*. We don't know that preparation can prevent storms, only that they'll help get the people ready for the storms. We don't know that all clouds announce thunderstorms, since we know that there are white and fluffy clouds as well as dark and angry clouds. The passage doesn't

tell us outright, but we can assume by the author's actions that there is little to do but wait once prepared for a storm. This is what the author does, according to the last sentence.

## Passage 25

1. C. *Main Idea*. The author makes it very clear that trees have many benefits, and can actually help reduce the cost of things like air conditioning. Of the many benefits provided by trees, the author doesn't take a stance as to which is most important. It is clear, especially from the opening and closing paragraphs, that the author feels that many people do not appreciate the trees for what they do for us.
2. B. *Supporting Idea*. The author mentions energy savings in the second paragraph. If trees are planted around buildings, they will be able to save people energy by shading buildings and reducing the cost air conditioning and energy consumption. The other answer choices are mentioned in the passage, but not in terms of saving energy.
3. A. *Tone/Language/Style*. The author uses the imagery of "giant underground nets" to describe what the network of roots underground looks like. The preceding sentence mentions "deep roots" that "hold soil together."
4. C. *Vocabulary*. In context, we know that the sentence is talking about fruits that are fit for humans to eat. This means that we are talking about an action that trees do to make, or produce, fruit. The word "bear" has another meaning, which is to support or carry. But, in this case, the idea of a tree producing fruit is more accurate.
5. D. *Inference*. The author describes how trees can feed people and help them make medicine (fifth paragraph), as well as how trees help prevent erosion (fourth paragraph). It doesn't mention explicitly how trees are scattered or dispersed in different climates.
6. A. *Organization/Logic*. The passage describes an opinion, or a particular point of view. The author supports this opinion with facts, and doesn't bother to disprove any dissenting or opposing views. There are many theories described, but none are discredited. The passage doesn't concern itself with a particular problem, instead describing benefits of trees and how they can help in a variety of different ways.

## Passage 26

1. D. *Main Idea*. The passage is about Justin's trip to the Golden Gate Bridge, and how it differed in person versus what he had read about in the guidebook. The main theme is about how Justin was consistently drawing a comparison between what he had read and what he experienced, not about whether or not San Francisco is beautiful, or which bridges in particular are the most beautiful.
2. B. *Vocabulary*. The word is used to point out that the color of the bridge is particularly unique, or distinctive. This isn't to say that it is necessarily bright or obvious (and it certainly wouldn't be familiar). Instead, it simply means that it is very different.
3. B. *Inference*. We don't know when Justin purchased his airplane ticket (if he even took a plane) or how long his friend had been living in San Francisco. Instead, all we know is that he had read about his trip in a travel guide (mentioned in line 6). This implies that Justin had prepared in advance. We don't know that Justin did any training on his bike.
4. A. *Supporting Idea*. The fifth paragraph describes the beauty of what Justin saw, and compares his expectations with reality. We know this because the sentence states that the "travel guide did little to prepare [Justin] for the graceful beauty of the bridge." The paragraph repeatedly states that "it didn't matter" what was said in the book, since the real experience was so different. The actual appearance of the Bridge is mentioned, but it is also mentioned in earlier paragraphs. Instead, the fifth paragraph uses the actual description as a way to compare it against what Justin was expecting and not expecting.
5. D. *Tone/Language/Style*. The phrase is used to describe how much the bridge stands out in contrast to its surroundings. The colors of the sunset aren't described until later in the passage.
6. C. *Organization/Logic*. The passage describes a personal experience, not an opinion. No travel tips are shared, nor are there directions to be followed.

## Passage 27



1. D. *Main Idea*. The author makes it clear that he doesn't believe that everyone needs to have a grass lawn, even though he himself enjoys lawns. He points out how bad lawns are for the environment, and how expensive they can be. He questions whether or not this norm is a good one. He doesn't believe, then, that gardening makes everyone happy, or that appearances are more important than other things.
2. D. *Tone/Language/Style*. The author is recalling fond memories from his childhood, which are far from regretful or apathetic. Instead, they are sentimental, and full of emotion.
3. A. *Vocabulary*. The word "dismayed" is negative (the prefix "dis-" gives us this clue), so we can rule out "encouraged" and "inspired." "Unfazed" means that someone is not moved one way or another. This is not the case, since the author clearly had positive feelings toward lawns as a child, and no longer does after hearing about how grass lawns have "many negative effects on the environment." Therefore, we could say the author is disappointed.
4. D. *Inference*. The author questions whether everyone should feel pressured to maintain grass lawns. Therefore, he doesn't feel that they are irreplaceable. He also says that we shouldn't fight nature to make sure we have lawns, which means not using irrigation to water lawns, but instead working with plants that are better suited to dry areas. This logic follows also for fertilizers, which shouldn't be used to unnaturally help grass grow. If grass lawns are being grown, however, he would advocate using lawn mowers to help prevent a lot of the air pollution that comes from maintaining and taking care of grass lawns.
5. B. *Supporting Idea*. The author mentions wasting water (lines 22-24), lawn mower pollution (lines 30-32), and time (lines 27-30) as being reasons why maintaining lawns is wasteful. He does not mention the fact that lawns are visually pleasing. This by itself is not a reason why lawns are wasteful, though it is used as justification for the other wasteful practices.
6. D. *Organization/Logic*. The third paragraph includes several facts and statistics to support the idea that maintaining grass lawns "can have many negative effects on the environment." The opinion was presented in the second paragraph. This paragraph focuses on supporting that opinion.

## Passage 28

1. C. *Main Idea*. The beauty of the moai and the fact that islands in the Pacific (including Easter Island) are frequently visited aren't really the focus of the passage. Instead, the passage describes a historical mystery – how the moai were transported around the island – which remains unsolved. The "walking" technique is just one of several theories.
2. A. *Tone/Language/Style*. The author's tone when describing the information about the moai is academic. That is, it is very unbiased, and factual. The author only reports the information, not using it to support any particular point of view. The author doesn't seem confused about this information, or particularly proud.
3. C. *Vocabulary*. Since the word "outlandish" is what critics considered the stories to be, then we know that outlandish can't be a positive word. So, we can rule out "true" and "reasonable." "Not unreasonable" is the same as "reasonable." We're left with "unbelievably strange," which makes the most sense if we substitute it back into the sentence.
4. A. *Supporting Idea*. For some, the walking experiment described in fifth paragraph proved that the moai were shuffled around the island. However, some people didn't agree, as we are told in the sixth paragraph. These people would have believed that there needed to be another explanation, since the moai were much heavier than the replicas used in the experiment. It's not necessarily the case that opponents to the walking experiment supported magic, or that they support the conveyor belt system theory.
5. C. *Inference*. Oral histories aren't described in this paragraph, nor is the weight of the stone. Instead, the paragraph describes how there were no trees on Easter Island – so how could there have been a conveyor belt system?
6. D. *Organization/Logic*. The reason there is a question of how the moai were transported around the island is because the moai were largely carved from the same rock quarry (lines 14-16). There were oral stories about how the moai walked around, but this isn't the archaeological reason why



we know they were transported around the island. If the Europeans had witnessed the construction and transportation of the moai, then we would know how they were moved. There are no inscriptions on the moai described.

#### Passage 29

1. C. *Main Idea*. The relationship between the author and Charlie was friendly, but they weren't friends. We know this from lines 20-21, where the author states that they "weren't exactly friends, but we didn't dislike each other, either." So, we know that a friendship isn't being recalled. No specific scientific theories or processes are discussed at all. Instead, we are told about a memory from school.
2. B. *Vocabulary*. To plead is to ask for something very emotionally or earnestly. Charlie was in a panic, and was looking to the author for help. We could say that Charlie was desperate to avoid the experience of dissecting a frog. Charlie wasn't approving or passive at all. Instead, Charlie was scared and frantic.
3. B. *Inference*. Charlie's surprise mentioned in these lines shows that the author's generosity to do all of the work was unexpected. We know that Charlie didn't enjoy the project, and that the author did enjoy the project. We don't know that the author's generosity was the result of his teacher's instruction. All we know is that the author and Charlie have argued about dividing up school work before, so the fact that the author would agree to do all of the work would be surprising to Charlie. Charlie's surprise at the nice gesture actually suggests that the Charlie thought the author was being nice, not the opposite.
4. B. *Organization/Logic*. The passage states that "Charlie bolted out of the classroom" (lines 55-58) after the author "lengthened the incision" and cut open the frog. We know that the frog was on the tray, and that Charlie was still with the author at the time, not yet having run out of the room. There were no mean words exchanged between Charlie and the author. On the contrary, we're told that the author was nice to Charlie (lines 40-44).
5. B. *Tone/Language/Style*. The author compares Charlie's face after seeing the author begin to dissect the frog with the color of the frog itself. We don't know what the color of the frog is, but we can assume it is something that a normal, healthy human skin color should not be. This means that this isn't Charlie's normal appearance, but instead his reaction to the beginning of the project. The author wasn't mean-spirited, nor was there an action described.
6. D. *Organization/Logic*. There is no particular point of view that the author is advancing with facts. Nor are there a number of different opinions which are being compared. Instead, the author describes events chronologically after first sharing more about the relationship between two students.

#### Passage 30

1. C. *Main Idea*. The passage is about how we go from plants to flour. The author describes various parts of this process, which includes harvesting, threshing, and winnowing. The tools used in each of these are mentioned as part of each process, but the focus of the passage is on the process itself, not on the tools.
2. A. *Organization/Logic*. The passage tells us grain is first harvested, then threshed, then winnowed. The passage doesn't mention drying in the sun. The passage tells us that grain can't be milled into flour before it is harvested, threshed, and winnowed.
3. A. *Supporting Idea*. The scythe and sickle are mentioned in the second paragraphs as examples of tools that helped people save time in the harvesting process. The scythe and sickle aren't mentioned in the context of milling flour, or in threshing or winnowing.
4. D. *Organization/Logic*. We're told in the third paragraph that the threshing process takes a long time. Just as people used tools like sickles and scythes to harvest, people used tools to thresh. In addition to the flail, people used animal labor to help save time.
5. D. *Tone/Language/Style*. The author keeps an impartial tone throughout the passage. He doesn't use language to suggest that he is fascinated or bored by the topic. Instead, he relays this information without opinion or bias, very methodically and logically.

6. C. *Vocabulary*. We can tell from context that winnowing was a time consuming process, especially since the process is likened to harvesting and threshing, which we know are time consuming processes. This is the reason why many tools were invented. Therefore, we can infer that something that is arduous is difficult and tiring, not enjoyable or particularly beautiful or artistic. An arduous task is not necessarily repetitive, though this could be the case.

## Numbers & Operations

### Integers

- C. Thanks to the commutative property, we can rewrite this expression as  $1,650 - 952$ . This gives us 698.
- B. Remember PEMDAS to observe proper order of operations: “Parentheses, Exponents, Multiplication and Division, and Addition and Subtraction.” Step one (parentheses) gives us  $6 \div 3 + 4 \times 3$ . Step two (multiplication and division) gives us  $2 + 12$ . Step three (addition) gives us 14.
- C. A and D are both evenly divisible by 3, and can be eliminated immediately. Simply use long division on B and C.  $55 \div 3 = 18$  remainder 1.  $65 \div 3 = 21$  remainder 2.
- C. To square a number, multiply it by itself (for example,  $3 \times 3 = 9$ ). Any number ending in 3 or 7, upon squaring, will leave a units digit of 9. Any number ending in 9 will leave a units digit of 1.
- A. The least number of ducks Elmo could have would be  $4 + 4 \times 4 = 20$ , and the most would be  $4 + 8 \times 4 = 36$  (remember to include the 4 original ducks). Thus,  $36 - 20 = 16$ .
- B. When multiplying large numbers, we start by multiplying the right most digits. In this case, this is the units digits of 4 and 9, which gives us a product of 36. The units digit then ends in 6, so we need only look for the answer choice with 6 in the units digit.
- D. The leopard sleeps 36 hours over the three days, since  $12 \times 3 = 36$ . The leopard slept 10 hours on the first day, so  $36 - 10 = 26$  hours of sleep remain for the leopard over the next two days.
- C. If 6 cats can catch a total of 12 mice in 1 day, then 12 cats can catch a total of 24 mice in 1 day (multiply both quantities by 2).  $96 \div 24 = 4$ , so it would take 12 cats 4 days to catch all 96 mice.
- C. Start with the largest number and use process of elimination. Clearly 99 is not prime; it is divisible by 9 and therefore by 3 as well. To test whether or not 89 is prime, you only need to test to see if it is evenly divisible by 2, 3, 5, or 7 (every other even number – like 4 – is covered by 2, since every even number is divisible by 2). So, 2 and 5 do not divide 89 evenly, since the last digit is neither even nor equal to 5. The sum of the digits is 17, which is not a multiple of 3. And  $7 \times 12 = 84$ , which is 5 less than 89, so it is not a multiple of 7 either. Since 89 is not a multiple of 2, 3, 5, and 7, then it is prime.
- D. Adding together  $1 + 2 + 3 \dots$  would take a long time. The sum of all positive integers up to some number  $n$  can be determined by the formula  $\frac{n(n+1)}{2}$ . In this case, let  $n = 15$ :  

$$\frac{15(15+1)}{2} = \frac{15(16)}{2}$$
 This gives us 120. If you don’t know this formula, you can work with a technique called pairing. Pair the smallest number in the range with the largest, and so on. So,  $1 + 15$ ,  $2 + 14$ , etc. If you keep doing this, you’ll find that there are 7 pairs, all of which add up to 16. So, we know the sum is  $16 \times 7 = 112$ . 8 is left over, so  $112 + 8 = 120$ .
- A. Column A is greater because the smallest possible choices are 1 and 3, and their product is 3, already greater than 2. The products get bigger after that.
- B. Column A is equal to  $1 + 2 + 4 + 8 + 16 = 31$ . Column B is equal to  $2 \times 2 \times 2 \times 2 \times 2 = 32$ .

### Fractions

- A. The shaded portion represents a total of 4 of the 10 divided parts of the circle. This can be expressed as  $\frac{4}{10}$ , which simplifies to  $\frac{2}{5}$ .

2. B. When multiplying fractions, simply multiply across the numerator and denominator and simplify. In this case,  $\frac{5}{6} \times \frac{1}{4} = \frac{5}{24}$ .
3. D. When dividing fractions, simply multiply the dividend  $\left(\frac{7}{9}\right)$  by the reciprocal of the divisor  $\left(\frac{5}{6}\right)$ . In this case:  $\frac{7}{9} \times \frac{6}{5} = \frac{42}{45}$ . We can simplify by dividing both the numerator and denominator by 3 for  $\frac{14}{15}$ .
4. C. To add fractions, both denominators must be the same. Find the lowest common denominator if possible, or else multiply the two denominators together to find a common denominator. In this case, the two are the same:  $2 \times 3 = 6$ . So, we want the denominator of both fractions to be 6. If we multiply the denominator by a number, we must multiply the numerator by the same number, or the value of the fraction changes. So:  $\frac{3}{3}\left(\frac{1}{2}\right) + \frac{2}{2}\left(\frac{2}{3}\right) = \frac{3}{6} + \frac{4}{6}$ . Thus,  $\frac{7}{6}$ .
5. A. To subtract fractions, both denominators must be the same. Find the lowest common denominator if possible, or else multiply the two denominators together to find a common denominator. In this case, the two are the same:  $3 \times 5 = 15$ . If we multiply the denominator by a number, we must multiply the numerator by the same number, or the value of the fraction changes. So:  $\frac{3}{3}\left(\frac{4}{5}\right) - \frac{5}{5}\left(\frac{2}{3}\right) = \frac{12}{15} - \frac{10}{15}$ . Thus,  $\frac{2}{15}$ .
6. D. We can rewrite the expression with a common denominator:  $\frac{16}{16} + \frac{8}{16} + \frac{4}{16} + \frac{2}{16} + \frac{1}{16} + \frac{1}{16}$ . This gives us  $\frac{32}{16} = 2$ .
7. D. Create improper fractions first by converting the mixed fractions into improper fractions.  $3\frac{2}{3} \times 2\frac{1}{2} = \frac{11}{3} \times \frac{5}{2}$ . Then, simply multiply across the numerators and denominators:  $\frac{11}{3} \times \frac{5}{2} = \frac{55}{6}$ . Reduce if necessary, converting back to a mixed fraction if necessary.
8. A. A common denominator of 81 gives us  $\frac{27+9+3+1}{81}$ , or  $\frac{40}{81}$ . This simplifies to just under  $\frac{1}{2}$ , which means  $x$  is between 0.40 and 0.50.
9. D. If  $\frac{1}{4}$  of the pencils are used, then  $\frac{3}{4}$  are not.  $\frac{3}{4} \times 20 = 15$ .
10. B. If  $\frac{1}{3}$  of the bag has been eaten, then  $\frac{2}{3}$  has not. So,  $\frac{2}{3} \times 10 = \frac{20}{3}$ . This simplifies to  $6\frac{2}{3}$ .  
There are 6 whole pieces of chocolate left (as well as  $\frac{2}{3}$  of another piece).
11. A. Since there are 12 baseballs in total, there must be  $12 - 7 = 5$  hockey pucks in total. This means there are a total of  $12 + 5 = 17$  baseballs and hockey pucks. The question asks about the fraction of the duffel bag that contains hockey pucks, which must be  $\frac{5}{17}$ .
12. C. After Alex ate,  $\frac{2}{3}$  of the pie was left.  $\frac{2}{3}$  of 15 is 10. After Bob ate  $\frac{2}{5}$  of 10 slices, which accounts for 4 slices, only 6 are left.
13. A.  $\frac{1}{3} + \frac{2}{5} = \frac{5}{15} + \frac{6}{15}$ . This means that  $\frac{11}{15}$  of the punch was either coconut water or cranberry juice, leaving  $\frac{4}{15}$  other ingredients.

14. C. Since the recipe makes one cup of granola, and we know Gavin made 6 cups of granola, we can multiply the sum of oil and honey in the recipe by 6:  $6 \times \left(\frac{1}{3} + \frac{1}{6}\right)$ . This results in  $\frac{6}{3} + \frac{6}{6} = 3$ .
15. C. Two ways to solve this question might be to perform long division on each fraction, or else memorize the decimal form of each fraction. Another way would be to recognize that, when fractions are written in lowest terms, the decimal form of the fraction will terminate only if the denominator's prime factorization contains nothing but 2's and/or 5's. Only 8, in the denominator of  $\frac{1}{8}$ , fits this criteria (the prime factorization of 8 is  $2^3$ ).
16. C. If there are only cats and dogs being sold in the store, and  $\frac{4}{9}$  of the total represents dogs, then  $\frac{5}{9}$  of the total represents cats. This means that the number of cats must be a multiple of 5 (otherwise we would end up with a fractional number of cats). Of the 4 answer choices, only 20 is a multiple of 5. We can check this by substitution: if we know that there are 20 cats, and there are 5 cats for every 9 animals, then there must be  $4 \times 9 = 36$  animals in total.  $36 - 20 = 16$  dogs, which is 4 less than the 20 cats.
17. A. If we suppose that  $x$  represents the total number of pages in the book, then we could say that  $x = \frac{1}{2}x + 90 + \frac{1}{8}x$ , since  $\frac{1}{2}x$  represents what was read on Monday, 90 what was read on Tuesday, and  $\frac{1}{8}x$  what remains to be read. First, solve for  $x$  by combining like terms. To add fractions, create a common denominator:  $x = \frac{4}{8}x + 90 + \frac{1}{8}x$ . This simplifies to  $x = \frac{5}{8}x + 90$ , and then to  $\frac{3}{8}x = 90$ . Multiply both sides by 8 and divide both sides by 3 to isolate  $x$ . This gives us for  $x = \frac{720}{3} = 240$ . The question asks how many pages are left, which we know is  $\frac{1}{8}x$ , or  $240 \div 8 = 30$ .
18. A. Use long division to convert Column B into an improper fraction:  $\frac{18}{7} = 2\frac{4}{7}$ . Now we can ignore the leading 2's and compare the remaining fractions:  $\frac{7}{12}$  and  $\frac{4}{7}$ . Perform long division, or create a common denominator of 84 and compare:  $\frac{49}{84} > \frac{48}{84}$ . Therefore, Column A is larger.
19. D. Determine the value of Column A by creating a common denominator and simplifying:  $\frac{6}{6} - \frac{3}{6} - \frac{2}{6} = \frac{1}{6}$ . The equation in Column B actually has two solutions: one that is positive and one that is negative. We solve the equation of Column B by taking the square root of both sides of the equation:  $\sqrt{x^2} = \frac{\sqrt{1}}{\sqrt{36}}$ . This gives us  $x = \pm\frac{1}{6}$ . This is true because squaring  $\frac{1}{6}$  gives us  $\frac{1}{36}$ , as does squaring  $-\frac{1}{6}$ , since multiplying a positive by a positive and a negative by a negative give us positive values. Therefore, the value in Column A is actually greater than or equal to the value in Column B, which means we cannot know for sure whether Column A or Column B are greater.
20. A. Use long division to recognize that Column A is equal to 0.18181818 (repeating). There are two distinct digits: 1 and 8. Use long division to recognize that Column B is equal to 0.77777 (repeating). There is only one distinct digit: 7. Therefore, Column A is the greater quantity.
21. B. The decimal form of  $\frac{1}{7}$  contains a pattern of 6 repeating digits: 0.142857... Therefore, the value in Column B is greater.



## Decimals

1. B. The tenths place in  $W$  and  $X$  are greater than those in  $Y$  and  $Z$ . In  $X$ , the hundredths place is 2, whereas it is 0 in  $W$ . Remember that with decimals, tenths are greater than hundredths, and hundredths are greater than thousandths, etc. The bigger the value in each place, the greater the value of the decimal.
2. A. Make sure the decimal points are aligned with each other and that values are carried appropriately. To be sure, write as  $12.600 - 2.770 + 0.004$ .
3. B. Note that no actual calculation needs to be performed to solve this question. The digits in each answer choice are the same; only the decimal place is in a different place. We know that, approximately,  $1 \div 1 = 1$ .
4. C. To perform  $0.16 \div 0.008$ , or the equivalent  $0.160 \div 0.008$ , move the decimal point three places to the right in both the dividend and divisor to obtain  $160 \div 8$  which is 20.
5. C. Either perform long division ( $5.000 \div 8.000$ ), or note that  $\frac{1}{8} = 0.125$ , and that  $\frac{4}{8} = \frac{1}{2} = 0.5$ , the sum of which is 0.625.
6. D. The mixed number  $4\frac{1}{5}$  is equal to 4.2 (because  $1 \div 5 = 0.2$ ). Multiplying 1.25 by 4.2 gives us a product of 5.25. Remember to carry values and adjust decimals appropriately.
7. D. When dividing by a decimal, we can simplify by moving the decimal point of both the dividend and divisor the same number of spaces to make a whole number. In this case, 1.01 becomes 101, and 0.01 becomes 1 (we moved the decimal over to the right two times). This gives us  $101 \div 1 = 101$ . When multiplying by a decimal, we can first pretend there is no decimal, multiply as if the numbers were integers, then add back the decimal to the product. In this case, we have  $101 \times 0.02$ . Ignoring the decimal:  $101 \times 2 = 202$ . Then, adding back the decimal gives us 2.02.
8. C. Multiply both sides of the expression by  $n + 1$  to arrive at  $n = 0.95(n + 1)$ . Distribute the 0.95 to get  $n = 0.95n + 0.95$ . Simplify by combining like terms to arrive at  $0.05n = 0.95$ . To make things easier, multiply both sides by 100 to get  $5n = 95$ . Simply divide by 5 to arrive at  $n = 19$ .
9. B. The question asks what we need to multiply 5 by in order to get 6. Algebraically, this can be represented as  $5 \times x = 6$ , or  $5x = 6$ . To solve for  $x$ , we divide both sides by 5. This gives us  $x = \frac{6}{5}$ .  
We can see that as a mixed number, this would be  $1\frac{1}{5}$ , which is equivalent to 1.2.
10. D. The question asks us what number times 16 will give us 6. 6 is less than half of 16, so we can eliminate everything immediately except for D. Or, we can represent the question algebraically as follows:  $16 \times x = 6$ , or  $16x = 6$ . Solve for  $x$  by dividing:  $x = 6 \div 16$ , or  $x = \frac{6}{16}$ , which simplifies to  $x = \frac{3}{8}$ . Using long division, we can find that this equals 0.375. Alternatively, if we know that  $\frac{1}{8} = 0.125$ , then we know that  $3 \times 0.125 = 0.375$ .
11. B. We can set up a proportion to solve this question:  $\frac{25 \text{ arn}}{22 \text{ dollars}} = \frac{11 \text{ arn}}{x \text{ dollars}}$ . Cross multiply to arrive at  $25x = 242$ , then use long division to find  $x$ . This is  $242 \div 25 = 9.68$ .
12. C. We can use long division to find that  $\frac{8}{25} = 0.32$ . Or, using long division, we can find that  $\frac{1}{25} = 0.04$ , and  $8 \times 0.04 = 0.32$ .
13. B. There are several ways to solve this question, one of which is to convert the decimals into fractions. This gives us  $0.\overline{33} = \frac{1}{3}$ . Since we know that  $0.25 = \frac{1}{4}$ , we can set up an equation to find

the equivalent of 2.25. This would be  $2.25 = \frac{1}{4}x$ . When we multiply both sides by 4, we end up with  $x = 9$ . Thus,  $2.25 = \frac{9}{4}$ . Now, multiply the fractions:  $\frac{1}{3} \times \frac{9}{4} = \frac{9}{12}$ , which simplifies to  $\frac{3}{4}$  which is less than 1.

14. B. Find the value described in Column A. Since Andrew had two different coins, he can't have two of the same coin in his pocket. Since a quarter, dime, nickel, and penny have values of 0.25, 0.10, 0.05, and 0.01, respectively, we know that the largest combination of coins will be a quarter and a dime, or  $0.25 + 0.10 = 0.35$ . The smallest combination would be a penny and a nickel, or  $0.01 + 0.05 = 0.06$ . The difference, then, is  $0.35 - 0.06 = 0.29$ , which is less than 0.48 in Column B.

15. A. Since both columns include 250 in the expression, we only need to look at the decimals to see that the value in Column A will be bigger. This is because the decimals in Column A are bigger, which means that when multiplied by a whole number, the resulting product will be larger.

16. A. We don't know the original number of frogs, but we can find out by representing the original number of frogs with a variable,  $x$ . Then, since the 15 frogs that escaped represents 0.75 times the original number of frogs, we know that  $15 = 0.75x$ . We then divide both sides by 0.75.

Alternatively, if we know that  $0.75 = \frac{3}{4}$ , we can solve as follows:  $15 = \frac{3}{4}x$ . Multiplying both sides by  $\frac{4}{3}$  gives us  $20 = x$ . This is greater than 11.25 in Column B.

17. A. We know that  $0.5 = \frac{1}{2}$ ,  $0.25 = \frac{1}{4}$ , and  $0.75 = \frac{3}{4}$ . Since dividing by a fraction is the same as multiplying by its reciprocal, we can rewrite the expression as  $\frac{800}{1} \times \frac{2}{1} \times \frac{1}{4} \times \frac{3}{4}$ . This simplifies to

$\frac{1,600 \times 3}{16} = 100 \times 3 = 300$ . Since the expression only contains multiplying and dividing, we can work in any order that we like.  $10 \times 1.2 = 12$ , and  $12 \times 20 = 240$ . What's left then is  $240 \div 1.5$ .

Either use long division, or convert 1.5 to  $\frac{3}{2}$  and multiply by the reciprocal:  $\frac{240}{1} \times \frac{2}{3} = \frac{480}{3} = 160$ .

18. D. In Column A,  $80n$  is between  $80 \times 0.25 = 20$  and  $80 \times 0.75 = 60$ , but we don't know what it is, exactly. In Column B,  $20m$  is between  $20 \times 2.5 = 50$  and  $20 \times 3.5 = 70$ . Since there's a range of 20 to 60 in Column A, and a range of 50 and 70 in Column B, we don't know for sure which value is greater.

19. A. Assess each column separately. In Column A, we know that  $n$  must be between 5 and 10, which means  $n$  must be either 6, 7, 8, or 9. We can test each fraction that results ( $\frac{1}{6}, \frac{1}{7}, \frac{1}{8}, \frac{1}{9}$ ) by using long division. If we do, we find that only  $\frac{1}{8}$  terminates (at 0.125). Therefore,  $n = 8$ .

Column B, we are given an equation. Simply solve for  $m$  by multiplying both sides of the equation by 30. This gives us  $m = 6$ . Therefore, Column A is greater.

## Percents

- A. A is not equal to  $\frac{22}{16}$ ; although  $\frac{44}{32}$  is equal to  $\frac{22}{16}$  as a fraction, as a percentage,  $\frac{44}{32}\%$  is one hundredth as much as  $\frac{22}{16}$ . Therefore, they are not equal. The other choices represent simplified, mixed-number, or percentage equivalents.
- B. The cat caught a mouse on 3 out of 7 of the days in the week, which means that it did not catch one on 4 of the 7 days. This is slightly more than half, so only choice B makes sense.
- C. First, find the percentage of questions answered incorrectly. This must be  $100\% - 80\% = 20\%$ , since 80% were answered correctly. To find the total number of questions, divide the number of

questions answered incorrectly by the percentage of questions answered incorrectly. This becomes  $10 \div 0.2 = 50$ .

4. D. Read this question carefully! Since we are told that 15 left-handed people are men, then 10 left-handed people must be women. This means that  $\frac{10}{50}$  women are left-handed, so  $\frac{40}{50}$  are right-handed. This reduces to 0.8, or  $0.8 \times 100 = 80\%$ .
5. B. Let  $x$  represent the number that is being decreased. A decrease of 30% can be represented as either  $x - 0.3x$  or  $0.7x$ . For simplicity, let's use the latter format. If the new number ( $0.7x$ ) is decreased by another 30%. This means either  $0.7x - (0.3)(0.7x)$  or simply  $(0.7)(0.7x)$ . Again, for we'll use the latter for simplicity. If we multiply  $0.7 \times 0.7$ , we arrive at  $0.49x$ . But this gives us only the value, not the percentage decrease. We would have to decrease by  $0.51x$  to get to  $0.49x$ , so the answer is 51%. Substitute 100 in for  $x$  to prove this.
6. B. Let  $x$  represent the original number. An increase to  $x$  of 10% can be represented as  $1.1x$ . A decrease to this number of 10% can be represented as  $(0.9)(1.1x)$ . Simplifying gives us  $0.99x$ . The difference between  $x$  and  $0.99x$  is  $0.01x$ , or 1%.
7. C. Remember that in each month, there is a 50% increase in population based on the previous month. We know that there are 80 songbirds to begin with, and that 50% of 80 can be represented as  $80 \times 0.5 = 40$ . So after the first month, there are  $80 + 40 = 120$  songbirds at the end of the first month (alternatively,  $80 \times 1.5 = 120$ ). At the beginning of the second month, there are 120 songbirds, which increases again by 50%. This can be represented as  $120 + 120(50\%)$ , or  $120 + 60 = 180$  songbirds at the end of the second month. At the beginning of the third month, there are 180 songbirds, which increases again by 50%. This can be represented as  $180 \times 1.5 = 270$ .
8. A. The rectangle's original area is  $20 \times 20 = 400 \text{ ft}^2$ . The new length can be expressed as  $20 \times 1.25 = 25$ . The new width can be expressed as  $20 \times 0.85 = 17$ . The new area is  $425 \text{ ft}^2$ . This means the change was  $425 \text{ ft}^2 - 400 \text{ ft}^2 = 25 \text{ ft}^2$ .
9. B. We can express a percentage decrease in a number as that number multiplied by 100% minus the percentage decrease. In this case, we know that the length is decreased by 20%, and the width is decreased by 30%. This means we know we are multiplying the length by  $100\% - 20\% = 80\%$ , and the width by  $100\% - 30\% = 70\%$ . Since area is length times width, we know that we can multiply 80% by 70% to get 56%. This means the new area is 56% of the old area, which means there was a  $100\% - 56\% = 44\%$  decrease in area.
10. C. Let  $x$  represent the number of pieces of garbage that Jordan picked up. In terms of  $x$ , Jerry picked up  $x + 100$  pieces of garbage. Together, we know they picked up 500 pieces of garbage. Therefore,  $x + x + 100 = 500$ . Solve for  $x$ :  $2x = 400$ , and  $x = 200$ . But this is the number of pieces of garbage that Jordan picked up; Jerry picked up 100 more pieces, or 300 pieces of garbage. This represents  $3 \div 5 = 60\%$ .
11. C. The Luthors lost 12 out of their first 15 games, which means they won 3 of their first 15 games. Since they won their next 5 games played, the total number of games won becomes  $3 + 5 = 8$ , and the total number of games played becomes  $15 + 5 = 20$ . Therefore, the percentage of games won can be found by dividing the number of games won by the total number of games played:  $\frac{8}{20} = \frac{2}{5} = 0.4$ , or 40%.
12. D. We must work backwards to determine the number of chocolates originally in the box. After David and Eric took chocolates, 27 remained. Convert percents to decimals, and let  $x$  represent the number of chocolates available to Eric (left after David took chocolates). If we do, we can represent the information algebraically as  $x - (0.25)x = 27$ . Therefore,  $0.75x = 27$ , and  $x = 36$ , which is the number of chocolates left after David took some amount. Similarly, let  $y$  represent the number of chocolates available to David (the number originally in the box), and represent the information algebraically. This gives us  $y - (0.4)y = 36$ . This means  $0.6y = 36$ , and  $y = 60$ .

13. A. There was a total of  $9 \times 4 = 36$  at-bats. This means the team went  $\frac{12}{36}$  which is equal to  $\frac{1}{3} \cdot \frac{1}{3}$  is can be represented as approximately 33%.
14. C. We must work backwards to determine the number of pages in the book to start. After Francis wrote for two weeks, 30 pages remained. Convert percents to decimals, and let  $x$  represent the number of pages left after the first week. If we do, we can represent the information algebraically as  $x - (0.25)x = 30$ . Therefore,  $0.75x = 30$ , and  $x = 40$ , which is the number of pages left after the first week. Similarly, let  $y$  represent the number of pages initially in the book, and represent the information algebraically. This gives us  $y - (0.2)y = 40$ . This means  $0.8y = 40$ , and  $y = 50$ .
15. A. The triangle's original area is  $(10 \times 25) \div 2 = 125$  ft<sup>2</sup>. The new base can be expressed as  $10 \times 0.9 = 9$ . The new height can be expressed as  $25 \times 1.2 = 30$ . The new area is 135 ft<sup>2</sup>. This means the change was  $135$  ft<sup>2</sup> -  $125$  ft<sup>2</sup> =  $10$  ft<sup>2</sup>.
16. B. First, find the percentage of free-throws Sharon missed. This must be  $100\% - 75\% = 25\%$ , since 75% were made. To find the total number of free-throws, divide the number of free-throws missed by the percentage of free-throws missed. This becomes  $20 \div 0.25 = 80$ . Then, subtract the number of total free-throws by the number of free-throws missed to get the number of free throws Sharon made,  $80 - 20 = 60$ .
17. B. As a decimal,  $\frac{1}{5} = 0.2$  (find this by performing long division). As a percent, this is  $0.2 \times 100 = 20\%$ . We can see that this is less than  $22.\overline{22}\%$ , since  $22.\overline{22}\% - 20\% = 0.\overline{22}\%$ . Or, we could recognize that  $22.\overline{22}\%$  as a decimal equals  $0.\overline{22}$ . The values in Column A and Column B have the same value in the tenths places, but not in the hundredths place.
18. C. Since we are looking for the percentage of rotten apples, we divide the number of rotten apples by the total number of apples in the barrel:  $8 \div 40 = 0.2$ . To find the percentage, multiply by 100 for  $0.2 \times 100 = 20\%$ . The columns represent the same quantity.
19. C. There is exactly one dollar on the table:  $0.05(1) + 0.10(2) + 0.25(3) = 1$ . The three quarters total 75 cents, so  $75 \div 100 = 75\%$ . The columns are equal.
20. B. Out of the 20 casts in the shelter, 8 are Burmese cats. This is represented as  $\frac{8}{20}$ , which simplifies to  $\frac{2}{5}$ . Find the decimal form by long division, or remember that  $1 \div 5 = 0.2$ , and  $2 \times 0.2 = 0.4$ . To convert to a percent, multiply by 100, which gives  $0.4 \times 100 = 40\%$ . The number of non-Burmese cats is the total number of cats (20) minus the number of Burmese cats (8). This means there are  $20 - 8 = 12$  non-Burmese cats. Of this amount, 6 are Persian. We can represent this as  $\frac{6}{12}$ , which simplifies to  $\frac{1}{2} = 0.5$ , or  $0.5 \times 100 = 50\%$ , which is greater than 40%.
21. B. There are 6 multiples of 3 less than 20: 3, 6, 9, 12, 15, and 18. Since there are 20 positive integers in total (1-20), then 6 out of this 20 represents  $6 \div 20 = 0.3$ , or 30%, of numbers in the range. This is less than 33%.
22. A. Column A must be less than  $400 \times 0.75 = 300$  and more than  $400 \times 0.5 = 200$ . Column B must be more than  $120 \times 1.25 = 150$  and less than  $120 \times 1.5 = 180$ . All possible values in Column A are larger than anything possible in Column B.
23. C. First, find the number of children who speak each language.  $50 \times 40\% = 50 \times 0.4$ . This equals 20 French speaking children.  $50 \times 30\% = 50 \times 0.3$ . This equals 15 Spanish speaking children. We're told that 20 children speak neither language. In total, this would mean that there are  $20 + 15 + 20 = 55$  children, but we know there are only 50 children., so the remaining 5 must speak both languages.  $5 \div 50 = 10\%$ .
24. A. There are  $6 + 2(6) = 18$  yellow fish in total. If the maximum number of blue fish, 23 fish, were in the pond, there would be  $6 + 9 + 12 + 23 = 50$  fish in both ponds combined.  $18 \div 50 = 36\%$ ,



which is more than Column B. Since 36% is the minimum for the percentage of yellow fish in both ponds (if there were fewer blue fish, then the percentage of yellow fish would be greater), Column A must represent the greater quantity.

### Factors, Multiples, Primes

- A. Use prime factorization to break down 64 into its prime factors. In this case, we end up with 2 & 32, which becomes 2 & 16, which becomes 2 & 8, which becomes 2 & 4, which becomes 2 & 2. In total, this is 6 2's, which makes  $x = 6$ .
- A. The greatest common factor (GCF) is the largest shared factor of each number in a set (in this case, the set is 3 and 17). List the factors of each number (the numbers that multiply together and result in that number). In this case, the factors of 3 are 1 and 3, and the factors of 17 are 1 and 17. The greatest number shared in these two sets of factors is 1.
- D. The least common multiple (LCM) is the smallest integer that is evenly divisible by all of the numbers in a set (in this case, the set is 6, 7, and 11). The safest way to solve this is to break each number into its prime factors. We do this by finding the smallest prime numbers that multiply together to give us the each number. In this case, 7, and 11 are already prime, so there is no factoring that needs to be done.  $6 = 2 \times 3$ , or  $2^1 \times 3^1$ . Then, multiply together the prime number values with the greatest exponents: Thus,  $2^1 \times 3^1 \times 7^1 \times 11^1 = 462$ .
- D. Find the prime factorization of both numbers:  $135 = 3^3 \times 5^1$  and  $75 = 3^1 \times 5^2$ . Then find the highest exponent of each prime factor present in the prime factorization. These are  $3^3$  and  $5^2$ . Multiply the results together:  $27 \times 25 = 675$ .
- A. With large numbers, utilize prime factorization to determine the GCF instead of writing every pair of factors.  $84 = 2 \times 2 \times 3 \times 7$  (or  $2^2 \times 3 \times 7$ ) and  $144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3$  (or  $2^4 \times 3^2$ ). Simply multiply all of the common prime factors to arrive at the GCF:  $2 \times 2 \times 3 = 12$ .
- D. Break down each number into its prime factors. This gives us  $216 = 2^3 \times 3^3$  and  $144 = 2^4 \times 3^2$ . Find the least exponent of each prime factor in both numbers' prime factorizations. These are  $2^3$  and  $3^2$ . Take the product to determine the greatest common factor. Their product is  $8 \times 9 = 72$ .
- B. 90 has only one factor of 2 in its prime factorization, which is  $2 \times 3^2 \times 5$ . 4 is  $2^2$ , which is not present in 90's prime factorization. Therefore, 4 is not a factor.
- A. If the sum of a number's digits is a multiple of 3, then that number is divisible by 3. Adding the digits of 1,383 gives us 15, as adding the digits of 1,353 gives us 12, both of which are multiples of 3, which means they are divisible by 3, and therefore are not prime. 1,313 is clearly divisible by 13 ( $13 \times 101$ ). Only 1,303 is left.
- C. 25 can be expressed as  $5^2$ . When we raise an exponential term by another power, we multiply those exponents together. Thus,  $(5^2)^3 = 5^6$ . Note that we can multiply exponents together like this only when the base is the same.
- D. We can list every factor of 36, or use prime factorization. The complete list of factors is 1, 2, 3, 4, 6, 9, 12, 18, 36. Using prime factorization, we find that  $2 \times 2 \times 3 \times 3 = 36$ . This is also written as  $2^2 \times 3^2 = 36$ . For each distinct factor ( $2^2$ , for example), add one to the exponent and multiply the result together. The two exponents are 2 and 2. Adding 1 to each gives us 3 and 3. The product is  $3 \times 3 = 9$ .
- B. Every prime number only has two factors – 1, and itself. Every number here except for 9 is prime, which means every number except 9 has only 2 factors, which is an even number of positive integer factors. 9 has 1, 3, and 9 as its factors, which is an odd number of factors. We can also list every factor for each number. 7 has factors 1 and 7 only. 11 has factors 1 and 11 only. 13 has factors 1 and 13 only. Only 9 has an odd number of factors.
- A. Use prime factorization to break down 81 into its prime factors. Nine times nine makes 81, and each 9 is made up of two threes. This means that 81 factors to  $3^y$ . This means  $y = 4$ .
- C. The greatest common factor (GCF) is the largest factor of each number in a set (in this case the set is 10, 25, and 205). With the rules of divisibility, we can see that the one factor that these three numbers share, other than 1, is 5. Therefore, 5 is the greatest common factor.

14. B. The least common multiple (LCM) is the smallest integer that is evenly divisible by all the numbers in a set (in this case, the set is 140 and 35). The safest way to solve this is to break each number into its prime factors. We do this by finding the smallest prime numbers that multiply together to get each number.  $140 = 2^2 \times 5 \times 7$  and  $35 = 5 \times 7$ . Then, multiply together the prime number values with the greatest exponents: Thus:  $2^2 \times 5 \times 7 = 140$ .
15. B. Using rules of divisibility, we can see that 100 is not divisible by 3,  $1 + 0 + 0 \neq 3$ .
16. C. We can list every factor of 24, or use prime factorization. The complete list of factors is 1, 2, 3, 4, 6, 8, 12, and 24. This means there are 8 positive unique factors of 24.
17. C. Use prime factorization to find the LCM in each column.  $8 = 2^3$  and  $27 = 3^3$ . Since the highest exponential power for each prime factor is just the prime factorization itself, we take  $2^3 \times 3^3$ , which is just  $16 \times 27 = 432$ . The prime factorization of the numbers in Column B give us  $24 = 2^3 \times 3$  and  $54 = 3^3 \times 2$ . The highest exponents for 2 and 3 are  $2^3$  and  $3^3$ , which are equal to 16 and 27, just as in Column A. Therefore, the two columns have equal values.
18. A. Use prime factorization to find the GCF in each column. The prime factorization of Column A is  $36 = 2^2 \times 3^2$  and  $27 = 3^3$ . Multiplying the common prime factors results in  $3 \times 3 = 9$ . The prime factorization of Column B is  $30 = 2 \times 3 \times 5$  and  $24 = 2^3 \times 3$ . Multiplying the common prime factors results in  $2 \times 3 = 6$ . Therefore, the value in Column A is greater than the value in Column B.
19. B. The largest prime number between 10 and 20 is 19. The prime factorization of  $2,592 = 2^5 \times 3^4$ . So  $x = 5$ , and  $y = 4$ . Therefore,  $xy = (5)(4) = 20$ . Therefore, the value in Column B is greater.
20. D. There are four prime numbers between 1 and 10: these are 2, 3, 5, and 7. The prime factorization of  $100 = 2^2 \times 5^2$ . So  $x = 2$ , and  $y = 2$ . Therefore,  $xy = (2)(2) = 4$ . There are prime numbers below 4 (2 and 3) and above 4 (5 and 7) in Column A, so we can't be sure which one we're picking. The value in A could be greater or less than the value in Column B.

### Estimation

- A. Notice how all the numbers can be easily rounded to the 10s and 100s places. For these questions, we can round these numbers and simplify our estimations. When rounding, we can find  $400 \div (10 \times 20) = 400 \div 200 = 2$ .
- D. We can round these values to the closest 10 places, which gives us:  $(300 \div 30)^2$ . From there, we can divide to get an integer, which we then square.  $10^2 = 100$ .
- B. Round the numbers so they can divide more easily. Notice that 248 is close to 250 and 53 is close to 50, so the rounded numbers are easily divisible. The equation therefore becomes  $5^2 - 2^2$ , or  $25 - 4$ , which is closest to 20.
- D. We know that the square root of 25 is 5, since  $5 \times 5 = 25$ . Since we don't know what the square root of 40 is (there is no integer that, when multiplied by itself, gives us 40), we must estimate. We know that  $6 \times 6 = 36$ , and  $7 \times 7 = 49$ . Since 40 is between 36 and 49, the square root of 40 must be between 6 and 7. Therefore,  $5 + 6 = 11$ , and  $5 + 7 = 12$ .
- A. We can round these values to the closest 10 places, which gives us:  $\sqrt{200 \div 50}$ . From there, we can divide to get an integer, which we can then take the square root of.  $\sqrt{4} = 2$ .
- C. Before we can find the cost, we need to find the approximate area of the lawn. Using the medians of the length (13 feet) and width (9 feet) ranges, we can estimate an area:  $13 \times 9 = 117$ , or approximately 120 square feet. Multiply by \$2 per square foot to find the approximate answer.  $120 \times 2 = 240$ .
- C. If we divide the area (approximately 160 sq. ft.) by the number of tiles (approximately 20), we can find the area of the tiles.  $160 \div 20 = 8$ .
- A. The increase to the length of the garden can be rounded to 7 ft. So the new dimensions of the garden are  $12 + 7 = 19$  ft. long and  $5 + 5x$  ft. wide. The new area is  $19(5 + 5x) = 200$  ft<sup>2</sup>. Round 19 to 20, and simplify for  $100 + 100x = 200$  ft<sup>2</sup>. This simplifies to  $100x = 100$ , and  $x = 1$ .

9. B. This question requires that we use the formula for volume of a rectangular prism:  $\text{volume} = \text{length} \times \text{width} \times \text{height}$ . We are searching for the volume of water, which can be approximated using the values given. If we multiply the area of the base ( $\text{length} \times \text{width}$ , or 8, as we are already given) by the height, we can find how much rainwater is in the container. Using the median of the range (9), we can multiply  $9 \times 8 = 72$ , or approximately 75.
10. C. To find how much the customer spent on salami, we need to subtract the cost of the cheese from the total. To do this, we need to find the average cost of cheese per ounce. If we take the average of the range, we find that to be \$1.25. If we multiply this cost per ounce by the number of ounces, we discover that she spent a total of \$5 on cheese. That leaves him spending \$10 on salami.
11. D. For this triangle-area question, we are going to round each value to simplify the algebra. If we round the base to  $20x$  inches, the height to 5 inches, and the area to 300 inches, we can evaluate the expression:  $\frac{(20x)(5)}{2} = 300$ . By using orders of operations we can simplify to  $\frac{100x}{2} = 50x$ . From there, we can divide both sides by 50 to find that  $x = 6$ .
12. C. To find circumference, we need to use the formula:  $\text{circumference} = \text{diameter} \times \pi$ . For estimations, we can use 3 instead of  $\pi$ . If we do that and round the diameter to the nearest thousand, we can solve:  $8000 \times 3 = 24,000$ .
13. B. Estimate the average number of dozen donuts she can sell in a day, for each type of day (rainy or sunny). On an average sunny day, we can take the average of the range to find out that she sells about 30 dozen donuts. On an average rainy day, we can use the average of the range to find out that she sells about 12 dozen donuts. To find how much money she makes on each type of day, we multiply the number of dozen by the price per dozen donuts. Thus, she makes about \$300 on an average sunny day and \$120 on an average rainy day. Then multiply by the number of rainy and sunny days, respectively:  $(\$300 \times 4) + (\$120 \times 3) = \$1,560$ , which is closest to \$1,500.
14. B. For this question, we are dealing with a missing value in an addition equation. We can approximate a value within the ranges of swimming and running times by using medians. The median of the swimming times is approximately 53 minutes and the median of the running portion is 100 minutes. If we subtract these from the total time, we can solve for an approximation of her biking time:  $230 - 153 = 77$ . This is closest to 75 minutes.
15. C. To find profit, we need two elements: revenue and cost. The cost is already given by the range \$475-\$575, the median of which is \$525. On average, each of the 50 customers per day is spending  $(\$9.5 + \$12.5) \div 2 = \$11$  per sandwich. This gives us a total revenue of  $50 \text{ per day} \times \$11 \text{ per person per sandwich} \times 11 \text{ days} = \$6,050$ . Total profit will be  $\$6,050 - \$525 = \$5,525$ .
16. B. Randy writes approximately 900 words per day, or 4,500 words per week. He'll spend about 60 weeks writing, for a total of 270,000 words. If there are about 500 words on each page, we can round 270,000 to 250,000 since that is more easily divisible by 500, so his book will be approximately  $250,000 \div 500 = 500$  pages long. His editor will charge about \$12 per page, or  $500 \times 12 = \$6,000$ .
17. A. If we divide the area (approximately 150 sq. ft.) by the number of tiles (approximately 75), we can find the area of the tiles.  $150 \div 75 = 2$
18. B. This question requires that we use the formula for volume of a cylinder:  $\text{volume} = \text{Area of the base} \times \text{height}$ . We are searching for the volume of water, which can be approximated using the values given. If we multiply the area of the base (9) by the height, we can find how much water is in the container. Using the median of the range (13), we can multiply  $13 \times 9 = 117$ , or approximately 120.
19. C. Estimate the average number of pizza pies Andrew can sell in a day for each type of day (rainy or sunny). On an average rainy day, we can take the average of the range to find that he sells about 40 pizza pies. On an average sunny day, we can use the average of the range to find out that he sells about 11 pizza pies. To find out how much money he makes on each type of day, we multiply the number of pizza pies by the tip. Thus, he makes about \$200 on an average rainy day



and \$55 on the average sunny day. Then multiply by the number of rainy and sunny days respectively:  $(\$200 \times 2) + (\$55 \times 5) = \$650$ , which is closest to \$650.

### Numbers & Operations Mixed Practice

- B. (Integers) Eliminate answer choices based on the criteria in the question. The number must have a units digit of 7, so eliminate choice D. The number must also be prime, so use divisibility rules to determine which numbers are prime (only divisible by 1 and itself). 147 is divisible by 3 (the sum of the digits is divisible by 3), so choice C can be eliminated. Use divisibility rules to find that 137 is prime (not even, so not divisible by any even number, and the sum is not divisible by 3 or 9, so 137 is not divisible by those numbers either. 137 does not end in a 5 or 0, so is not divisible by 5 or 10. Division shows that 7 and 11 are also not factors of 137, making it prime. 7 is also prime, but the correct answer is 137 as it is larger than 7.
- D. (Integers) Use number of the lowest and highest values to find that  $1 + 19 = 20$ , then  $2 + 18 = 20$ , then  $3 + 17 = 20$ , etc., making 9 pairs of numbers that sum to 20 ( $9 \times 20 = 180$ ) with the number 10 left without a match ( $180 + 10 = 190$ ).
- B. (Fractions) When multiplying fractions, convert from a mixed number to an improper fraction, multiply across the numerator and denominator, and simplify. In this case,  $\frac{3}{8} \times \frac{7}{3} = \frac{21}{24} = \frac{7}{8}$ .
- D. (Fractions) If  $\frac{2}{5}$  are used, then  $\frac{3}{5}$  are not, so  $\frac{3}{5} \times 50 = 30$ .
- C. (Decimals) We can eliminate A, C, and D right away because 3 divided by 2 is about 1, so the answer should have one in the ones place. No calculation is necessary to solve this if 3.15 is estimated to 3, and 2.8 is estimated to 3. A number divided by itself is 1, so the correct answer will be a little larger than 1. This only leaves choice B.
- C. (Decimals) The question asks us what number times 4 will give 18. Divide 18 by 4 to find that there are 4.5 times fewer penguins than whales.
- D. (Percents) First, find the percentage of questions answered incorrectly. This must be  $100\% - 40\% = 60\%$ , since 40% were answered correctly. To find the total number of questions, divide the number of questions answered incorrectly by the percentage of questions answered incorrectly. This becomes  $30 \div 0.6 = 50$ .
- C. (Percents) Let  $x$  represent the original total amount of milk in the carton. Larry drinks 30%, leaving 70% of the milk, or  $0.70x$ . Then, Larry drinks 20% of the remaining milk, leaving 80% of the remaining milk, or  $(0.80)(0.70)x$ . This is equal to 112mL. So,  $0.56x = 112$ . Divide 112 by 0.56 to find that there was originally 200mL of milk in the carton.
- D. (Factors, Multiples, Primes) We can eliminate answer choice A because it is a number smaller than the two given numbers. In order for a number to be a multiple of a number, it must be equal to or greater than that number. Answer choice B is not divisible by 40, so it cannot be a LCM of 40 and 28. Likewise, Answer choice C is not divisible by 28, so it cannot be a LCM of 40 and 28. Hence, our answer is D, 280.
- C. (Factors, Multiples, Primes) 45,636 does not end in a 0 or a 5, so based on the divisibility rule of 5, the number will not be divisible by 5. Since it is not divisible by 5, 5 is not a factor of 45,636.
- C. (Estimation) Round the numbers so they can divide more easily. 362 rounds to 400, and 18 rounds to 20. Divide to find  $400 \div 20 = 20$ . Then,  $20^2 = 400$ . Subtract  $6^2$  or 36.  $400 - 36 = 364$ .
- C. (Estimation) To find profit, we need two elements: revenue (money made) and cost (money spent). The cost is already given by the range \$325–\$445, the mean of which is \$385. On average, each of the 50 customers per day spent  $(\$7.50 + \$10.50) \div 2 = \$9$  per magazine. This gives us a total revenue of 50 per day  $\times$  \$9 per person per magazine  $\times$  8 days = \$3,600. Total profit was  $\$3,600 - \$385 = \$3,215$ .
- D. (Factors, Multiples, Primes) 16 can be expressed as  $4^2$ . When we raise an exponential term by another power, we multiply those exponents together. Thus,  $(4^2)^5 = 4^{10}$ .



14. A. (Fractions) When dividing fractions, simply multiply the dividend  $\left(\frac{4}{6}\right)$  by the reciprocal of the divisor  $\left(\frac{2}{3}\right)$ . In this case:  $\frac{4}{6} \times \frac{3}{2} = \frac{12}{12}$ . We can simplify by dividing both the numerator and denominator by 12.
15. A. (Integers) When subtracting a negative number, rewrite it as addition:  $-237+783$ . To add a negative and a positive integer, subtract the smaller number from the larger ( $783-237=546$ ) and take the sign of the larger number. Since 783, the larger number, is positive, the result is positive 546.
16. B. (Fractions) Use long division to convert Column B into an improper fraction:  $\frac{67}{14} = 4\frac{11}{14}$ . Now we can ignore the leading 4's and compare the remaining fractions:  $\frac{6}{15}$  and  $\frac{11}{14}$ . Perform long division, or create a common denominator of 210 and compare:  $\frac{90}{210} < \frac{165}{210}$ . Therefore, Column B is larger.
17. A. (Percents) There are 6 multiples of 4 within 24: 4, 8, 12, 16, 20, and 24. Since there are 24 positive integers in total (1-24), then 6 out of this 24 represents  $6 \div 24 = 0.25$ , or 25%, of numbers in the range. This is greater than 24%.
18. C. (Factors, Multiples, Primes) Use prime factorization to find the GCF in the first column. The prime factorization of Column A is  $48 = 2^4 \times 3^1$  and  $60 = 2^2 \times 3^1 \times 5^1$ . Multiplying the common prime factors with the lowest exponent results in  $2^2 \times 3^1 = 12$ . That means, the GCF of 48 and 60 is 12. Use prime factorization to find the LCM in the second column. The prime factorization of Column B is  $4 = 2^2$  and  $6 = 2^1 \times 3^1$ . Multiplying the unique prime factors with the greatest exponent results in  $2^2 \times 3^1 = 12$ . The LCM of 4 and 6 is 12. Therefore, the value in Column A is equal to the value in Column B.

## Algebraic Concepts

### Solving Algebraic Equations

- C. Remember that to solve for a variable, it must be isolated by itself on one side of the equal sign. We do this by performing various operations to the equation to eliminate terms from the side of the equation with the variable. We eliminate terms by canceling them out; positive values are subtracted from, negative values are added to, coefficients are divided to equal one, and divisors are multiplied to equal one. Remember that what is done on one side of the equation sign must be done on the other. To eliminate the 10 from the left hand side, add 10 to both sides of the equation. This gives us  $3x = 21$ . Then, to eliminate the 3 from the left hand side, divide by sides of the equation by 3. This leaves us with  $x = 7$ .
- A. Since a denominator can't be 0, we can cross out choice B immediately. We can simplify the equation to  $25 = \frac{5}{y} - 25$ , and then to  $50 = \frac{5}{y}$  and then  $50y = 5$ . Dividing both sides by 50 gives us  $y = 0.1$ .
- B. We are asked to solve for  $r$ , but because there is another variable,  $q$ , and we do not have additional information, we cannot determine a numerical value for it. So, we can give the value of  $r$  in terms of  $q$ . We do this by isolating  $r$  on one side of the equation. Since  $q$  is positive, we can subtract  $q$  from both sides of the equation to arrive at  $r = 9 - q$ .
- D. If we multiply 75 by 2, we get 150. Since we multiplied one side of the equation by 2, we do so to the other side. So,  $8y \times 2 = 16y$ .
- D. Multiply both sides by  $x$  to arrive at  $xy = 5$ .
- A. We must isolate  $m$  on one side of the equation. Address each term individually. First, we want to get rid of fractions by multiplying the entire equation by the denominator. So, multiply both

sides of the equation by 3. This gives us  $3(10) = \frac{3}{3}m + 3(3)m$ , or  $30 = m + 9m$ . Then, combine like terms.  $9m + m = 10m$ . Therefore,  $30 = 10m$ . Divide both sides by 10, to arrive at  $m = 3$ .

7. A. Remember order of operations. First, add 4 to both sides for  $x^2 = 9$ . Then, take the square root of both sides to find that  $x = 3$ . Note that when we take the square root of something, the result can be either positive or negative (since squaring something will always result in a positive number). Check your answer by substituting  $-3$  in for  $x$ .
8. C. Recognize that there is an “invisible” 1 in front of the first term,  $x (x \times 1 = x)$ . 1 can also be expressed as  $\frac{4}{4}$ . Thus, the equation could be rewritten as  $\frac{4}{4}x - \frac{1}{4}x = 12$ . Now, we can combine like terms and arrive at  $\frac{3}{4}x = 12$ . To cancel out the  $\frac{3}{4}$ , multiply both sides by the reciprocal:

$\left(\frac{4}{3}\right)\frac{3}{4}x = \frac{4}{3}(12)$ . This leaves us with  $x = \frac{48}{3}$ , which simplifies to  $x = 16$ .

9. C. Both sides of the equation are expressed in a similar form, where some number is being multiplied by  $a$  and then that same number is added afterward. We can substitute values from the answer choices to help. If we do, we find that only C is true.
10. A. The arithmetic is very difficult to do. Instead, recognize that the expression in question is half as much as the expression written in the equation. If a denominator is doubled, the value of the fraction is cut in half. This means that the amount of 16 must be half as much, as well.
11. C. The equation is just two equivalent fractions, which is called a proportion. We solve by cross multiplying. First, cross multiply for  $3(2x - 4) = 4(6 + 3y)$ . Distribute the coefficients for  $6x - 12 = 24 + 12y$ . Reduce for  $6x = 12y + 36$  and divide by 6 on both sides for  $x = 2y + 6$
12. B. Distribute the fraction on the outside of the parentheses to each of the fractions inside the

parentheses. This gives us  $\frac{xy}{yx} + \frac{xx}{yz}$ . Notice that in the first term, both the  $x$  and the  $y$  are in the

numerator and denominator, which means they cancel out to 1 both times:  $\frac{x}{x} = 1$ , as  $\frac{y}{y} = 1$ . So, 1

$\times 1 = 1$ . This means we are left with  $1 + \frac{xx}{yz}$ , which can also be rewritten as  $1 + \frac{x^2}{yz}$ .

13. A. We can recognize that if we subtract  $y$  from both sides of the equation, we would arrive at  $3x + 3y = 0$ . This is similar in form to  $9x + 9y$ . In fact, we could multiply the equation by 3 to find the value of  $9x + 9y$ . Doing so would give us  $3(3x + 3y) = 3(0)$ . This simplifies to  $9x + 9y = 0$ .
14. C. The first step in solving any algebraic equation is to combine like terms. Here, we can subtract  $x$  from both sides and add  $2y$  to both sides. This gives us  $6x - 6y = 1$ . Since we are asked to find the value for  $12x - 12y$ , we know we can simply multiply the entire equation  $6x - 6y = 1$  by 2 to find the answer. This gives us  $12x - 12y = 2$ .
15. A. To find the value of the expression, simply modify the equation until one side of the equation looks like the expression. In order to make the left side of the equation equal to  $13x - 3$ , we must subtract 12 from both sides of the equation. This would give us  $13x + 9 - 12 = 7 - 12$ . Thus,  $13x - 3 = -5$ .
16. C. First, simplify the equation by distributing the  $q$  to arrive at  $aq + bq = 60$ . We are given  $aq = 10$ , so substitute that value into the equation to get  $10 + bq = 60$ . Solve for  $bq$ , which equals  $60 - 10 = 50$ .
17. D. If we subtract 5 from both sides, we arrive at  $1 = \frac{1}{x}$ . If we subtract 6 from both sides, we arrive at  $0 = \frac{1}{x} - 1$ . If we multiply both sides by  $x$ , we arrive at  $6(x) = x\left(\frac{1}{x} + 5\right)$ , which we then distribute to get  $6x = 5x + 1$ . The only equation we cannot arrive at is D, which involves adding an  $x$  to the left side of the equation, but not on the right side.

18. D. When a number is squared, it is multiplied by itself. In this case,  $y^2 = y \times y$ . When a term has a coefficient, such as 2 in  $2y$ , this means that the term has a value of that many times the number. In this case,  $2y = y + y$ . Choice D gives the proper expanded form of the expression.
19. A. Remember order of operations. The entire portion in parentheses ( $3 \times (1 + 2)$ ) is divided by  $(4 - 1)$ . The only two expressions to show that are A and B. Note that the 3 in the first parentheses should be distributed to the  $(1 + 2)$ , not simply multiplied by the 1.
20. B. Find the value of Column A by first squaring both sides. This leaves us with  $x + 1 = 81$ . Subtract 1 from both sides for  $x = 80$ .
21. A. Find the value for Column A by substituting  $-3$  into the equation. This gives us  $-3^2 = 9$ , so  $y = 9$ . Repeat this process for Column B, using  $x = 2$ . This gives us  $2^2 = 4$ .
22. B. If we simply solve for  $y$ , we find that  $y = x + 5$ . This means that no matter what the value of  $x$ ,  $y$  will be 5 more than  $x$ . Substitute any number for  $x$  to find that this is true.
23. A. The fraction  $\frac{1}{n}$  can also be written as  $1 \div n$ . We know that if we divide 1 by 1, we get 1. If we divide 1 by a number greater than 1 (for example, 2), we get a number that is smaller than 1 – in this case,  $\frac{1}{2}$ . However, if we divide 1 by a number less than 1 (but greater than 0, as the question tells us), we are dividing 1 by a decimal or a fraction. If  $n = \frac{1}{2}$ , then  $1 \div \frac{1}{2}$  could be written as  $\frac{1}{1} \div \frac{1}{2}$ . To divide fractions, we multiply by the reciprocal, giving us  $\frac{1}{1} \times \frac{2}{1} = 2$ . So, if we divide 1 by a number greater than 0 but less than 1, we get a number that is bigger than 1.
24. B. Cross multiply for  $6(x + 1) = 3$ . Divide both sides of the equation by 6 for  $x + 1 = 0.5$ . Subtract 1 from both sides for  $x = -0.5$ .
25. A. There are several ways to solve this question. The easiest way is to recognize that  $\frac{x}{7}$  is equal to  $\frac{2x}{14}$ , which would give us common denominators in the equation. This means we can simply set the numerators equal to one another as  $2x = x - 2$ . Combine like terms for  $x = -2$ .
26. C. Find the value of Column A by substituting 2 for  $x$ . This gives us  $2^2 + 1 = 5$ . Then, add 5 to both sides for  $y = 10$ . In Column B, substitute 106 in for  $y$ . This gives us  $106 - 5 = x^2 + 1$ . Combine like terms for  $100 = x^2$ , and take the square root of both sides for  $x = 10$  or  $x = -10$ . Because the question tells us that  $x > 0$ , we can rule out  $-10$ . Therefore, both values in Column A and Column B are equal.
27. D. Find the value for Column A by substituting  $-1$  into the equation for  $y$ .  $(-1)^2 = 1$ . Find the value for Column B by substituting 1 for  $z$ . This gives  $y^2 = 1$ . Take the square root to find that  $y$  can be either positive or negative 1. The relationship cannot be determined.

### Ratios, Proportions, & Scale Factor

- D. Proportions expressed in this format (two fractions set equal to each other) can be solved by cross multiplying and solving for the variable, in this case,  $x$ . This gives us  $(4)(6) = (3)(x)$ , or  $24 = 3x$ . Dividing by 3 on both sides gives us  $8 = x$ .
- C. Proportions expressed in this format (two fractions set equal to each other) can be solved by cross multiplying and solving for the variable, in this case,  $x$ . To make it easier, we can simplify the fraction on the left from  $\frac{24}{32} = \frac{3}{4}$ . If we rewrite the equation, we get  $\frac{3}{4} = \frac{36}{x}$ . Cross multiply for  $144 = 3x$ , and  $x = 48$ .
- C. We are looking for the total sum of twez that Stephen will get. Since he has 100 yorz, and every 5 yorz gives 1 twez, then he can get  $100 \text{ yorz} \div 5 \text{ yorz per twez} = 20 \text{ twez}$ . Similarly, he has 20 jimp, and every 0.2 jimp gives 1 twez, then he can get  $20 \text{ jimp} \div 0.2 \text{ jimp per twez} = 100 \text{ twez}$ . Therefore, he has  $100 + 20 = 120 \text{ twez}$  in total.

4. D. 1 glass of water is equal to  $3 \times \frac{1}{3}$  glasses of water. 3 glasses of water is equal to  $3 \times 3 \times \frac{1}{3}$  glasses of water. Therefore, there is a total of  $9 \times \frac{1}{3}$  glasses of water. If he drinks 9 times, it means he has eaten  $6 \times 9 = 54$  hotdogs.
5. B. If circle  $R$  has a diameter of 10 cm., then a circle similar to it with a scale factor of  $\frac{2}{5}$  has a diameter of  $10 \times \frac{2}{5} = 4$  cm. The formula for the circumference of a circle is  $2r\pi$ , where  $r$  is the radius. Notice that  $2r$  describes the diameter, which is already given to us. Therefore, the circumference of the circle similar to circle  $R$  is  $4\pi$  cm.
6. D. Set up a proportion to solve this question reliably.  $\frac{3 \text{ tickets}}{8 \text{ points}} = \frac{39 \text{ tickets}}{x \text{ points}}$ . Cross multiply and solve for  $x$ :  $(39)(8) = 3x$  simplifies to  $312 = 3x$ , and  $x = 104$ .
7. D. We know that 3 ounces of oil + 2 ounces of vinegar = 5 ounces of dressing. The question asks how much oil is needed to make 20 ounces of dressing. We know that this amount is  $20 \div 5 = 4$  times as much dressing as the original recipe. Therefore, multiply both oil and vinegar by 4 to find the total amount of oil needed. This is  $3 \times 4 = 12$  ounces of oil.
8. A. Set up a proportion to solve this question reliably.  $\frac{25 \text{ lbs. butter}}{300 \text{ cakes}} = \frac{x \text{ lbs. butter}}{6 \text{ cakes}}$ . Cross multiply and solve for  $x$ :  $(25)(6) = 300x$  simplifies to  $150 = 300x$ , and  $x = 0.5$ .
9. C. Pay close attention to the units used in the question. We are given a scale of 1 inch on a model representing 3 yards on a real ship. We are told that the model is 5 feet long, and we know that each foot contains 12 inches. Create a proportion to help solve this question reliably.
- $\frac{1 \text{ inch}}{3 \text{ yards}} = \frac{60 \text{ inch}}{x \text{ yds}}$  Cross multiply and solve to find that  $x = 60 \times 3$ , or  $x = 180$ .
10. B. Set up a proportion to solve this question reliably.  $\frac{30 \text{ feet}}{0.25 \text{ inches}} = \frac{40 \text{ feet}}{x \text{ inches}}$ . Cross multiply and solve for  $x$ :  $(40)(0.25) = 30x$  simplifies to  $10 = 30x$ , and  $x = \frac{1}{3}$ .
11. B. A ratio tells us the relationship between two amounts. Here, we know that for every 5 giraffes, there are 9 polar bears (or, for every 9 polar bears, there are 5 giraffes). The ratio itself tell us that there could be a total of 5 giraffes + 9 polar bears = 14 giraffes and polar bears. If there are 36 polar bears, this is  $36 \div 9 = 4$  times as many as is given in the ratio. Therefore, there must be 4 times as many giraffes too, for a total of  $4 \times 5 = 20$  giraffes. But the question asks for the total of polar bears and giraffes, so  $36 + 20 = 56$ . Using a proportion:  $\frac{9 \text{ polar bears}}{5 \text{ giraffes}} = \frac{36 \text{ polar bears}}{x \text{ giraffes}}$ . Cross multiply and solve for  $x$ :  $(36)(5) = 9x$  simplifies to  $180 = 9x$ , and  $x = 20$ . Remember, this only tells us the number of giraffes.
12. A. We are not given figures to work with, but we know that  $XY$  will correspond to  $AB$ , that  $YZ$  will correspond to  $BC$ , and that  $XZ$  will correspond to  $AC$ . Once we know this, we need only set up a proportion to solve:  $\frac{XY}{YZ} = \frac{AB}{BC}$ . Substitute the given values:  $\frac{6}{4} = \frac{3}{BC}$ . Cross multiply and solve for  $BC$ :  $(6)(BC) = 12$  simplifies to  $BC = 2$ .
13. A. Since the rectangles are similar, we know that corresponding sides are proportional. Since this is the case, we can set up a proportion as follows:  $\frac{AB}{WX} = \frac{AD}{XY}$  (since  $XY = WZ$ , and  $WZ$  is the corresponding side to  $AD$ ). Plug in the values known and solve for the unknown:  $\frac{12}{8} = \frac{6}{XY}$ . Cross multiply and solve for  $XY$ :  $(12)(XY) = 48$  simplifies to  $XY = 4$ .



14. A. There are two right triangles shown in the figure: one with legs measuring 6 cm and  $x$  cm, and one with legs measuring  $6 + 2 = 8$  cm and 3 cm. We know that the two triangles are similar to each other because they share a base (the longest leg) and two of the same angle measurements (the left-most angle as well as a right angle formed by the bottom-right angle). This means that we can set up the following proportion:  $\frac{6 \text{ cm}}{8 \text{ cm}} = \frac{x \text{ cm}}{3 \text{ cm}}$ . Cross multiply and solve for  $x$ :  $(8)(x) = 18$  simplifies to  $8x = 18$ , and then  $\frac{18}{8} = \frac{9}{4}$ .
15. A. Solve for  $x$  by cross multiplying to get  $(6)(6) = 5x$ , or  $36 = 5x$ . Divide both sides by 5 for  $x = \frac{36}{5}$ . Perform long division, or recognize that  $7 \times 5 = 35$ , which is less than 36. So,  $\frac{36}{5}$  is greater than 7.
16. B. Set up a proportion to solve this question reliably.  $\frac{60 \text{ ft object}}{50 \text{ ft shadow}} = \frac{5 \text{ ft object}}{x \text{ shadow}}$ . Cross multiply and solve for  $x$ :  $(5)(50) = 60x$  simplifies to  $250 = 60x$ . Perform long division to solve for  $x$  to get  $x = 4\frac{1}{6}$ , or recognize that  $x$  must be less than 5, since  $5 \times 6 = 30$  and  $6x$  is only 25.
17. A. Though we can find the total number of miles run for both columns, we don't need to in order to answer this question. We know that 2 slices of pizza contains  $300 + 300 = 600$  calories. We know that 1 slice of pizza and 1 hamburger contains  $300 + 200 = 500$  calories. Since Geoff must run the same distance for every 200 calories, we can see that he has to run more if he eats 2 slices of pizza. He would have to run 2 miles in this case, but less than 2 miles if he ate a slice of pizza and a hamburger.
18. A. In Column A, cross multiply and solve for  $x$ . This gives us  $(12)(9) = 4x$ , or  $108 = 4x$  and  $x = 27$ . In Column B, cross multiply and solve for  $y$ . This gives us  $(12)(3) = y^2$ , or  $36 = y^2$  and  $x = 6$  or  $-6$ .
19. C. The shortest side of triangle X is 3, and the longest is 5. With a scale factor of  $\frac{7}{3}$ , a similar triangle would have a short side of  $\frac{7}{3} \times 3 = 7$ . With a scale factor of  $\frac{7}{5}$ , a similar triangle would have a long side of  $\frac{7}{5} \times 5 = 7$ . Both amounts are equal.
20. A. Since 1 kehl = 5 tae, if we have 3 kehl, we have  $3(1 \text{ kehl}) = 3(5 \text{ tae})$ , or a total of 15 tae. We are told that 3 verv = 10 tae, which is less than 15 tae.
21. A. Since the two triangles are similar, and we know that side  $AC$  corresponds to side  $DF$ , we know that the other corresponding sides of the triangle have the same proportion as  $AC$  to  $DF$ . Therefore, the length of side  $EF$  will always be greater than the length of side  $CB$ .
22. C. The ratio of sliced apple packages to pieces of candy is 1:7. Since she gives out 13 packages of sliced apples, then she must have given out  $7 \times 13 = 91$  pieces of candy. This is a total of  $13 + 91 = 104$  pieces of candy and sliced apple packages.
23. B. There are several ways to find the value of Column A. We could set up a proportion and solve for the actual distance between the two towns that are 5 cm apart on the map,  $x$ :  $\frac{3 \text{ cm}}{700 \text{ km}} = \frac{5 \text{ cm}}{x \text{ km}}$ . Cross multiply to find  $(5)(700) = 3x$ , or  $3,500 = 3x$ . We can perform long division, or simply substitute 1,200 for  $x$  to find that  $(3)(1,200) = 3,600$ , which is greater than 3,500. So, the actual distance is actually less than 1,200 km.
24. B. Determine the value of Column A. Since there are 6 hours in a school day, and 1 is reserved for lunch, there are 5 remaining for classroom learning and for exercise. The ratio of exercise to classroom learning (in minutes) is 15:60, or 1:4. This means for every 4 hours of classroom learning, there is 1 hour of exercise. Since there are a total of 5 hours of non-lunch time in the

school day, we know that there is 1 hour of exercise. Column B describes the amount of time students spend in lunch or exercising, which is  $1 + 1 = 2$  hours.

25. A. If there are 60 oranges, then the scale factor is 12, so there would be  $12 \times 4 = 48$  bananas. If there are 60 bananas, then the scale factor is 15, so there would be  $15 \times 3 = 45$  apples. Therefore, the value in Column A is greater.

### Functions & Patterns

- B. If Siloam has  $x$  dollars, and Jefferson has twice as much, then Jefferson has  $x + x = 2x$  dollars. We can check this by substituting any number in for  $x$ . For example, if Siloam has \$5, then  $x = 5$ . If Jefferson has twice as much, then he has  $5 + 5 = 10$ , which is  $5 \times 2 = 10$ .
- A. The key phrase to pay attention to is “four years less than twice as long as Mrs. Simons.” Since Mrs. Simons has been teaching for  $x$  years, “twice as long as Mrs. Simons” would be  $2x$ . Four less than this amount would be represented as  $2x - 4$ .
- C. If Brent mowed  $m$  lawns, then Jessica mowed  $m + 4$  lawns. Together,  $m + m + 4$  lawns, or  $2m + 4$  lawns.
- B. If we are unsure of how to proceed, substitute any odd integer in for  $a$ ; for example, let  $a = 1$ . The “next odd integer greater than  $a$ ” must be 3, so  $3 - 1 = 2$ . Therefore,  $a + 2$  gives us the next odd integer.
- B. In dollars, a quarter has a value of \$0.25, a dime has a value of \$0.10, and a penny has a value of \$0.01. To find the total value, we add together the value of each group of coins, not multiply them all together.
- D. If there are  $x$  toppings, then the cost for toppings is  $\$0.35 \times x$  (the number of toppings). This means a salad has a base cost of \$5 plus a cost of  $\$0.35x$ , or  $5 + 0.35x$ .
- A. Translate the information provided into an equation. Since we don’t know what the original number is, we can use  $x$  to represent this number. We are told that if the number is tripled ( $3x$ ) and the result is decreased by 5 (minus 5), the result is 43. Thus:  $3x - 5 = 43$ . This simplifies to  $3x = 48$ , or  $x = 16$ .
- C. Let  $x$  represent the unknown number. “Four times a number” can be represented as  $4x$ , and “7 less” than this is  $4x - 7$ . If this is equal to 41, then  $4x - 7 = 41$ , then  $4x = 48$ , and  $x = 12$ . However, the question asks for 3 times this number, so  $3(12) = 36$ .
- B. Let  $x$  represent Loris’s age. In terms of  $x$ , Micaela is  $2x + 2$  years old (“two more than twice Loris’s age”). We are told that Micaela is actually 22 years old. Therefore,  $22 = 2x + 2$ . Combine like terms and solve for  $x$ :  $20 = 2x$ , and  $x = 10$ . Therefore, Loris is 10 years old.
- C. Let  $x$  represent the number of books that Herene read. In terms of this, Ansuz read  $3x + 4$  (“four more than three times”) books. This means that  $28 = 3x + 4$ . If we solve for  $x$ , we find that  $24 = 3x$ , and  $x = 8$ .
- D. If Norman jogged  $x$  km, and Martin jogged 8 times as much as this, then Martin jogged  $8x$  km. In total, this is  $x + 8x = 63$ , or  $9x = 63$ . Therefore,  $x = 7$ . The question asks for  $8x$ , or 56.
- C. We know that Jacob (J) has 4 more books than Kelly (K) has, however many that might be. Since we don’t know how many books Kelly has, we can represent that value with a variable. Let’s use  $x$ , in this case. If  $K = x$  books, then  $J = x + 4$  books. The total of  $J + K = x + x + 4$ . This total is equal to 50, so the equation we get is  $x + x + 4 = 50$ . To solve for  $x$ , combine like terms and simplify:  $2x = 46$ , and  $x = 23$ . Remember that  $x$  represents K; the question asks for J, or  $x + 4$ . Thus,  $23 + 4 = 27$ .
- D. Represent unknown values as variables. In this case, let  $x$  represent the number of articles of clothing that a human can sew together. A machine can sew together “10 times as many” articles of clothing, so  $10x$ . Together, in a certain period of time, the machine and the human can sew together  $x + 10x$  articles of clothing, or a total of  $11x$  articles of clothing. We’re told that in one day, the machine and human together sew together 440 articles of clothing, so  $440 = 11x$  and  $x = 40$ . Remember that the question asks for how many articles of clothing were sewn together by the machine, so  $10x = 10(40) = 400$ .
- B. A nickel has a value of 0.05 dollars, and a dime has a value of 0.10 dollars. The value of  $x$  nickels is  $0.05x$  (the value of a coin times the number of coins there are). The number of dimes is

$x + 5$ , and the value of this is  $0.10(x + 5)$ . Distribute this and find that  $0.10x + 0.5$ . Together, the total value is  $0.05x + 0.10x + 0.5$ , or  $0.15x + 0.5$ .

15. A. We are given a ratio, where there are 5 pennies to every 1 quarter. This means there must be 0.05 for every 0.25, or a total of 0.30. Since the total value of coins is 5.40, we need to find how many times 0.30 fits into 5.40, which can be expressed as  $5.40 \div 0.30 = 18$ . Therefore, there are 18 sets of the six coins (5 pennies and 1 quarter). This means there are 18 quarters.
16. C. We are given a ratio, where there are 7 dimes to every 1 quarter. This means there must be  $7 \times 0.10 = 0.70$  for every 0.25, or a total of 0.95. Since the total value of coins is 4.75, we need to find how many times 0.95 fits into 4.75. The long division is difficult, but we can use estimation to help us know that the answer must be 5. If we test this out, we find that  $0.95 \times 5 = 4.75$ . Therefore, there are  $5 \times 7 = 35$  dimes in her pocket.
17. B. If Kevin has  $x$  quarters, then he has  $x - 2$  nickels. The value of this can be represented as  $0.25x + 0.05x - 0.1 = 1.1$ . This simplifies to  $0.3x = 1.2$ , or  $x = 4$ . This is the number of quarters, so he has  $4 - 2 = 2$  nickels.
18. C. In a triangle, there are a total of  $180^\circ$ . Since angle  $R$  is  $90^\circ$ , then  $P + Q = 180 - 90$ . In total,  $P + Q = 90^\circ$ . We're told that  $P = 2Q + 9$  ("angle  $P$  is 9 more than two times angle  $Q$ "), so we can substitute this into the previous equation:  $2Q + 9 + Q = 90$ , which simplifies to  $3Q + 9 = 90$ . Solving for  $Q$ , we get  $3Q = 81$ , and  $Q = 27$ . Remember that the question asks for  $P$ , which is  $90 - 27 = 63$ .
19. C. There are  $360^\circ$  in a quadrilateral. Since we are only given the ratio of the angles, we can use variables to help us determine the measurement of the angles. If  $x$  represents the smallest angle, then the next largest is  $2x$ ,  $3x$ , and  $4x$ . Since we know the sum must be 360, then we have an equation:  $x + 2x + 3x + 4x = 360$ . This simplifies to  $10x = 360$ . Therefore,  $x = 36$ .
20. D. There are  $180^\circ$  in a triangle. If  $x$  is the smallest angle, then the other angles are  $3x$  and  $5x$ . Therefore, there are  $x + 3x + 5x = 180$  degrees in this triangle. Simplifying, we arrive at  $9x = 180$ , and eventually,  $x = 20$ . However, the question asks for the largest angle:  $5x$ . Therefore,  $5(20) = 100$ .
21. D. We can set up the equation  $8x = 6x + 10$ , where  $x$  represents the number of hours they each ran. Subtracting  $6x$  from both sides and then dividing by 2 results in  $x = 5$ , or 5 hours.
22. B. Translate the information provided into an equation. Since we don't know what the original number is, we can use  $x$  to represent this number. We are told that if the number is doubled ( $2x$ ) and the result is increased by 8 (plus 8), the result is 30. Thus:  $2x + 8 = 30$ . This simplifies to  $2x = 22$ , or  $x = 11$ .
23. C. Let  $x$  represent Ethan's age. In terms of  $x$ , Janice is  $3x + 4$  years old ("Four more than three times Ethan's age"). We are told that Janice is actually 55 years old. Therefore,  $55 = 3x + 4$ . Combine like terms and solve for  $x$ :  $51 = 3x$ , and  $x = 17$ . Therefore, Ethan is 17 years old.
24. D. If Jane ran  $x$  laps, and Betty ran 4 times as much as this, then Betty ran  $4x$  laps. In total, this is  $x + 4x = 75$ , or  $5x = 75$ . Therefore,  $x = 15$ . The question asks for  $4x$ , or 60.
25. C. If Raphael has  $x$  flashcards, then Mike has  $x + 18$  flashcards. Together this makes  $x + x + 18$  or  $2x + 18$ . The question states that this total is 60, so  $2x + 18 = 60$ . Subtract 18 to get  $2x = 42$ , so  $x = 21$ . The question asks for the number of flashcards Mike has, so this is  $21 + 18 = 39$ .

### Slope

1. C. This equation is presented in the format  $y = mx + b$ , where  $m$  is the slope of the line. In this case,  $m = 3$ .
2. B. This equation is presented in the format  $y = mx + b$ , where  $m$  is the slope of the line. In this case,  $m = \frac{1}{4}$ .
3. B. Parallel lines have the same slope. This equation is presented in the format  $y = mx + b$ , where  $m$  is the slope of the line. The slope of the line given is  $-\frac{3}{4}$ , so the correct answer is also  $-\frac{3}{4}$ .

4. C. In this equation, which is presented in the format  $y = mx + b$ ,  $m = 3$ . A line that is perpendicular to another line has a slope of the negative reciprocal of the original slope. In this case, since the slope is 4, the negative reciprocal is  $-\frac{1}{3}$ .
5. B. Parallel lines share the same slope because they never intersect. In this equation, which is presented in the format  $y = mx + b$ ,  $m = -\frac{1}{3}$ . Therefore, only  $y = -\frac{1}{3}x + 6$  works.
6. A. In this equation, which is presented in the format  $y = mx + b$ ,  $m = 4$ . A line that is perpendicular to another line has a slope of the negative reciprocal of the original slope. In this case, since the slope is 4, the negative reciprocal is  $-\frac{1}{4}$ .
7. C. The change in distance over time is often expressed as the slope of a linear equation. If distance is  $y$  and time is  $x$ , we can write a formula:  $y = 70x$ . This means that after  $x$  minutes, Michael would have driven  $70x$  miles (since he is driving 70 miles per hour).
8. C. This equation is presented in the format  $y = b + mx$ , where  $m$  is the slope of the line. Therefore, 10.25 is the slope of the line in this equation. The slope shows the change in the  $y$  (cost) over the change in  $x$  (hours). This means that this is an increase in cost of 10.25 over each hour.
9. D. The slowest elevator will descend the fewest floors (shown by the change in the  $y$ -axis) over the greatest amount of time (shown by the change in the  $x$ -axis). Elevator A changes 1 floor in 1 second, so its slope is  $-1$ . Elevator B changes 1 floor in 2 seconds, so its slope is  $-\frac{1}{2}$ . Elevator C changes 1 floor in 2 seconds, so its slope is also  $-\frac{1}{2}$  (be sure to pay attention to the scale on both axis!). Elevator D changes 1 floor in 4 seconds, so its slope is  $-\frac{1}{4}$ .
10. C. Since we're told that time is represented as  $x$ , and bacteria population is given as  $y$ , we can substitute values of  $x$  from the table into each equation to see which is true. If we use  $x = 1$  (when time is 1), we can see that only choices B and C work. If we use  $x = 2$ , we can see that only choice C is true, since  $2(2) + 2 = 6$  (whereas  $4(2) = 8$ ).
11. C. We are told that  $c$  and  $m$  represent the total cost of the call and the number of minutes, respectively. 0.30 represents the minimum cost of the call (if  $m = 0$ , then the call would still cost \$0.30). \$0.10 represents the additional cost of the call per minute, since this increases as  $m$  increases.
12. A. Only choice A shows us that the minimum amount of money Carly can earn is \$20, which increases at \$0.25 for every newspaper delivered. We can see that at 40 newspapers, Carly has earned  $40 \times 0.25 = 10$  additional dollars on top of the \$20 minimum, for a total of \$30.
13. D. The line is decreasing, which means it has a negative slope. Only choices C and D have negative slopes. The chart shows us that when the container is full, at time 0, it contains 8 mL of water. This is because the point (0,8) tells us that at the beginning (when time is 0), the container contains 8 mL of water.
14. C. Since  $y$  represents distance and  $x$  represents time, then we know that the slope of both equations tells us how fast each car is traveling. This is because as  $x$  increases,  $y$  increases, and the amount of the increase is the same for both cars. It doesn't matter that Ash starts  $50 - 10 = 40$  miles farther along the highway than Tori, as given by the  $y$ -intercepts.
15. A. Convert the equation in Column B into the form  $y = mx + b$  by isolating  $y$ . The equation becomes  $6y = -3x + 9$ . We see then that  $m$  will be  $\frac{-3}{6} = -\frac{1}{2}$ .
16. C. Over two weeks (weeks 0-2), we see that Plant X grows  $45 - 42 = 3$  inches. Over four weeks (weeks 6-10), we see that Plant Y grows  $40 - 34 = 6$  inches. So, Plant X grows at a rate of 3 in.  $\div$



2 wks. = 1.5 inches per week, and Plant Y grows at a rate of  $6 \text{ in.} \div 4 \text{ wks.} = 1.5$  inches per week. Both values are equal.

### Algebraic Concepts Mixed Practice

- C. (Solving Algebraic Equations) Multiply the first equation  $4x + 6y = 12$  by 4 to get the second equation  $16x + 24y = 48$ .
- B. (Solving Algebraic Equations) Subtract  $m$  on both side of the equation to get  $-n = 12 - m$ . Divide negative 1 on both side to get  $n = m - 12$ .
- B. (Ratios, Proportions and Scale Factor) A ratio tells us the relationship between two amounts. Here, we know that for every 7 paintings, there are 8 sculptures. The ratio itself tell us that there could be a total of 7 paintings + 8 sculptures = 14 paintings and sculptures. If there are paintings, this is  $49 \div 7 = 7$  times as many as is given in the ratio. Therefore, there must be 7 times as many sculptures too, for a total of  $7 \times 8 = 56$  sculptures. But the question asks for the total of paintings and sculptures, so  $49 + 56 = 105$ .
- A. (Ratios, Proportions and Scale Factor) Set up a proportion to solve this question reliably.  

$$\frac{8 \text{ points}}{2 \text{ questions}} = \frac{40 \text{ points}}{x \text{ questions}}$$
 Cross multiply and solve for  $x$ :  $(40)(2) = 8x$  simplifies to  $80 = 8x$ , and  $x = 10$ .
- B. (Functions and Patterns) Substitute any even integer in for  $x$ ; for example, let  $x = 2$ . The “next even integer greater than  $x$ ” must be 4, so  $4 - 2 = 2$ . Therefore,  $x + 2$  gives us the next even integer.
- B. (Functions and Patterns) Let  $x$  represent the number of laps that Ada ran. In terms of this, Isla ran  $2x + 6$  (“six more than two times”) laps. This means that  $20 = 2x + 6$ . If we solve for  $x$ , we find that  $14 = 2x$ , and  $x = 7$ .
- D. (Slope) The line is decreasing on the graph, so the slope must be negative. This eliminates answer choice A and B. Find a coordinate point on line, let’s say  $(0, 10)$ . Plug it into the equations for choice C and D to see that only choice D is true.
- A. (Slope) A line that is perpendicular to another line has a slope of the negative reciprocal of the original slope. In this case, since the slope is 2, the negative reciprocal is  $-\frac{1}{2}$ .
- D. (Functions and Patterns) If there are  $x$  children, then the cost for children ticket is  $\$7.55x$ . This means the total cost would be  $\$15$  cost of the adult ticket plus a cost of  $\$7.55x$ , or  $15 + 7.55x$ .
- C. (Solving Algebraic Equations) Both  $y^2$  and 9 are perfect squares. The square root of  $y^2$  is  $y$  and the square root of 9 is 3. Use the formula for difference of squares  $a^2 - b^2 = (a+b)(a-b)$  to factor the polynomial into  $y^2 - 9 = (y + 3)(y - 3)$ .
- B. (Ratios, Proportions and Scale Factor) If circle  $Q$  has a diameter of 12 cm., then a circle similar to it with a scale factor of  $\frac{1}{4}$  has a diameter of  $12 \times \frac{1}{4} = 3$  cm. The formula for the circumference of a circle is  $d\pi$ , where  $d$  is the diameter. Therefore, the circumference of the circle similar to circle  $Q$  is  $3\pi$  cm.
- B. (Slope) Since we’re told that time is represented as  $x$ , and egg population is given as  $y$ , we can substitute values of  $x$  from the table into each equation to see which is true. If we use  $x = 2$  (when time is 2), we can see that only choices B works, so that is the correct answer.
- B. (Functions and Patterns) We know that Lauren (L) has 5 more than 2 times the folders Amy (A) has, however many that might be. Since we don’t know how many folders Amy has, we can represent that value with a variable. let’s use  $x$ , in this case. If  $A = x$  books, then  $L = 5 + 2x$  books. The total of  $L + A = x + 5 + 2x$ . This total is equal to 26, so the equation we get is  $x + 5 + 2x = 26$ . To solve for  $x$ , combine like terms and simplify:  $3x = 21$ , and  $x = 7$ . Remember that  $x$  represents A, so our answer is 7.

14. B. (Slope) The slope of the given function is  $\frac{2}{5}$ . The slope of the line perpendicular to the function above is the negative reciprocal of the function's slope, so the slope is  $-\frac{5}{2}$ . The slope of the line parallel to the function given is the same slope,  $\frac{2}{5}$  and  $\frac{2}{5} > -\frac{5}{2}$ .
15. C. (Ratios, Proportions and Scale Factor) Set up a proportion and solve for the actual height of the building,  $\frac{2 \text{ in}}{150 \text{ ft}} = \frac{20 \text{ in}}{x \text{ ft}}$ . Cross multiply to find  $(20)(150) = 2x$ , or  $3,000 = 2x$ . Divide 2 on both side of the equation to find  $x=1500 \text{ ft}$ . For column B, set up a proportion using the conversion scale  $1 \text{ yd}=3 \text{ ft}$ ,  $\frac{1 \text{ yd}}{3 \text{ ft}} = \frac{500 \text{ yd}}{x \text{ ft}}$ , to find that  $(3)(500) = 1,500 \text{ ft}$ , which is the same as the value in column A. So, the two amounts are equal.
16. D. (Solving Algebraic Equations) When  $x$  is a positive integer,  $y > x$ , but when  $x$  is a fraction,  $x > y$ , so more information is needed to define the relationship.

## Measurements

### Formulas

- D. If we insert these values into the formula, we end up with an algebraic equation:  $35 = \frac{1}{2} \times L \times$   
14. Simplify for  $35 = 7 \times L$ , then divide both sides by 7 to arrive at  $5 = L$ .
- C. The square and the triangle share a side. If the square has an area of 16 units<sup>2</sup>, then each side of the square is 4 units in length. If one side of the triangle is 4 units in length, and the total perimeter of the triangle is 12 units, then the remaining two sides must have a combined length of 8 units. Therefore, 3 sides of the square  $\times$  4 units per side + 8 units from the triangle = 20 units for the outlined solid trapezoid.
- C. Since the area of the trapezoid is given, simply plug in known values into the formula given and solve.  $150 = \frac{h(b_1 + b_2)}{2}$  becomes  $150 = \frac{10(x + 20)}{2}$ . This simplifies to  $300 = 10(x + 20)$  and then  $30 = x + 20$ , which eventually gives us  $x = 10$ .
- D. For this question, we can plug the given values into the formula, where  $A = 120$  and the two bases are 6 and 10. Thus:  $A = h\left(\frac{(6+10)}{2}\right)$ . This simplifies to  $120 = 8h$ , or  $h = 15 \text{ in}^2$ .
- A. Substitute the given values into the equation and isolate  $b_1 + b_2$ . When we do that we end up with:  $26 = 4\left(\frac{(b_1 + b_2)}{2}\right)$ . This can be simplified to  $26 = 2(b_1 + b_2)$ , and eventually  $13 = b_1 + b_2$ .  
Therefore, the sum of the bases is 13.
- B. The floor of the swimming pool is a circle. Circles have an area given by the formula  $\pi r^2$ . Since  $r$  (the radius) is given to us as 5 ft., we substitute and simplify to get  $3.14(5)^2 = \sim 78.5 \text{ ft}^2$ . We know that each gallon of paint can only cover  $25 \text{ ft}^2$ . Since 3 gallons of paint will only cover  $75 \text{ ft}^2$ , we need a 4<sup>th</sup> gallon to cover the remainder.
- C. To find the circumference of a circle, we need its diameter (or twice its radius). We can find the radius using the area formula:  $A = \pi r^2$  and when given the values, we can substitute into the equation.  $150 = 3r^2$ , which simplifies to  $50 = r^2$ . Taking the square root of both sides gives us  $7 \approx r$  (since  $7^2 = 49$ ). From here, we can use the circumference formula:  $C = 2r\pi$ . This gives us  $C = 2 \times 7 \times 3 = 42 \text{ in}$ .
- A. The volume of a rectangular prism (a three-dimensional figure with 6 faces) can be represented as length ( $L$ ) times width ( $W$ ) times height ( $H$ ). We don't know what  $H$  is, but we do know that  $L \times W = 24$ . So, we know that  $24 \times H = 48$ . Dividing both sides by 24 tells us that  $H = 2 \text{ in}$ .

9. A. We need the radius of the sphere to find the volume, so we divide the diameter by 2 to arrive at 3 m. Then, plug the radius into the equation for  $\frac{4}{3}\pi(3)^3$ , which gives us  $\frac{4}{3}\pi(27) = 36\pi$ .
10. D. The volume of a rectangular prism (a three-dimensional figure with 6 faces) can be represented as length ( $L$ ) times width ( $W$ ) times height ( $H$ ). A cube is a special type of rectangular prism where  $L = W = H$ . Since this is the case, we know that a number multiplied by itself 3 times must give us the volume, which is  $27 \text{ cm}^3$ .  $3 \times 3 \times 3 = 27$ . This means each side has a length of 3 cm. Each face, therefore has an area of  $3 \times 3 = 9 \text{ cm}^2$ . Since there are 6 faces,  $9 \text{ cm}^2 \times 6 = 54 \text{ cm}^2$ .
11. D. The volume of a trapezoidal prism =  $(h)(l)\frac{(b_1 + b_2)}{2}$ . We are given all measurements of the prism except for  $h$ . Substitute known values into the formula:  $60 = (h)(5)\frac{(2+4)}{2}$ . Simplify and solve for  $h$ :  $120 = (h)(5)(6)$ , which simplifies to  $4 = h$ .
12. A. The volume of a pyramid is equal to one-third the area of the base times the height (or,  $V = \frac{lw}{3}$ ). To solve for the volume of a square pyramid, we need to know the area of the base. We are told that the base has a 4-inch side, so  $4 \times 4 = 16 \text{ in}^2$ . From there, we can plug the known values into the volume formula to get  $V = \frac{16 \times 9}{3}$ . This simplifies to  $V = 48 \text{ in}^3$ .
13. D. To find the surface area, substitute known values into the formula:  $2^2 + \frac{4(2)(4)}{2} = 20 \text{ mm}^2$ .
14. A. The figure given only shows the dimensions of the pyramid's base. We need to use these values, as well as the height to find the volume. The area of the base is  $\frac{1}{2}bh$ , or when using the given values,  $\frac{1}{2}(6 \times 4) = 12 \text{ ft}^2$ . We can use this value to solve for the volume:  $V = (12 \times 10) \div 3 = 40 \text{ ft}^3$ .
15. A. The base of the cylinder is a circle which has an area of  $\pi r^2$ . Therefore, the formula for a cylinder is  $\pi r^2 h$ . Plug in known values to arrive at  $108 = 3r^2(9)$ . Simplify for  $108 = 27r^2$  and eventually  $4 = r^2$ . Taking the square root of both sides gives us  $2 = r$ .

## Units

- A. There are 1,000 milligrams in a gram, so to convert from a milligram to a gram, we divide the amount of milligrams by 1,000.  $85 \div 1,000 = 0.085$ .
- B. There are 1,000 milligrams in a gram, and 1,000 grams in a kilogram. So, to convert from a milligram to a kilogram, we divide the amount of milligrams by  $1,000 \times 1,000$ , or 1,000,000.  $3,500,000 \div 1,000,000 = 3.5$ .
- B. There are 1,000 mL in 1 liter. Therefore, to change a number from milliliters to liters, move the decimal point three places to the left (divide by 1,000). This makes  $355 \text{ mL} = 0.355 \text{ L}$ .
- D. Each hour the kite will descend 2 yards. Since there are 3 feet in a yard, this means each hour the kite will descend  $2 \times 3 = 6$  feet. The kite is descending a total of 30 feet, and  $30 \div 6 = 5$  hours from 4:00p which is 9:00pm.
- A. Convert the wall measurements into yards:  $6 \text{ ft.} \div 3 \text{ ft. per yd.} = 2 \text{ yds.}$   $12 \text{ ft.} \div 3 \text{ ft. per yd.} = 4 \text{ yds.}$  The wall has a total area of  $2 \times 4 = 8 \text{ yds}^2$ .
- D. Always convert the units to the answer units before multiplying to find the area. This makes the dimensions of the blanket  $5 \times 12 = 60$  inches and  $4 \times 12 = 48$  inches. Now, multiply the inches together to find the total number of square inches she needs:  $60 \times 48 = 2,880$  square inches.
- A. A decrease of 6 inches per hour for 6 hours means a total of  $6 \times 6 = 36$  inches of water decreased over 6 hours.  $36 \text{ inches} \div 12 \text{ inches per foot} = 3 \text{ feet}$ . This means the depth of water beneath the buoy measures  $16 - 3 = 13$  feet (6 hours later).

8. B. We see how many meters Jake has run by multiplying:  $100 \times 17 = 1,700$  m. There are 1,000 m in 1 km, so we can convert:  $1,700 \text{ m} \div 1,000 = 1.7$  km
9. A. There are 12 inches in 1 foot. We can calculate Marisa's height in inches:  $(5 \times 12) + 8 = 68$  inches. From there, we can convert her height in inches to centimeters by using the scale factor:  $68 \times 2.5 = 170$  cm. There are 100 centimeters in a meter, so we can solve:  $170 \text{ cm} \div 100 = 1.7$  m.
10. C. In 100 hours, the snail has moved  $6.1 \times 100 = 610$  centimeters. There are 100 centimeters in a meter, so the snail has moved 6.1 meters in that time.
11. A. We need to see how many 1-inch tiles Darcy will need, and can find that by dividing the actual value by the scaled value:  $640,000 \div 6,400 = 100$ . When converting between area units (like square feet), divide by the square of the unit-conversion value. In this case, to convert from inches to feet, divide by 12; to convert from square inches to square feet, divide by  $12^2$ , or 144. This becomes  $100 \div 144$ . Though the closest value is 0.7, we don't need to actually do the calculation, as 0.7 is the only value less than 1.
12. C. There are several ways to solve this question. First, we can rewrite the volume of the vase as  $2.5 \times 10,000,000 = 27,000,000 \text{ mm}^3$ . The easiest way might be to convert each expression into millimeters. There are 10 mm for every 1 cm. Therefore  $0.3 \text{ cm} \times 10 \text{ mm} = 3 \text{ mm}$  and  $3 \times 3 \times 3 = 27 \text{ mm}^3$ .  $3.0 \text{ cm} \times 10 \text{ mm} = 30 \text{ mm}$  and  $30^3 = 27,000 \text{ mm}^3$  (notice this is the same as choice D). So, we are left with  $30 \text{ cm} \times 10 \text{ mm} = 300 \text{ mm}$ , and  $300^3 = 27,000,000 \text{ mm}^3$ . The volume of the vase is less than the volume of the box, which is the only way the vase could fit inside the box.

## Geometry

### Geometric Objects

1. B. Since the area of a square is given as length  $\times$  length (or length<sup>2</sup>), then we know that the length times itself must give us an area of 64. The only number to do so is 8, since  $\sqrt{64} = 8$ . The perimeter is therefore  $8 \times 4 = 32$ .
2. D. A square has 4 equally long sides. If the perimeter is 64, then each side is  $64 \div 4 = 16$  cm long. The area then would be  $16^2 = 256 \text{ cm}^2$ .
3. D. There are 6 rows of 3 cubes each. We know this because each row ends in a grey face, and there are 6 cubes with a grey face on the right-hand side. This makes for a total of  $6 \times 3 = 18$  individual cubes.
4. C. Since each square in the grid has an area of  $3 \text{ in}^2$ , and the shaded region accounts for 11 squares, then the total area of the shaded region is  $11 \times 3 = 33 \text{ in}^2$ .
5. A. Since there are 11 squares in the shaded region, then the area of each square must be  $44 \div 11 = 4 \text{ ft}^2$ . The question asks for the length of each square's side. This is  $\sqrt{4} = 2$  ft.
6. D. The total area of the large square is  $10 \times 10 = 10^2 = 100$ . The area of the smaller square cut away from the larger one is  $2 \times 2 = 2^2 = 4$ . We are given several expressions. Only choice D gives us an expression resulting in the proper value of 96.
7. D. Since the circle is inscribed in the square, we know that the circle touches the square at exactly 4 points (one on each side of the square). Since the area of the square is given as the length  $\times$  length, we need to find the length of a side of the square. The length of the side of the square is equal to the diameter of the circle. Since the area of a circle is  $\pi r^2$ , where  $r$  is the radius, we know that  $36\pi = \pi r^2$ . We can divide both sides of the equation by  $\pi$  to end up with  $36 = r^2$ . Solve for  $r$ :  $\sqrt{36} = \sqrt{r^2}$ , and find that  $r = 6$ . The diameter is  $2r$ , or 12. Therefore, each side of the square is 12 cm., and the area must be  $12 \times 12 = 144 \text{ cm}^2$ .
8. C. The total area of the square is  $5 \times 5 = 25 \text{ units}^2$ . The total area of the circle is  $\pi r^2 = \pi(2)^2 = 4\pi$ . Therefore, the shaded portion is  $25 - 4\pi \text{ units}^2$ .
9. D. Since ACDF is made up of two squares, we know that the area of each square is  $200 \div 2 = 100$ . That means that each side of the square has a length of  $\sqrt{100} = 10$ . The area of the unshaded portion will be the total area of ACDF ( $200 \text{ cm}^2$ ) minus the area of the shaded circle. Since we know that the side BE = the diameter of the circle = 10, we know that the radius is half this value



(since the diameter is twice the radius). This means the radius = 5, and the area of the shaded circle equals  $\pi(5)^2 = 25\pi$ . Therefore, the area of the unshaded portion of ACDF will be  $200 - 25\pi^2$ .

10. D. Since each side of the large square has sides of 6 in, each gray triangle has legs with length 3. Notice that there are 4 equally sized triangles, and that 2 of the triangles put together make a complete square. Therefore, we can make 2 gray squares out of the 4 gray triangles. Since the side of each triangle is 3, the side of each gray square is 3. Therefore, the area of each gray square is  $3^2 = 9$ , and since there are two of these small gray squares,  $2 \times 9 = 18 \text{ in}^2$ .
11. D. The hatch marks show that all three sides of the triangle have an equal length. This is the definition of an equilateral triangle. In an equilateral triangle, all angles have an equal measure, and since there are  $180^\circ$  in a triangle, and 3 angles, each angle is  $180 \div 3 = 60^\circ$ . This means that the triangle is not obtuse or right. Note that an equilateral triangle is sometimes considered to be a special case of an isosceles triangle, which has at least 2 sides of equal length.
12. B. An isosceles triangle is one that has at least two sides of equal length. Since there is no hatch mark on the bottom side, we know that it is not the same length as the other two sides. If there are two sides with an equal length, then the angles opposite those sides have an equal measure. Since the third side is not the same length as the other two, the angle opposite that side does not have the same angle measurement.
13. A. Since there are no hatch marks, we do not know whether or not the sides have the same length. The triangle does not have sides with an equal length, which is the definition of a scalene triangle.
14. D. An obtuse triangle has one angle that is greater than  $90^\circ$ . This means that the sum of the other two angles must be less than  $90^\circ$ . This automatically rules out an equiangular triangle. An obtuse triangle can never be an equilateral triangle, because equilateral triangles are also equiangular.
15. D. In an equilateral triangle, all angles have an equal measure, and since there are  $180^\circ$  in a triangle, and 3 angles, each angle is  $180 \div 3 = 60^\circ$ . The definition of an acute angle is an angle which has a measure less than  $90^\circ$ , and an acute triangle only has acute angles.
16. D. A quadrilateral with two pairs of parallel sides is a parallelogram. A rhombus is a parallelogram that also has 4 sides of equal length. A rectangle is a parallelogram that also has 4 right angles. A trapezoid only has one pair of parallel sides.
17. C. The figure only has one pair of parallel sides, so it is not a parallelogram, and is instead a trapezoid. A rhombus is a type of parallelogram.
18. B. A parallelogram is any quadrilateral with two pairs of parallel sides. This is the case for squares, rectangles, and rhombuses. Therefore, D is true. A rectangle is any quadrilateral with four sides, and four right angles. This is also the case for every square, except that square also have four sides of equal length. Therefore, A is true. Since a rhombus has four sides of equal length and two pairs of parallel sides, squares are rhombuses that happen to have four right angles; not every rhombus has four right angles. Therefore, C is true. Only B is false.
19. C. A trapezoid (even an irregular one) is a quadrilateral, as is a rhombus. In all quadrilaterals, the sum of the angles inside the quadrilateral will always equal  $360^\circ$ .
20. C. The area of the shaded region can be found using the following equation:  $(12 \times 5) - (5 \times 2)$ . This simplifies to  $60 - 10 = 50$ .

### Coordinates

1. A. Draw the triangles resulting from each answer choice. Doing so shows us that only (1,1) gives us a right triangle.
2. C. An isosceles trapezoid has two sides that are parallel while the remaining two sides are equal in length. The only way for such a trapezoid to be formed is if the fourth vertex is in the fourth quadrant (bottom right quadrant of the grid). The only coordinates in the fourth quadrant are (3,-2).
3. D. An obtuse triangle has one angle greater than  $90^\circ$  and two angles less than  $90^\circ$ . Draw the triangles resulting from each answer choice. (2,0), (5,-3), and (6,-3) gives us acute triangles. Only (-6,-2) gives us an obtuse triangle.

4. D. Choice A represents a rotation around the origin. Choice C represents a rotation around another point. Choice B represents a reflection over the  $x$ -axis. Only Choice D represents a translation.
5. A. Point A is in the first quadrant (the top-right most quadrant), where all points are given as  $(x,y)$ . Point B is in the third quadrant (the bottom-left most quadrant), where all points are given as  $(-x,-y)$ . The second quadrant (the top-left most quadrant) contains points given as  $(-x,y)$ , and the fourth quadrant (the bottom-right most quadrant) contains points given as  $(x,-y)$ .
6. D. Sketch a coordinate grid to help picture the rectangle. If we do so, we find that  $(-2,7)$  and  $(-2,-1)$  make up the right-most side of the rectangle, and that  $(-5,7)$  and  $(-2,7)$  make up the top-most side of the rectangle. The left-most and bottom-most side of the rectangle must be at  $(-5,-1)$ . Notice that this coordinate shares the  $x$ -coordinate from the top-left most vertex, and that the  $y$ -coordinate shares the bottom-right most vertex.
7. A. Draw a coordinate grid to help picture the square. If we do so, we find that  $(2,3)$  and  $(7,3)$  make up the top-most side of the square, and that  $(2,3)$  and  $(2,-2)$  make up the left-most side of the square. The right-most and bottom-most side of the square must be at  $(7,-2)$ . Notice that this coordinate shares the  $x$ -coordinate from the top-right most vertex, and that the  $y$ -coordinate shares the bottom-left most vertex.
8. D. Draw a coordinate grid to help picture the rectangle and determine the length of each side. We can see then that the difference between the greatest and the least of the  $y$ -coordinate values gives us  $3 - 1 = 2$ , and that the difference between the greatest and the least of the  $x$ -coordinate values gives us  $3 - 0 = 3$ . This means  $2 \times 3 = 6$ .
9. D. If the center of the circle is the origin  $(0,0)$ , and a point is drawn from there to point  $(-3,0)$ , then we know the distance from the origin to the edge of the circle is 3, since we move 3 units over to the left along the  $x$ -axis. Note that area and distance can never be negative, so even though the point is at  $(-3,0)$ , the distance is still 3 units from the origin. The area of circle is  $\pi r^2$ , or  $\pi(3)^2$ .
10. D. When a point is rotated around the origin by  $180^\circ$ , then the coordinates of the new point will have the opposite sign of the coordinates of the old point. In this case, 17 and 53 are both positive, so they become  $-17$  and  $-53$  after rotation.
11. A. Any time we rotate a point  $90^\circ$  clockwise around the origin, we see that a point  $Q$  located at  $(h,k)$  ends up at a point  $Q'$  located at  $(k,-h)$ . We swap the numbers, and then switch the sign of the  $y$ -coordinate. Point  $Q$  is located in the second quadrant, and a rotation  $90^\circ$  clockwise means that  $Q'$  must be in the first quadrant. Only choices A and B are in the first quadrant. Notice that if we drew a line from point  $(-10,19)$  to the origin and from point  $(10,19)$  to the origin we would get a “v” rather than a right angle (which looks like an “L”). Only if we draw a line from point  $(-10,19)$  to the origin and from point  $(19,10)$  to the origin would we get an “L” shape.
12. D. A counterclockwise rotation of  $90^\circ$  is the same as a clockwise rotation of  $270^\circ$ . This means three clockwise rotations of  $90^\circ$  each. Since each  $90^\circ$  clockwise rotation changes point  $(h,k)$  to point  $(k,-h)$ , we need only repeat this three times.  $(17,53)$  first becomes  $(53,-17)$  in the first rotation.  $(53,-17)$  becomes  $(-17,-53)$  in the second rotation.  $(-17,-53)$  becomes  $(-53,17)$  in the final rotation. Note that one  $90^\circ$  counterclockwise rotation changes point  $(h,k)$  to point  $(-k, h)$ .

### Measurement & Geometry Mixed Practice

1. D. (Formulas) To find the circumference of a circle, we need its diameter (or twice its radius). We can find the radius using the area formula:  $A = \pi r^2$  and when given the values, we can substitute into the equation.  $225\pi = \pi r^2$ , which simplifies to  $225 = r^2$ . Taking the square root of both sides gives us  $15 = r$  (since  $15^2 = 225$ ). From here, we can use the circumference formula:  $C = 2\pi r$ . This gives us  $C = 2 \times 15 \times \pi = 30\pi$  in.
2. C. (Formulas) The base of the cylinder is a circle which has an area of  $\pi r^2$ . Therefore, the formula for a cylinder is  $\pi r^2 h$  as given. The question asks for the “closest value,” so estimate  $\pi$  to 3. Plug in known values to arrive at  $384 = 3r^2(8)$ . Simplify for  $384 = 24r^2$  and divide to find  $16 = r^2$ . Taking the square root of both sides gives us  $4 = r$ . The radius is half of the diameter, so  $r \times 2 = d$  or  $4 \times 2 = 8$  in.

3. C. (Geometric Objects) Since each side of the large square has sides of 10 in, each gray right triangle has legs with length 5 in. Notice the hypotenuse is the same as the side of the smaller square. Using the Pythagorean theorem  $a^2 + b^2 = c^2$ , you can find the hypotenuse of the right triangle.  $5^2 + 5^2 = c^2$ . Solve the squares,  $c^2 = 25 + 25$  and eventually  $50 = c^2$ . Notice the area of the smaller square is also just the square of the hypotenuse of the triangle, so that means, the area of the smaller square is  $50 \text{ in}^2$ .
4. B. (Geometric Objects) Since a rhombus is a polygon with four sides of equal length and two pairs of parallel sides, squares are rhombuses.
5. B. (Coordinates) Any time we rotate a point  $90^\circ$  counterclockwise around the origin, we see that point  $J$  located at  $(h, k)$  ends up at point  $J'$  located at  $(-k, h)$ . We swap the numbers, and then switch the sign of the  $x$ -coordinate. Since point  $J$  is located at  $(5, -12)$ ,  $J'$  will be located at  $(12, 5)$ .
6. E. (Coordinates) Point  $E$  is exactly 5 units below Point  $C$   $(7, 1)$ .
7. C. (Formulas) Notice one of the legs is 5 units and the hypotenuse is 13 units. This indicates one of the Pythagorean Triples 5-12-13, which means the missing leg is 12 units. The alternative solution to find the missing leg is to use the Pythagorean Theorem:  $a^2 + b^2 = c^2$ , where  $a$  and  $b$  are the legs of the right triangle and  $c$  is the hypotenuse. To find the second leg, subtract  $a^2$  ( $5^2$ ) from  $c^2$  ( $13^2$ ) to get  $b^2$ , which is 144.  $b^2 = 144$ , so  $b = 12$ . The area of a triangle is the base multiplied by the height, divided by two:  $\frac{5 \times 12}{2} = 30$ .
8. D. (Units) Set up an equation to find out how many meters the cheetah will run in one hour,  $\frac{35 \text{ meters}}{1 \text{ second}} \times \frac{3600 \text{ seconds}}{1 \text{ hours}} = 126,000 \text{ meters per hour}$ . Hence, in 4 hours, the cheetah has run:  $126,000 \text{ meters} \times 4 \text{ hours} = 504,000 \text{ meters}$ . To convert to kilometers, set up a proportion.  $\frac{504,000 \text{ meters}}{x} = \frac{1,000 \text{ meters}}{1 \text{ kilometer}}$ . Cross-multiply and simplify to get  $x = 504$  kilometers.
9. B. (Units) 1 gallon = 8 pints, so to convert 16 pints into gallons, divide by 8. So,  $16 \text{ pints} \div 8 \text{ pints/gallon} = 2 \text{ gallons}$ .
10. A. (Formulas) Since line segment  $AB$  does not pass through the center, the diameter must be longer than line segment  $AB$ . If the diameter is longer than 8, the circumference must be longer than  $8\pi$  ( $C = \pi d$ ).  $10\pi$  is the only answer choice that is longer than  $8\pi$ .
11. C. (Coordinates) Draw a coordinate grid to help picture the rectangle. Points  $(4, 6)$  and  $(4, 1)$  forms a side with a length of 5 units. A rectangle has equal opposite sides. Hence, 5 units up from the point  $(2, 1)$ , which is  $(2, 6)$ , will form a rectangle with the given coordinates.
12. B. (Geometric Objects) To find the area of the unshaded region or the smaller square, multiply the side length by itself, which results in an area of  $4^2 = 16$ . To find the area of the unshaded region, use the area of the larger square to subtract the smaller square. Using the same area formula, compute the area of the larger square to get  $6^2 = 36$ . To find the area of the shaded region, subtract the two areas:  $36 - 16 = 20$  and  $20 > 16$ , which means the area of the shaded region is greater.
13. C. (Units) Set up a proportion to convert 0.01 kiloliter into liters.  $\frac{1 \text{ kiloliter}}{1,000 \text{ liters}} = \frac{0.01 \text{ kiloliter}}{x \text{ liters}}$ . Cross multiple to find that  $x = 100$  liters, making it is equal to the amount in column B.

## Data & Probability

### Probability

- C. There are 7 green markers in the box. There are a total of 14 markers in the box. If someone reaches into the box and picks out one marker, there is a 7 in 14 chance of getting a green marker. Therefore, there is a  $7 \div 14 = 0.5$  (or 50% or  $\frac{1}{2}$ ) chance of this happening.
- B. While the actual results might vary, there are only ever two possible outcomes for a coin toss: heads or tails. This means that there is a  $\frac{1}{2}$  chance that heads will turn up, no matter how many times a coin is flipped.
- C. Since we know that 7 students were born in a month with exactly 30 days in it, and there are 21 students in the class in total, then there are  $21 - 7 = 14$  students who were born in months with more or less than 30 days. This is  $\frac{14}{21} = \frac{2}{3}$ .
- A. The probability that the elephant eats bananas on any given day is 40%. Since there is a total of 100%, the days the elephant DOES NOT eat bananas is every other day, or  $100\% - 40\% = 60\%$ . We could also add up the percentages of the rest of the diet:  $10\% + 20\% + 30\% = 60\%$ , which equals  $\frac{3}{5}$ .
- D. The probability of it not raining on Saturday is  $\frac{1}{4}$ , which is the same for Sunday.  $\frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$ . Either perform long division ( $1.0000 \div 16.0000$ ) or recognize that  $\frac{1}{16}$  is half of  $\frac{1}{8} = 0.125$ . The result is 0.0625.
- C. 40% of 30 can be expressed as  $30 \times 0.4 = 12$ . However, we're told that the chance of picking a green marble is LESS than 40%. This means we can't have 12 marbles, and the next integer below 12 is 11.
- A. The outcome of one dice roll is independent of the outcome of the other. However, everyone will lose the game if Jurvis rolls two dice and the combined value of that roll is 2. This can only happen if both dice rolls result in 1. On one dice, the chance of this happening is  $\frac{1}{6}$ . However, this has to coincide with rolling a 1 on the other dice, as well. Therefore,  $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$ .
- A. Assume there are  $x$  white chocolates. In terms of  $x$ , there are  $2x$  dark chocolates and  $6x$  milk chocolates. Since we are looking for the chance of picking a white chocolate out of the bag, we put  $x$  in the numerator and the total number of chocolates in the denominator. This gives us  $\frac{x}{x + 2x + 6x} = \frac{x}{9x}$ . We can simplify by dividing both the numerator and denominator by  $x$  which leaves us with  $\frac{1}{9}$ .
- A. There are 60 seconds in one minute, and 60 minutes in one hour. Therefore, during 3 hours (from 7 P.M. to 10 P.M.), there are  $3 \times 60 = 180$  minutes. The commercial will air during 6 of the 180 minutes, which means the chance of seeing a commercial at least once is  $\frac{6}{180} = \frac{1}{30}$ .
- C. Since there are 6 sides on each cube, and there are 2 cubes, there are a total of  $6 \times 6 = 36$  possible combinations. This is because each number on one cube can be paired with each number on the other cube to form a product. Since we're only looking for products that are prime, we know that we are only interested in numbers that have themselves and 1 as factors. This means



only 2, 3, and 5. There are only 6 ways to do this: (2,1), (3,1), (5,1), (1,2), (1,3), and (1,5).

$$\frac{6}{36} = \frac{1}{6}.$$

11. B. Each roll has 10 possible outcomes. However, Nguyen is looking for a number greater than 6, which is only 7, 8, 9, and 10. This is 4 out of the 10 possible outcomes. So the chance of rolling a number greater than 6 is  $\frac{4}{10} = \frac{2}{5}$ . Since we want this to happen twice in a row, we multiply the probabilities together:  $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$ .
12. C. There are a total of 10 instruments. The instruments that are not keyboards are the 2 flutes and the 3 guitars, so there are 5 instruments that are not keyboards. Therefore, the probability that she will pick an instrument that is not a keyboard is 5 out of 10, or 1 out of 2.
13. A. There is a 3 out of 4 chance that it will freeze on Friday, so that means that there is a 1 out of 4 chance that it will NOT freeze. There is a 1 out of 3 chance it will rain on Sunday, so that means that there is a 2 out of 3 chance that it will NOT rain on Sunday. Multiply to find the probability that it will NOT freeze Friday and NOT rain Sunday:  $\frac{1}{4} \times \frac{2}{3} = \frac{1}{6}$ .
14. C. There is a  $\frac{1}{6}$  chance that each roll will result in a 4 on this number cube. Since there are 60 rolls, multiply  $\frac{1}{6} \times 60 = 10$ .
15. C. Choose numbers to make this question more approachable. If there is one gold coin, there is also one silver coin, and there are 3 bronze coins. This is a total of 5 coins. The probability of picking a silver coin is 1 out of 5.
16. D. There are a total of 8 sides, and the numbers less than 5 are 4, 3, 2, and 1, for a total of 4 numbers that are less than 5. The chance of rolling a number less than 5 for each roll will be 4 out of 8, or  $\frac{1}{2}$ . This is true for each roll, so for both rolls result in a number less than 5 will be  $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$ .
17. D. There are 10 tiles, and there are 4 prime numbers from 10-19 (11, 13, 17, and 19). This means that the probability that Leo will pick a prime number is 4 out of 10 or 0.40.
18. C. The small square has an area of  $4 \times 4 = 16 \text{ in}^2$ . The larger square has an area of  $6 \times 6 = 36 \text{ in}^2$ . The shaded area is the area in between these two squares, so we subtract the two areas to find the area of the shaded region:  $36 - 16 = 20 \text{ in}^2$ . Since the total area is 36, the probability is  $\frac{20}{36} = \frac{5}{9}$ .
19. D. Quickly sketch a coordinate grid and draw the vertices. To satisfy the criteria in the question, we look for coordinates with both coordinates having a value 3 or greater. This gives us a smaller square with vertices (3,3), (3,5), (5,3), and (5,5). The area of this square is 4, since its side length is 2. The area of the larger square is 25, so the probability of choosing a point with both coordinates greater than 3 is  $4 \div 25$ .
20. C. There are 6 pairs of shoes in total in Sarah's closet. 3 are sneakers, and 3 are either rain boots or high heels. Therefore, the chance of picking sneakers is 50%, as is the chance of picking either rain boots or high heels.
21. C. Any number divisible by 2 is an even number. There are an equal number of odd and even numbers in the set, so the chance of picking two odds is equal to the chance of picking two evens.
22. C. The probability of eating a hot pepper is 1 out of every 10.
23. A. There are 13 spades in a deck of 52 cards. This means there is a  $\frac{13}{52}$  chance of picking a spade. Since there are four suits in each deck of cards, and each suit has a queen, a king, and an ace,

- there are a total of  $3 \times 4 = 12$  queens, kings, and aces in the deck. This means there is a  $\frac{12}{52}$  chance of getting either a queen, king, or ace. Therefore, the value in Column A is greater.
24. A. If there are twice as many yellow candies as red candies, and the chance of picking out a red candy is  $\frac{1}{8}$ , then the chance of picking out a yellow candy is  $\frac{2}{8}$ . This means the chance of picking out a green candy must be  $\frac{5}{8}$  (since  $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = 1$ ).
25. C. Since the name starting with B is replaced into the hat before the name starting with C is picked, the two events are independent. Since we know what the probability of picking B is, and since we know the probability of picking both B and C, we can set up an equation to solve this. If  $x$  equals the probability of picking out a name that starts with C, then:  $\frac{2}{15} \times x = \frac{1}{45}$ . We can solve this by dividing both sides by  $\frac{2}{15}$ :  $x = \frac{1}{45} \times \frac{15}{2}$ . This simplifies to  $x = 15 \div 90$ , or  $x = \frac{1}{6}$ .
26. B. There are a total of 36 possible outcomes. This is because each die has 6 outcomes, and we have two dice. The only way to have a sum of 2 is if both dice show a 1. The chance of this is  $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$ . The only way for there to be a sum of 11 is if one die shows a 5 and the other shows a 6. But this can happen two different ways (imagine if we labeled the die A and B; A could be 5, and B could be 6, or A could be 6, and B could be 5). Therefore, there is a  $\frac{2}{36}$  chance of getting a sum of 11.
27. A. There are 8 multiples of 3 in the set of numbers: 3, 6, 9, 12, 15, 18, 21, and 24. There are 5 multiples of 5: 5, 10, 15, 20, and 25. This is 12 unique numbers that are circled or in purple, because 15 must not be counted twice. Therefore, 12 out of the 25 numbers are either circled or shaded, meaning there are 13 out of 25 numbers that are neither circled nor shaded in purple. Since  $50\%$  of  $25 = 12.5$ , and  $12.5$  is less than 13, the value in Column A is greater.

### Mean, Median, Mode, & Range

- B. In a set of numbers, the mode is the number that appears most frequently. In this set, the numbers represent number of hours spent doing homework:  $\{1, 2, 2, 2, 3, 3, 4\}$ . 2 appears a total of three times.
- A. In a set of values, the average (mean) is found by dividing the sum of all values in that set by the number of values in that set. We can use this formula:
 
$$\frac{\text{sum of values in set}}{\text{number of values in set}} = \frac{(x_1 + x_2 + x_3 \dots x_n)}{n} = \text{mean}$$
 , where  $x_1 + x_2 + x_3 \dots x_n$  represents the sum of the values in the set, and  $n$  represents the total number of values in the set. In this case, we are already told that the sum of all values is 15. We know that there are 5 values, as given by "five soccer games." This means  $15 \div 5 = 3$ .
- D. Use the formula  $\frac{\text{sum of values in set}}{\text{number of values in set}} = \frac{(x_1 + x_2 + x_3 \dots x_n)}{n} = \text{mean}$ . We know that the average (mean) is 88, and that there were 4 exams (i.e. 4 values in the set). Substituting, we arrive at  $\frac{(x_1 + x_2 + x_3 + x_4)}{4} = 88$ . We're asked to find the total (sum) of the four exam scores, which is the sum of value in the set, or the numerator in the equation. To do this,  $88 \times 4 = 352$ .
- D. To find the average, we divide the sum of the values in the set by the number of values in the set. In this case, the sum of weights is  $5 + 5.5 + 6 + 6.5 + 7 = 30$ . There are 5 balls in total, so  $30 \div 5 = 6$ .

5. B. Use the formula  $\frac{\text{sum of values in set}}{\text{number of values in set}} = \frac{(x_1 + x_2 + x_3 \dots x_n)}{n} = \text{mean}$ . We know that the average should be 90, and that there will be 5 quizzes. In terms of the formula, this becomes  $\frac{(x_1 + x_2 + x_3 + x_4 + x_5)}{5} = 90$ . Thus, the sum of the quiz scores must be  $90 \times 5 = 450$ .
6. C. The median is the value in the middle of the set. When there is an odd number of values in a set, there will always be a middle value given in the set. In this case, there is an even number of values. When this happens, we find the mean (average) of the two values in the middle of the set. In this case, this is the average of the 3<sup>rd</sup> and the 4<sup>th</sup> highest values. These are 2 and 3. The average of 2 and 3 is  $(2 + 3) \div 2 = 2.5$ .
7. B. An equilateral triangle has three sides of the same length. We are given the measurement of one side of each of the 5 different triangles. To find the average perimeter, we must find the sum of the perimeters and divide by the number of perimeters we're given. The perimeters are  $3 \times 3 = 9$ ,  $4 \times 3 = 12$ , etc. The total perimeter of all 5 triangles is  $9 + 12 + 15 + 18 + 21 = 75$ . Since there are 5 perimeters (one for each triangle), the average is  $75 \div 5 = 15$ .
8. D. First, list the values in the range in order from least to greatest:  $\{5, 10, 15, 20, 25, 25, 30\}$ . The middle value is 20. The range is the difference between the least and greatest numbers. In this case, this is  $30 - 5 = 25$ . Therefore,  $20 + 25 = 45$ .
9. C. Use the formula  $\frac{\text{sum of values in set}}{\text{number of values in set}} = \frac{(x_1 + x_2 + x_3 \dots x_n)}{n} = \text{mean}$ . We know in this case that there are 7 students with an average of  $x$ . In the formula, this becomes  $\frac{\text{sum of values in range}}{7} = x$ . We're asked to solve for the total height (sum of values in set), so this is simply  $7 \times x$ , or  $7x$ .
10. D. Use the formula  $\frac{\text{sum of values in set}}{\text{number of values in set}} = \frac{(x_1 + x_2 + x_3 \dots x_n)}{n} = \text{mean}$ . Substitute values:  $\frac{(x + 3x + 5x)}{3} = 45$ . This simplifies to  $9x = 135$ , and  $x = 15$ .
11. C. Use the formula  $\frac{\text{sum of values in set}}{\text{number of values in set}} = \frac{(x_1 + x_2 + x_3 \dots x_n)}{n} = \text{mean}$ . Substitute in known values:  $\frac{(274 + 86)}{4} = \text{mean}$ . This simplifies to  $360 \div 4 = 90$ .
12. A. If a set of 5 numbers has an average of 13, then let  $x$  represent the sum of the numbers in that set:  $\frac{x}{5} = 13$ . This means  $x = 65$ . If we change the set of numbers by adding one number, we increase the average to 18. This can be represented algebraically as  $\frac{65 + x}{6} = 18$ . Solving for  $x$ , we simplify to  $65 + x = 108$ , and  $108 - 65 = 43$ . The number that must be added is 43.
13. D. We know that the mean is the sum of a set of values divided by the number of values in the set. Here, there are 5 values in the set, so we know the denominator of the equation. We also know the mean. So, we can express the information given as  $\frac{(3 + 4 + 5 + 10 + x)}{5} = 5.6$ , where the numerator represents the sum of values in the set. We multiply both sides by 5 for  $3 + 4 + 5 + 10 + x = 28$ . We then combine like terms and find that  $x = 6$ .
14. B. Since we are asked to calculate several values, list the values in the set from least to greatest:  $\{4, 6, 6, 6, 7, 7, 7, 8, 8, 8, 8, 8, 9, 9, 9, 10, 10, 10, 10\}$ . From this we can see that the mode is 8, the median is also 8, and the range is  $10 - 4 = 6$ . We can see that the mode is equal to the median, so C is false. We can also see that the range of the scores is less than the median, so A is false. Similarly, the range is less than the mode, so D is false. Therefore, B must be true.

15. C. The information presented can be represented as  $\frac{x}{20} = 15$ , where  $x$  is the total number of box-tops collected, 20 is the number of students, and 15 is the per-person average number of box-tops collected. Thus,  $x = 15 \times 20 = 300$ , which is the total number of box-tops collected.
16. C. The mean of the recommended daily values is  $(10\% + 20\% + 25\% + 45\%) \div 4$ . This is  $100\% \div 4 = 25\%$ . This is equal to the recommended daily value in cereal C.
17. B. Remember that a set of values can have more than one mode, in the event that more than one value appears the same number of times in the set. In this case, 5 appears three times, and 3 also appears 3 times. This means there are two modes: 3 and 5. The mean of the modes is  $(3 + 5) \div 2 = 4$ . Thus, the value in Column B is greater.
18. B. Parker's current average can be represented as  $\frac{x}{4} = 11$ , where  $x$  represents the total number of points scored over four games.  $x = 11 \times 4 = 44$ . If Parker scores an additional 16 points in a 5<sup>th</sup> game, the new average will be  $\frac{44+16}{5} = 12$ .
19. B. The mode is 1, since no other digit appears more than once. To find the average, we divide the sum of the values in the set by the number of values in the set. In this case, the sum is  $3 + 1 + 4 + 1 + 5 + 9 = 23$ . There are a total of 6 values in the set, so we divide 23 by 6. We don't have to find the exact value, since we know the average must be somewhere between 3 and 4 (since  $3 \times 6 = 18$ , and  $4 \times 6 = 24$ , and 23 is between these two values). Regardless, the mode is less than the mean.
20. D. Since we don't know what the values in the set are, we can't possibly determine the mode. The best we can do is figure out what the sum of the values in the set is, though we don't even need to do this to solve the question. We don't need to find  $x$ .
21. B. First, find the set, which is  $\{4, 8, 12, 16, 20, 24\}$ . The range is the difference between the greatest and least value, or  $24 - 4 = 20$ . The median is the middle value, but since there are an even number of values in the set, we take the average of the two that are in the middle. This is  $(12 + 16) \div 2 = 14$ . We also know that the mean is 14, since  $4 + 8 + 12 + 16 + 20 + 24 = 84$ , and  $84 \div 6 = 14$ . Therefore,  $14 + 14 = 28$ , which is greater than 20.
22. B. Since there are an odd number of values in the set, we take the middle value. This is  $\frac{1}{3}$ , since  $\frac{1}{2} > \frac{1}{3}$  and  $\frac{1}{4} < \frac{1}{3}$ . The mean of the three fractions is found first by adding the fractions. This is accomplished by finding a common denominator, which is 12. Multiply the numerator and denominator of each fraction by the same integer to convert the fractions to have a denominator of 12. This is  $\frac{6}{6}\left(\frac{1}{2}\right) + \frac{4}{4}\left(\frac{1}{3}\right) + \frac{3}{3}\left(\frac{1}{4}\right)$ , which simplifies to  $\frac{6}{12} + \frac{4}{12} + \frac{3}{12} = \frac{13}{12}$ . We divide this by the number of values in the set, which is 3.  $\frac{13}{12} \div 3 = \frac{13}{12} \times \frac{1}{3}$ , which simplifies to  $\frac{13}{36}$ . Since the median is  $\frac{1}{3}$ , which is also equal to  $\frac{12}{36}$  (if we multiply both the numerator and denominator by 12), we can see that the mean of  $\frac{13}{36}$  is greater.

### Interpreting Data

- A. The segment representing rent must be larger than the segment representing food, which must be larger than the segment representing utilities, which must be larger than the segment representing clothes: rent > food > utilities > clothes. Only the graph in choice A represents these relationships.
- C. Saturday represents 50% of the graph. If we let  $x$  represent the total number of students, then we know that  $0.5x = 24$ , and  $x = 48$ . Friday represents 25% of the graph, or 12 students.



Therefore, Sunday + Monday + Thursday must account for the remaining 12 students. Sunday accounts for half of these 12 students, or a total of 6 students.

3. D. Though we can't know exactly what percentage of the total circle C represents, we can eliminate all other answer choices except for D. This is because if 400 students took the test in total, the 200 students would represent half the circle. Similarly, if 600 students took the test, then 200 students would represent a third of the circle, and if 800 students took the test, then 200 students would represent a fourth of the circle. None of these scenarios is true. However, if 1,000 students took the test, then 200 students would represent a fifth of the circle, which appears to be true.
4. A. Each point has a value. On the  $x$ -axis, the value is the amount of time spent studying. On the  $y$ -axis, the value is the score achieved on the exam. Each point represents a student's score as well as the amount of time they spent studying.
5. D. Find 100 on the  $y$ -axis, which represents score on the exam. Trace over to the right until hitting a point, then trace down to the  $x$ -axis to find the number of hours spent studying, which is 5.
6. B. A score of 45 and 85 are much lower/higher than the existing points along the 0 and 1 hour gridlines. 75 is also much higher than the data there, which is closer to 60 and 65. Though there are no exact points shown at 30 minutes, we can infer from the data that it would be closest to 65.
7. B. To find the expected value using a line of best fit, find the corresponding value along the line of best fit. Note that this is the expected value, not the actual value of data recorded and represented by the points on the graph. In this case, we find  $20^\circ$  on the  $x$ -axis, and trace over to the line of best fit. Then, we trace over to the  $y$ -axis to find the amount of money spent. In this case, the expected amount is \$40. This is different from the actual data point recorded at  $19^\circ$  and  $\sim$ \$38.
8. C. Each plot point on the scatter-plot represents a party. The fact that there are multiple points means there were parties of different sizes (number of diners in the party) and that they paid a different amount for dinner. Count the total number of points to find 17 total parties.
9. B. The median is the middle value in a range of numbers. We can visually and simultaneously work our way from top to bottom (more expensive to less expensive meals) and bottom to top (less expensive to more expensive meals) and cross out each point as we find the middle. Be careful not to work from left to right and right to left, as that tells us about the median size of the party. We see that there are 7 points below 150, and 8 points above 150. Cross out one of the points on the 150 line and we are left with another point on the 150 line. This is the median price paid.
10. B. For a party of two diners, we look at the gridline extending up from 2 on the  $x$ -axis. There are 3 points, one each at 150, 75, and 50. Notice that all numerators are the same, so we only need to look at the denominator. There are 3 data points, so we need to divide the total cost of dinner for all 3 parties by the number of parties, not the total number of people in the party, or the size of each party.
11. C. The  $y$ -axis represents the number of people, and the  $x$ -axis represents the weight groups of the signees. Each vertical bar represents the number of people who signed, divided into each weight grouping. Add the number of people represented by each bar except for the right-most bar:  
 $3 + 1 + 2 = 6$ .
12. C. The  $y$ -axis tells us the number of times a signee fell into each of the different weight ranges given by the  $x$ -axis. Therefore, work from left to right and right to left along the  $x$ -axis. From right to left: 5 signees fell into the 170-179 range. From the left: a total of 4 people fell into the 140-149 and 150-159 ranges. The 5<sup>th</sup> signee from the left fell into the 160-169 range. This leaves one more person in the 160-169 range.
13. C. Each column represents the number of signees who fell into a given weight range. Since this is the case, we know 5 people fell into the 170-179 range, while 6 fell into the other, so D is false. Each individual columns does not represent total weight or number of signees.

14. A. In a stem-and-leaf plot, the values in the leaf-portion of the chart represent individual data points. In this case, each data point represents the number of minutes spent exercising in a single day. Since there are 26 data points in the left-portion of the chart, there are 26-days' worth of data.
15. C. The median number in a range is the one that is exactly in the middle of the range. We need simply to cross off each value in the leaf portion beginning with the 2 in the 7 row working our way right, and the 9 in the 9 row working our way left. We work our way toward the middle number, each time to find that it is 87.
16. D. The highest score is the number with the largest value, which has a stem of 9 and a leaf of 9, representing a score of 99. The lowest, using similar logic, is 72. The difference is  $99 - 72 = 27$ .
17. D. Since we know that a single box costs \$0.90, and we know that the total order cost of a single box of pencils costs \$3.89, we know that  $3.89 - 0.90 = 2.99$ , or the cost of shipping. We can verify this using the other costs given. For 5 pencils,  $5(0.9) = 4.5$ , and  $4.5 + 2.99 = 7.49$ .
18. C. Take the difference between Weeks 4 and 3, and between Weeks 5 and 4. The first is  $1,965 - 1,710 = 255$ , and  $2,220 - 1,965 = 255$ .
19. D. There are three columns with miles less than 40. These columns reach up to different numbers of days: 3, 1, and 2, for a total of 6 days. The 40-49 column corresponds with 5 days. However, since each column represents a range of miles, we don't know exactly how many days the bus travelled more than 40 miles. There could be 5 days where the bus only travelled 40 miles, for example.
20. A. Divide the total cost by the number of ounces of tea. For Column A,  $8.8 \div 8 = 1.1$ . For Column B,  $21.6 \div 24$ . We don't need to do the actual calculation for Column B to see that Column A will be greater than Column B, since Column A is greater than 1, and Column B will be less than 1.

### Data & Probability Mixed Practice

1. A. (Probability) There are 9 green marbles and total 24 marbles in the box, so the probability of the first draw is  $\frac{9}{24}$ . Once Luna drew one green marble out from her first draw, she is not putting that marble back in the box. Therefore, the total number of marbles in the box now is 23 when she does her second draw. There are still 8 red marbles in the box, so the probability of the second draw being a red marble is  $\frac{8}{23}$ . To find the probability of the two draws without replacement, multiply the two probabilities,  $\frac{9}{24} \times \frac{8}{23}$ . Simplify and solve the fractions to get  $\frac{3}{8} \times \frac{8}{23} = \frac{3}{23}$ .
2. C. (Probability) There are 4 prime numbers within 1-10, which are 2, 3, 5, and 7. Therefore, the probability of rolling a prime number each time is  $\frac{4}{10}$  or  $\frac{2}{5}$ . Since she is rolling the dice twice, multiply the probability for the first and second roll:  $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$ .
3. D. (Mean, Median, Mode & Range) For the class average to be an 82 after Ben's test, the sum of the class scores must be  $82 \times 16 = 1,312$  because the sum = number of data  $\times$  data average. Using the same formula, find that the average before Ben's test was  $80 \times 15 = 1,200$ . The difference between the sum of the values before and after Ben took the exam was  $1,312 - 1,200 = 112$ , so Ben must get a score of 112 for the class average to become an 82.
4. C. (Mean, Median, Mode & Range) We know that the range is the maximum value minus the minimum value. Since the range and the minimum value is provided, we can find x,  $x = 16 + 2$ , which means x is 18. If x is 18, we can use that information and the rest of the given data to

calculate the mean of the data set. To do that, use the formula: mean

$$= \frac{\text{sum of data}}{\text{number of data}} = \frac{(2+6+8+12+18)}{5} = 9.2.$$

5. C. (Mean, Median, Mode & Range) The mode is the most frequently occurring value. For *A*, the mode is 4. For *B*, the mode is 8. For *C*, the mode is 3. For *D*, the mode is 6. Therefore, *C* has the mode with the smallest value.
6. B. (Interpreting Data) There are 48 hours on a weekend. If Nam spent 25% of time sleeping, that means he spent  $0.25 \times 48 = 12$  hours sleeping.
7. C. (Mean, Median, Mode & Range) We are given the measurement of one side of each of the 4 different squares. To find the average area, we must find the sum of the areas and divide by the number of areas we're given. The areas are  $4 \times 4 = 16$ ,  $5 \times 5 = 25$ , etc. The total area of all 4 squares is  $16 + 25 + 36 + 49 = 126$ . Since there are 4 areas (one for each square), the average is  $126 \div 4 = 31.5$ .
8. C. (Mean, Median, Mode & Range) The median is the middle value. List out all the pastries in order from the least numbers sold to the greatest number sold: "Macarons, Brownies, Tarts, Croissants, Cannolis, Cookies, Doughnut". The middle pastry is Croissants.
9. B. (Interpreting Data) The mode is the most frequent appearing value. Based on the stem-and-leaf plot, that value is 48.
10. B. (Interpreting Data) The median is the middle value. List the numbers out from least to greatest. The middle value is 45.
11. C. (Probability) The even numbers on a dice are 2, 4, and 6, so the probability of rolling an even number is  $\frac{3}{6}$  or  $\frac{1}{2}$ . The odd numbers on a dice are 1, 3, and 5, so the probability of rolling an odd number is also  $\frac{3}{6}$  or  $\frac{1}{2}$ . Therefore, the two probabilities are equal.
12. A. (Interpreting Data) The median is the middle value. List the numbers out from least to greatest and identify the middle value. In this case, it is 30. The mean is the sum of the values divided by the number of values which here is 152 divided by 5, which is a little more than 150 divided by 5 which is 30. This makes the value in column A greater.

## Final Practice Test (Form B)

### Verbal Reasoning

1. C. "Authentic" has a positive connotation, and means that something is undisputedly real or genuine. This is the opposite of fake. "The autograph was authentic, as proven by ink and handwriting analysis." Notice that in this case the thing being proven authentic is writing, but this may or may not be the case in other examples, and the thing may or may not be basic.
2. B. "Superior" contains the root word "super," which means "above" or "over." Something that is "superior" is over and above other things; in some way, it is better than other things. The thing being described might be food, in which case it may be delicious, but it might be of superior quality and not as delicious. "Superior" applies to many different situations, not just food. "The reproduction of the *Mona Lisa* is of superior quality." The painting is clearly not delicious.
3. D. "Accurate" has a positive connotation, as does "true" and "prized." The other choices have negative connotations. "The artist was so accurate in her painting of the dog that it looked like it might jump off of the canvas at any time." In this case, we can see that "prized" is not a good substitute for "true" or "accurate."
4. A. "Eligible" has a strong positive connotation, which is different from the negative connotation of "barred" or "forced." There is a subtle but important difference between "allowed" and "required." "Required" is similar to "forced," so we can rule this out as well. "The judges ruled that Carly was eligible to compete in the charity pie-eating contest." Carly does not have to compete, but is allowed to.
5. A. "Capacity" is a neutral word, so we might rule out "patience" and "envy" for having positive and neutral connotations. The word has a root word, "cap," which means "to hold." Thus we can

- guess that “capacity” has something to do with being about to hold something – perhaps an amount or quantity of something. “The jug did not have the capacity to hold more than a gallon.”
6. D. “Navigate” is similar to the word “navigation” and “navigator,” both of which have to do with finding the way from one place to the other. This doesn’t necessarily mean to escape, since people could have other reasons for going from one place to another. On a ship, for example, a navigator could steer a ship and guide it from one place to another.
  7. D. “Unruly” has a negative connotation. We see that “unruly” contains the word “rule,” which relates to law and order. Since the prefix “un-” precedes “rule,” we know that we are talking about the opposite of order and law, or something wild. “Unruly” does not necessarily mean “broken” or “timid.” “Tim, quiet and shy by nature, had unruly hair that refused to be combed.”
  8. A. “Subtle” includes the prefix “sub-,” which means “beneath,” but don’t let this confuse you. A sentence using the word helps us figure out its meaning. “The flavors were very subtle, and therefore difficult to distinguish individually.” We can see that none of the other words work as substitutes for “subtle” in this sentence.
  9. A. “Abode” refers to a place of residence, like a home. “Having a curfew means having to return to your abode by a certain time.” Replacing the other words don’t make sense here. A trick to remembering this word is that there is a material called “adobe,” which was often used to make homes, that looks similar to “abode.”
  10. D. “Melancholy” has a strong negative connotation, so we can rule out the other answer choices, since they all have positive connotations. “The sad song filled Melanie with melancholy.”
  11. A. “Potential” is used to describe something that has the possibility of happening. “Her teacher said that she had the potential to be a great artist.” In other words, her teacher said that it was “conceivable” (not impossible, lavish, or powerful) that she could be a great artist.
  12. B. “Volume” doesn’t have anything to do about with “figment,” which is used to describe imaginary things. “Volume” isn’t the same thing as “siren,” but rather describes a quality of it – how loud it is. This is the same as “silence.” Instead, “volume” can refer to the amount of something – the quantity of it. “The box has a volume of three cubic meters.”
  13. C. “Agile” includes the root word “agi,” which means “go,” “move,” or “do.” The suffix “-ile,” which means “having the qualities of,” follows “agi.” So, we can guess that “agile” has something to do with the ability or quality of being able to move or go or do. The word “nimble” has a similar meaning. “The rabbit is very agile, capable of making sharp turns quickly and skillfully.”
  14. C. “Assimilate” includes the root word “simil,” which means “like” or “resembling” (think “similar”). This root word is preceded by another, “as,” which means “toward” or “to.” So, if we assimilate something, we move toward likeness or similarity. This is most closely represented by the word “incorporate,” which means to include in part of something bigger. We know this because “incorporate” is made up of the roots “in,” which means “into” or “towards,” and “corp,” which means “body” or “whole.”
  15. C. “Haphazard” sounds like it has a negative connotation (the word “hazard” makes us think this), but it is actually used to refer to something random or lacking in organization. “Toby threw the cards up in the air, which then landed haphazardly on the floor.”
  16. D. “Orthodox” refers to something that is very traditional or conservative. This doesn’t mean that the thing is necessarily important, impressive, or perfect. “He was orthodox in his belief that dogs should sleep outside in the rain.”
  17. A. “Persuasive” is similar to the word “persuade,” which means to convince. If something is persuasive, then it is convincing, or very compelling (which is related to the word “compel,” or “force”). We can therefore rule out “unconvincing.” Something that is convincing is not necessarily stubborn or sophisticated, but it is something that cannot be denied (otherwise it wouldn’t be very convincing). “The argument was convincing because of its simple logic.”
  18. A. “Tirade” has a negative connotation, so we can rule out all of the other choices, which have positive connotations. “The politician went on a tirade and accused his opponent of corruption.”



19. B. “Vilify” has a strong negative connotation, which is mirrored in “criticize.” Using a sentence can help us choose between the two. “The newspaper vilified the candidate for his support of the unpopular law.” The newspaper can criticize the candidate.
20. D. “Glut” actually has a positive connotation, and is used to describe a situation where there is plenty and abundance. This is similar to the word “surplus”: “Thanks to their care and attention, the tomato crop thrived, and there was a glut of tomatoes at the end of the season.”
21. A. “Inundate” has a negative connotation, which is reflected in “devastate” and “bombard.” The word “inundate” is a word that is used to describe a situation where something is overwhelmed by an influx of something. For example, a bombardment. “The store was inundated with requests for its popular dessert, and was unable to make enough to satisfy everyone.”
22. D. We are looking for a verb that usually describes an action taken by a dog to a stick. We know that the opposite of what was expected happened, which was that it “wagged its tail and looked blankly.” So, we can infer that what was expected was that the dog would do the opposite of this, or retrieve (get) the stick. It could observe the stick from where it was. There’s no indication that the dog has the stick, so it couldn’t release it. “Incorporate” isn’t a verb usually used to describe something a dog does to a stick.
23. C. If carpooling has been encouraged, and if there are high taxes on gas, we can expect there to be fewer cars on the road, not more. We expect that the number of cars would decrease, or diminish. “Accelerate” isn’t a word that’s used to describe a number of things. “Develop” would mean that the number of cars is increasing. Similarly, “restore” would mean that the number of cars was returning to some other number, which is not supported here.
24. A. Divya was nice, but we can infer that Evan was the opposite because of the clues given in the sentence (“especially”; “could not understand why”). This means that Evan was not nice, or hostile. There’s no evidence to suggest that Evan is slow moving. He certainly wouldn’t be “vibrant” or “welcoming,” which is more like what Divya was like.
25. C. The sentence describes a difference between two types of smiles. If one is forced, or fake, then the other must be the opposite. The opposite of “forced” or “faked” would be “natural.” “False” is a synonym for “fake.” “Widespread” and “flexible” don’t make sense in context of describing a smile.
26. B. The beavers are described as being “tireless” in their work. This means that they are “industrious,” not “useless.” Whether beavers are private or not isn’t supported by the statement (in fact, the sentence says that they can be “observed”). If the beavers were content, they might not be so industrious or hardworking.
27. C. If Ned is able to “control” many decisions, then he must be powerful, or influential. So, we could say that his influence with the rich and powerful grew. If Ned was apprehensive (nervous) about the rich and powerful, his influence wouldn’t grow and he wouldn’t have the ability to control the decisions. This is the same if we substitute the word “mistrust” into the blank. The ability to keep things private (discretion) isn’t relevant to the sentence, which is about Ned’s influence and power.
28. A. The sentence describes how a museum’s collection is growing. This means the number of items in its collection is increasing. The only word that describes how the number of something might change is “acquiring,” which means “to obtain.” This is the opposite of “concealing” or “demolishing,” which would decrease the number of items in the collection. “Contaminating” (spoilage) doesn’t have anything to do with increasing or decreasing the number of something.
29. B. The sentence describes what happened after a storm. During the storm, buildings were “completely destroyed.” After the sentence, some verb is used to describe “new” buildings. So, we can infer that what happened after the storm was rebuilding, or construction. Notice that “renovate” means to improve on something, which must first exist. The more appropriate word here is “construct,” since there is nothing to renovate.
30. A. The sentence describes how parents were “surprised” by something. This is an “abrupt” change, since one that is “planned” would not be a surprise to anyone. A cautious change would

likely be slower and less surprising. We don't know whether or not the change is something they wanted the change to happen.

31. B. We can tell that Jeff needed to read the book for this report. Jeff's emotions, upon realizing that he won't be able to read everything that is required, is probably negative. The best word is "glum," which means that Jeff was running out of hope. It doesn't make much sense that Jeff would be encouraged (heartened), that he was nonchalant (casual; not caring), or careful.
32. A. "Critics" and "opponents" typically don't agree with people who share different views. This is compared with Kelly's "passionate" and "convincing" argument using the word "even," which tells us that the opposite of what we expect actually happened. So, we'd expect Kelly's opponents to disagree with her, but the "even" clues us in that they actually agreed (concurred) with Kelly. If the opponents "feuded" or "opposed" Kelly's view, then the word "even" wouldn't make sense.
33. D. The sentence compares how someone often arrives at "false conclusions." Because of this, the person likes to speak with strangers before they can say they know him or her. To make a decision without knowing is to presume. It doesn't make sense that to get to know someone better and speak with him or her that the person would "decline" to know a person or "fail" to do so. The phrase "intervene to know a person" doesn't make sense.
34. C. The athlete "refused to give up," which is very nearly the definition of remaining "steadfast." The athlete may have been careful, but this would not logically complete the sentence, which describes the athlete's perseverance. Being caring (humane) or awkward (ungainly) in this case is illogical given the purpose of the sentence.
35. C. Be careful of the phrase "not have seemed," since the word after it would therefore be the opposite of the clause that follows ("as such...days"). If it took many days to travel a certain distance, then traveling that same distance in just hours would be impossible, or infeasible. Since the phrase "not have seemed" precedes the blank, then we know that we should take the opposite meaning of the blank. So, we should choose "feasible" so that the sentence ends up describing something that is "not" feasible. Traveling some distance in hours instead of days would have seemed very much difficult or shocking (astounding). It would have seemed impractical, not "not" impractical.
36. D. The way the sentence is structured tells us that the two parts of the sentences are related. The last clause tells us that "hard work and persistence" is important to achieving goals. This means that if difficulties happen, one should "persevere," or be persistent or continue. To abstain would be to refrain or not participate, the opposite of continuing. Cooperating could be important, but it is nonsensical given the context in the rest of the sentence.
37. B. If Jonas is worth emulating (copying), then he must display or possess the positive values (otherwise what would people copy?). So, we know that Jonas doesn't dispute (argue with) these values, nor does he reject or simply remember them. Instead, he lives and embodies (exemplifies) them.
38. D. The sentence describes Lewis and Clark positively, saying that despite risk, they still chose to go on an expedition. Since this happened, it is not a fantasy. It is an example of, or testament of, the human spirit.
39. B. We are told that the supplies need to be distributed "as soon as possible." This means that it is urgent, or imperative, the opposite of inconsequential (which would mean it is not important). Though it is good (virtuous) that this happen, the context of the sentence tells us that we are discussing the importance of that fact, not how good it is. It we don't know that the supplies are "imminent" (pending, or happening soon).
40. A. Since the game is one-sided, we know that one team beat the other team without much competition. We could say that one team was "annihilated" (destroyed) by the other team. The losing team might be dumbfounded (shocked) or transformed, but we don't know that just based on this sentence. One team does not "practice" another; a team (or teams) practice.

## Quantitative Reasoning

1. *Data & Probability – Interpreting Data.* D. Since a circle has  $360^\circ$ ,  $90^\circ$  represents  $\frac{1}{4}$ , or 25% of the total circle. This means that 25% of 360 shoppers chose apples, and 75% did not.  $0.75 \times 360 = 270$ .
2. *Geometry – Coordinates.* B. The translation of a shape is also called a slide, which means shifting the position of a shape into a different part of the plane. The rotation of a shape is the moving of a shape around a certain point. A reflection is the flipping of a shape over an imaginary line. The dilation of a shape is the increasing or decreasing of a shape's area. In this case,  $ABC$  has been rotated around the origin.
3. *Numbers & Operations – Estimation.* D. Approximate by rounding each base to the nearest 10.  $81^2$  becomes  $80^2$ , and  $19^2$  becomes  $20^2$ . This leaves us with  $6,400 \div 400 = 16$ .
4. *Data & Probability – Mean, Median, Mode, & Range.* B. In a set of numbers ordered from least to greatest, the median is the middle number. If there are two numbers in the middle, then take the average of the two numbers. In this case, the numbers are the number of kilometers run per day, which are  $\{3, 4, 5, 6, 7\}$ . The middle number is 5.
5. *Geometry – Geometric Objects.* B. The hatch marks tell us that the quadrilateral has four sides of equal length. We can see that the angles are not right angles, so we can automatically rule out rectangles and squares. What is left is a rhombus, which has 4 sides of equal length.
6. *Data & Probability – Interpreting Data.* B. To find the expected value using a line of best fit, find the corresponding value along the line of best fit. Note that this is the expected value, not the actual value of data recorded and represented by the points on the graph. In this case, we find  $65^\circ$  on the  $x$ -axis, and trace up to until we hit the line of best fit. Then, we trace over to the  $y$ -axis to find that the expected value is 20 cones. This is not the same as the  $\sim 18$  or so cones actually recorded.
7. *Geometry – Coordinates.* B. In the format  $(x,y)$ ,  $x$  represents the  $x$ -coordinate, and  $y$  represents the  $y$ -coordinate. Together, these give us a point on an  $xy$ -plane. In this case, we know we are looking for the third point of a triangle which must have one right angle. Drawing the triangle resulting from each answer choice, we find that only  $(0,3)$  gives us a right triangle.
8. *Algebraic Concepts – Slope.* C. Even though there is no scale, we know that all of the running data is plotted on the same chart. This means that over the same period of time, the person who runs the farthest distance is running at the fastest pace. If we pick the first gridline along the  $x$ -axis (time), we can see that the greatest distance is run by Jamie. The least is run by Britney, and Oscar is in the middle.
9. *Numbers & Operations – Estimation.* A. Taking the square root of an integer only results in an integer when it is a perfect square – a number multiplied by itself. For example, 64 is a perfect square because  $8 \times 8 = 64$ . In this case, no integer multiplied by itself gives us 130. Since we know that  $10 \times 10 = 100$ , we know that the square root of 130 must be at least 10. We can try the answer choices:  $11 \times 11 = 121$ , and  $12 \times 12 = 144$ . 121 is closest to 130.
10. *Algebraic Concepts – Ratios, Proportions, & Scale Factors.* D. Since the two triangles are similar, we know that each side corresponds to the same side on the other triangle (for example,  $JK$  corresponds to  $GH$ ). Since we know that  $GI$  is 0.75 times as long as  $JL$ ,  $GH$  must also be 0.75 times as long as  $JK$ . This means  $JK = 12 \div 0.75 = 16$ .
11. *Algebraic Concepts – Slope.* A. The slope is positive, so we can rule out choices C and D since they have negative slopes (lines going down). The equation has a line with a  $y$ -intercept of 6, which only choice A has. Choice B has an  $x$ -intercept of 6, which means its  $y$ -intercept is actually negative.
12. *Numbers & Operations – Estimation.* C. Profit = revenue – cost. To estimate the profit, we need to find the average revenue and the average cost. To find the average revenue, we first find how much she makes per week, and then multiply that by the total number of weeks. We can round the selling price of jeans to \$100 and multiply by the average number of jeans sold:  $(150 + 60) \div 2 =$



105. This gives us  $100 \times 105 = \$10,500$  per week for 10 weeks, giving us a total of \$105,000 for the season. We are given the cost of between \$30,000 and \$50,000, which means the average is \$40,000. The difference between the total revenue of \$105,000 and the total cost of \$40,000 is \$65,000.
13. *Geometry – Geometric Objects*. A. The sum of all 3 angles in a triangle is  $180^\circ$ . If  $x$  is obtuse, it is greater than  $90^\circ$ . This means the remaining 2 angles must add up to a number less than 90 (since, for example,  $180 - 91 = 89$ ). Only 85 meets this requirement.
14. *Numbers & Operations – Factors, Multiples, Primes*. B. Since we know that the base of both exponential terms is 7, we know that we are multiplying 7 by itself a certain number of times to find 343.  $343 \div 7 = 49$ , and  $49 \div 7 = 7$ . So,  $343 = 7 \times 7 \times 7$ , or  $7^3$ . Since  $x$  and  $y$  are different positive integers, the only way for this to work is for one variable to represent 1 and the other to represent 2. This would give us  $7^1 \times 7^2 = 7^3 = 343$ . Therefore,  $1 \times 2 = 2$ .
15. *Data & Probability – Interpreting Data*. C. The  $y$ -axis represents Zhe's speed, and the  $x$ -axis represents the time spent while driving. Therefore, lines that slope upward represent increasing speed, lines that slope downward represent decreasing speed, and flat lines with no slope represents a constant speed. From minutes 7-9, there is a downward slope, indicating a decreasing speed. This doesn't mean that Zhe stopped (since that would mean a speed and  $y$ -axis value of 0), or that she was necessarily driving downhill. It only means that she was slowing down, as the  $y$ -axis values decrease from 30 to 10 from 7 minutes to 9 minutes.
16. *Algebraic Concepts – Slope*. C. There are two types of numbers in this question. One is variable, and one is constant. The \$20 paid is described as a "flat rate" (and is an algebraic constant). This means Chui will get paid \$20 no matter how many newspapers she delivers. The \$0.25 paid is variable, because it depends on the number of newspapers delivered. There is a variable,  $n$ , described in the question. We can represent the money Chui receives from delivering  $n$  newspapers as  $0.25n$ . Together, this becomes  $0.25n + 20$ , or  $20 + 0.25n$ .
17. *Geometry – Coordinates*. C. Draw a coordinate plane to help visualize the information. Since the center of the circle is at the origin, then the diameter of the circle must run through the origin. Since the radius is equal to half the diameter, we know that the length of the radius is equal to the distance from the origin to point  $(-2,3)$ , which is in the second quadrant. Therefore, the other end of the line must be in the fourth quadrant. The only answer choice in the fourth quadrant is  $(2,-3)$ .
18. *Numbers & Operations – Fractions*. C. Solve by performing long division:  $\frac{1}{4} = 0.25$ ,  $\frac{1}{3} = 0.33$ ,  $\frac{1}{6}$  is between 0.16 and 0.17, and  $\frac{3}{20} = 0.15$ .  $1 \div 5$ , which equals 0.2, is closest to  $\frac{1}{6}$ .
19. *Data & Probability – Interpreting Data*. D. There are a total of  $20 + 10 + 5 + 10 + 5 = 50$  7<sup>th</sup> graders who responded. 20 of them like chocolate, and 5 like mint. Since the question asks for the number of 7<sup>th</sup> graders who like one or the other, we know we can add these two numbers together:  $20 + 5 = 25$ . As a percentage of the 50 total students, this represents  $25 \div 50 = 0.5$ , or 50%. A circle contains  $360^\circ$ , so  $50\%$  of  $360^\circ = 180^\circ$ .
20. *Numbers & Operations – Factors, Multiples, Primes*. C. Think of the dollar amounts in terms of cents, the smallest possible denomination (so 2 dollars = 200 cents). The amount of cents Mary has must be a multiple of 8, since she has 2 times as much money as Carol and Carol has 4 times as much money as Susan. This is because, if Susan had just \$0.01, then Mary would have \$0.08. All answers represent cent amounts that are multiples of 8 except for 700, which if divided by 8 leaves a remainder (i.e. a part of a penny).
21. *Algebraic Concepts – Functions & Patterns*. B. The first five elements show how a single dot is added to the top of the figure first in the center column, then in the left column, then in the right column. Since the fifth element shows a single dot added to the center column, the next dot should be added to the left column.



22. *Numbers & Operations – Fractions*. A. When Allan first drinks his lemonade, he leaves  $\frac{1}{3}$  of 96 fluid ounces, which is  $96 \div 3 = 32$  fluid ounces. He then mixes  $\frac{1}{4} \times 32 = 8$  fluid ounces of lemonade with 16 fluid ounces of water. Since we are asked what portion of the final mixture was lemonade, we can represent this as  $\frac{8}{8+16} = \frac{8}{24} = \frac{1}{3}$ .
23. *Numbers & Operations – Fractions*. C.  $\frac{1}{6} + \frac{1}{3} = \frac{1}{6} + \frac{2}{6}$ . This simplifies to  $\frac{3}{6} = \frac{1}{2}$ . The reciprocal of  $\frac{1}{2}$  is 2. The value in both columns are equal.
24. *Geometry – Coordinates*. A. A point lies in a quadrant if neither of its coordinates is zero (which would mean that the point actually lies on an axis). In a coordinate plane, the first quadrant is in the top right of the coordinate plane, while the third quadrant is in the bottom left of the coordinate plane. This means that all coordinates of points in the first quadrant have positive values, and all coordinates of points in the third quadrant have negative values. If we were to add together the  $x$  and  $y$  coordinates of a point in the third quadrant, it will always be negative. Similarly, if we were to add together the  $x$  and  $y$  coordinates of a point in the first quadrant, it will always be positive.
25. *Algebraic Concepts – Slope*. A. We can see that line  $AC$  has a positive slope. We know that a line perpendicular to line  $AC$  would have a negative slope (we can sketch this to prove it). Therefore, the value in Column A will always be greater than the value in Column B.
26. *Numbers & Operations – Decimals*. A. Though we don't know the value of  $\sqrt{4.9}$ , we know that  $\sqrt{4} = 2$ . Since  $\sqrt{4.9} > \sqrt{4}$ , we know that  $\sqrt{4.9} > 2$ . In Column B,  $0.7 < 1$ . Therefore,  $\sqrt{4.9} > 0.7$ .  
**Educator's Note:** On this test, most questions involving square roots will involve the principal square root (the unique nonnegative square root of a nonnegative real number), which is as per the ERB's guidance. We have included question #26 in the *Solving Algebraic Equations* section of this book to demonstrate that there are both positive and negative roots.
27. *Algebraic Concepts – Ratios, Proportions, & Scale Factors*. A. Since the two rectangles are similar, the sides of one rectangle corresponds to the sides of the other. Since  $QR$  is greater than  $UV$  (since  $x > 0.7x$ ), we know that  $SR$  must be greater than  $TU$ . This means  $SR > 10$ , which means it is greater than 7.
28. *Data & Probability – Probability*. B. The probability of getting any matching pair is  $\frac{1}{5}$ . This is because once a first shoe is chosen, there is exactly one shoe out of the remaining 5 that will match it.  $1 \div 5 = 0.2$ , or 20%.
29. *Algebraic Concepts – Ratios, Proportions, & Scale Factors*. B. The value of Column A can be determined by setting up a proportion:  $\frac{15 \text{ min}}{2 \text{ mi}} = \frac{x \text{ min}}{6 \text{ mi}}$ . Cross multiply and solve for  $x$ . This gives us  $(15)(6) = 2x$ , which simplifies to  $90 = 2x$  and  $x = 45$  minutes. The value of Column B can also be determined by setting up a proportion:  $\frac{40 \text{ min}}{2 \text{ mi}} = \frac{x \text{ min}}{3 \text{ mi}}$ . Cross multiply and solve for  $x$ . This gives us  $(40)(3) = 2x$ , which simplifies to  $120 = 2x$  and  $x = 60$  minutes. Therefore, the amount in Column B is greater.
30. *Algebraic Concepts – Solving Algebraic Equations*. C. Find the value of each column separately. For  $x$ , combine like terms by subtracting 3 from both sides, leaving us with  $\frac{x}{5} = 25$ . Then, multiply both sides by 5 for  $x = 125$ . For  $y$ , add 4 to both sides, leaving us with  $7y = 35$ . Divide both sides by 7 for  $y = 5$ . Substitute 5 in for  $y$  to get  $25(5) = 125$ .

31. *Geometry – Geometric Objects*. B. Find the dimensions of the pool. Since the path is 3 ft. wide all around, then the width of the pool is  $18 - 3 - 3 = 12$  ft. and the length is  $10 - 3 - 3 = 4$  ft. The area is  $48 \text{ ft}^2$ . The area of the entire pool and path is  $18 \times 10 = 180$ . The area of the path, then, is  $180 - 48 = 132 \text{ ft}^2$ .
32. *Data & Probability – Mean, Median, Mode, & Range*. A. If Jerry drives at an average speed of 20 miles per hour, then to go 5 miles would take about 15 minutes (since  $\frac{20 \text{ miles}}{60 \text{ minutes}} = \frac{5 \text{ miles}}{x \text{ minutes}}$ , and  $300 = 20x$ , and  $x = 15$ ). This is the value of Column A. If Jerry drives at an average speed of 10 miles per hour, then to go 1 mile would take about 6 minutes (since  $\frac{10 \text{ miles}}{60 \text{ minutes}} = \frac{1 \text{ mile}}{x \text{ minutes}}$ , and  $60 = 10x$ , and  $x = 6$ ). This is the value of Column B.
33. *Data & Probability – Probability*. A. Initially, Percy has a 1 in 5 chance of choosing the winning door. However, after he opens the first door, there are only 4 closed doors remaining. His chance of choosing the winning door after the second time is 1 in 4. Therefore, his total chance of winning the boat is  $\frac{1}{5} \times \frac{1}{4} = \frac{1}{20}$ . This represents a greater chance of winning than  $\frac{1}{25}$ .
34. *Numbers & Operations – Factors, Multiples, Primes*. A. The value of Column B is 2, since  $64 = 2^6$  only has one prime factor (2). The value of A is  $x$ , which must be prime (2, 3, 5, 7, etc.). We can substitute one of these prime values in for  $x$  and see if  $x$ ,  $x + 2$ , and  $x + 4$  are prime. This only works if we plug in 3. Thus,  $3 > 2$ .
35. *Numbers & Operations – Percents*. B. Since there are 4 pennies, then there must be 3 nickels, 2 dimes, and 1 quarter. This means there is a total of 64 cents in the collection. Since a quarter is 25 cents, this means that the percentage of the total amount of money in quarters is  $25 \div 64$ . This is more than the 10% in Column A.
36. *Numbers & Operations – Integers*. C. Do not spend time finding the product of each column. We can easily multiply the units digits together:  $5 \times 4 \times 3 \times 2 \times 1 = 120$  to find that it ends in zero. We know that Column B is a multiple of 4 (as 24 itself is a multiple of 4) and 25 (since 25 is a term in the expression). Therefore, the product of the expression in Column B must be a multiple of 100 ( $4 \times 25$ ). This means it must end in two zeros (a zero each in the tens and units places).
37. *Data & Probability – Interpreting Data*. C. In Column A, there are a total of  $5 + 10 = 15$  sample responses, which account for a total of 20% of the total number of men in the group. This means there are a total of  $15 \div 0.2$  men in the group. Since  $0.2 = \frac{1}{5}$ , and we are dividing by this fraction, we know we can multiply by the reciprocal for  $15 \times 5 = 75$ . This is the total number of men in the group. Since the total number of men in the group is 5 times as much as the sample size, then we can expect that there will be 5 times as many people who will vote FOR and AGAINST. For the latter, this is  $10 \times 5 = 50$  people. Following the same logic, we know that the number of women who are expected to vote FOR is  $20 \div 0.4$ , or  $20 \times \frac{5}{2} = \frac{100}{2}$ , or 50. The two values are equal.

### Reading Comprehension

1. C. *Main Idea*. The passage goes through the steps of setting up and taking down tipis. It doesn't describe or compare different types of shelters, or go much into how they were used by different groups.
2. B. *Vocabulary*. Something that is done with efficiency is done quickly and capably, or "without much difficulty." It usually requires experience to do something efficiently. The number of people is not important to whether or not something is done efficiently.
3. B. *Inference*. In context, we can tell that the passage has to do with building and taking down tipis, which are forms of tents. The passage never mentions building permanent settlements or towns. It also doesn't mention doors. The passage discusses seams in the second-to-last paragraph, but only in context of how it was covered, not pegged.

4. A. *Organization/Logic*. These paragraphs provide step-by-step instructions about how to build a tipi. No opinion is shared, nor is any one particular event being described.
5. D. *Supporting Idea*. Simplicity and durability are mentioned in lines 51-52. Comfort is mentioned in line 6. The passage never mentions whether tipis were expensive or inexpensive.
6. C. *Tone/Language/Style*. The passage is not a story about a particular event, nor does it tell about a person's life. It is also not an advertisement, seeking to persuade. Instead, it is telling a series of facts, like an encyclopedia.
7. D. *Main Idea*. The passage does not make a judgement about which museum is the best. It describes how there are parts of the Met which are as much art as things that are in the galleries, giving examples of the patio of the Castle of Vélez Blanco and the Temple of Dendur. This means that art isn't simple to define, but could be all around, even a building.
8. B. *Organization/Logic*. The first paragraph describes the physical appearance of the Met, including its architecture (though it does not compare one style with another). Following paragraphs detail the Met's art collection. There is no mention of the Met's origin.
9. A. *Vocabulary*. To entice is to attract or to lure. If we substitute other answer choices into the sentence, none of them make sense. Endless halls of antiques could impress visitors (not repel, or discourage) visitors, but "impress" is not a word that is used to mean "cause," as "entice" does here.
10. C. *Tone/Language/Style*. Notice that the rest of the paragraph describes the variety of times and places where the art comes from. There's no mention that every visitor likes every piece of art, or that one particular time period or style of art is better or worse than others.
11. A. *Supporting Idea*. The passage never discusses whether the art should go back to original owners. Instead, the passage describes how the museum itself is art.
12. A. *Inference*. The passage supports the idea that the building and its architecture are art. The author also gives examples where one can walk through and be surrounded by art. The author doesn't suggest that classical art is the most respected.
13. C. *Main Idea*. The passage describes the weather and the foods in context of a particular holiday, Thanksgiving. It describes what makes Thanksgiving so special. Though it does mention Christmas, the focus is on Thanksgiving day itself.
14. C. *Vocabulary*. The rest of the sentence tells us that the parents send the children out with a "warning." This tells us that "chiding," which is similar to a scolding, probably has a similar meaning.
15. C. *Inference*. The author doesn't pick Christmas or Thanksgiving over the other. Food is mentioned as an important ritual, but this doesn't mean it's the most important part of the holiday. However, the passage focuses more on family – how everyone can take the day off, spending time with cousins and aunts and uncles.
16. B. *Supporting Idea*. The author never mentions that he spends time with friends, only that he enjoys hosting (lines 16-18), eating delicious food (lines 36-46), and spending time with family (lines 11-14).
17. A. *Tone/Language/Style*. The author explicitly states that he enjoys the food. When an author writes that the opposite of what he means, this is sarcasm.
18. C. *Organization/Logic*. The passage doesn't describe a process, theory, or guess. Instead, it states the author's personal point of view and shares detailed examples to support that view.
19. B. *Inference*. The passage is primarily concerned with describing the negative side effects of sitting for long periods of time. The author doesn't mention that these problems will be permanent, since he talks about how exercise can help stop, prevent, or improve some of the issues. The author doesn't make a value judgement about whether some problems are worse than others.
20. B. *Main Idea*. The second paragraph describes what happens to the body over time, not immediately. The immediate side effects are described later in the next paragraph.



21. D. *Supporting Idea*. The passage makes a connection between the fact that relying on machines too much causes people to sit, and that sitting leads to health issues. Stretching and proper posture are offered as a partial solution, not as one that can fix everything forever.
22. A. *Inference*. In the third paragraph, the author writes that “when it comes to bone strength, sitting has the opposite effect of exercise.” This means that exercise could improve bone strength. The idea that sitting is an exercise would be opposite of this, and never mentions that stretching is the most effective form of exercise (or that too much exercise could be bad).
23. A. *Vocabulary*. “Alleviate” means to “reduce” or “mitigate” – to make something less bad (think about a headache, and how medication might alleviate the symptoms). If the word was “deteriorate” or “impair,” the meaning of the sentence wouldn’t make sense given the context of the rest of the paragraph, which describes how some exercise can improve health issues.
24. D. *Organization/Logic*. The passages doesn’t describe a particular procedure or memory. There are many terms used throughout the passage, but these are used to support a theory. This theory concerns sitting and exercise, and what should be done about the problem of sitting. The passage also discusses what solutions are possible.
25. D. *Main Idea*. The passage describes the founding of the first national park, and the reasons for its creation. It goes on to describe how national parks are valuable not just for their resources, but as a place for people to enjoy. It also describes how people can make money in ecotourism, without destroying the environment.
26. A. *Vocabulary*. The paragraph describes how Hayden believed that companies would “despoil” the natural beauty. The passage uses the example of Niagara Falls, which had been “taken advantage of.” The word “exploit” means exactly that – to take advantage, in this case, of the natural beauty. This doesn’t mean necessarily that the businesses would pollute, steal, or package nature.
27. C. *Inference*. The quotation from Hayden describes how if no action is taken, natural beauties will be destroyed. This explains how conservationists believed it was urgent to make national parks a reality. This describes the events surrounding the creation of Yellowstone, not the Rocky Mountain national park.
28. C. *Inference*. The author doesn’t believe that all businesses are greedy and short-sighted, since he appears to support the ecotourism business. The author also praises governments for creating national parks, and cites the United States as an example of a leader in this area. The author implies that businesses favor short-term money making over long-term, since that is the reason why they opposed national parks.
29. C. *Supporting Idea*. The businesses wanted to sell resources (trees are an example given in the passage) that were found in national parks. There is no mention of supporting war. The businesses would not have wanted to exploit nature if they enjoyed it more than making money or if they were concerned about preserving land for future generations.
30. D. *Supporting Idea*. The author believes people need to take action by setting aside land, not leaving it alone. The author also mentions Niagara Falls negatively, saying that it has been spoiled. It is true that Yellowstone is one of the first national parks in the world, but this doesn’t support or justify environmental preservation by itself. The author offers the example of ecotourism as a way to make money while at the same time preserving the environment.
31. B. *Main Idea*. The author takes a strong stance against only learning in the classroom, providing examples of how learning outside of the classroom is just as important. She doesn’t actually say that education today should be the same as in the past, but that there are aspects of the past that we can learn from. Time is not mentioned as a factor in this passage. Instead, the passage describes the many benefits and reasons why one should participate in extracurricular activities besides just for applications.
32. A. *Tone/Language/Style*. The author’s tone here is very negative, suggesting that many people see extracurricular activities as requirements or formalities that must be observed when applying to schools. This view does not take into consideration whether or not students enjoy participating in those activities, or whether or not there are other benefits. This viewpoint supports the idea that






parents just want students to do the activities so that they will have the short-term benefits of being able to include these activities on their school applications.

33. *D. Inference.* Paragraph 2 describes the education in ancient Athens, and how people were schooled in class and outside, along with physical activity. They would not have supported book or classroom learning alone, and would have encouraged physical training as part of education. Instead, an ancient Athenian as described in this passage would agree with the author that not enough learning happens outside in the real world, where extracurricular activities can take place.
34. *D. Vocabulary.* In context of the paragraph, this sentence talks about how hands-on experiences are valuable. The way the sentence is written describes how people “should not” discount, or “write off” (think “undervalue” or “not appreciate”) the importance of hands-on education. If there was no “should not,” then “think more of” would make sense. The word “discount” in this context does not mean to pay less than something.
35. *C. Supporting Idea.* In these paragraphs, the passage discusses the various benefits of participating in extracurricular activities. One aspect of this is friendship, but the paragraphs describe more than that. They do not describe the importance of extracurricular activities on applications or suggest that extracurricular activities are more valuable than classroom learning (or that one type of extracurricular activity is more important than others).
36. *C. Organization/Logic.* The passage doesn’t describe scientific principles or offer a personal experience. It does share a particular point of view, which is supported with various examples and ideas. The different types of extracurricular activities are offered to support the argument, not simply to list and describe different kinds.

### Mathematics Achievement

1. *Measurements – Formulas.* B. The area of a parallelogram is given by the formula base ( $B$ ) times height ( $H$ ). Since we are given these values, simply plug in and simplify:  $11 \times 18 = 198$ .
2. *Algebraic Concepts – Solving Algebraic Equations.* B. On the left side of the equation, we can factor out 4 from both terms. To determine what this factor looks like, we ask ourselves “what do we multiply by 4 in order to get each term?”  $4x$  is simply  $x$  multiplied by 4.  $-4$  is simply  $-1$  multiplied by 4. So, we get  $4(x - 1)$  from  $4x - 4$ . Only choices A and B show us this. On the right side, we see that  $y$  is a common factor of both terms, so we need simply factor out the  $y$  to end up with  $y(5 + 1)$ , since 5 times  $y$  is  $5y$ , and 1 times  $y$  is  $y$ .
3. *Geometry – Geometric Objects.* D. The total area of the bathroom is  $12 \times 9 = 108 \text{ ft}^2$ . Each square has a length of  $12 \div 4 = 3 \text{ ft}$ . (also  $9 \div 3 = 3$ ). This means the area of each tile is  $3 \times 3 = 9$ . Since there are 8 unshaded areas,  $8 \times 9 = 72 \text{ ft}^2$ .
4. *Data & Probability – Probability.* D. If the probability of a coin landing on heads is  $\frac{1}{2}$ , then the probability of it happening three times in a row is  $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8}$ . This is because the probability of each coin toss is an independent event (the outcome of one toss has no impact on the outcome of another toss).
5. *Algebraic Concepts – Ratios, Proportions, & Scale Factors.* A. Since 3 bananas = 1 apple, and 1 apple = 6 grapes, then we know that 3 bananas = 6 grapes. To find out how many bananas are equal to 1 grape, we divide both sides of the equation by 6, ending up with  $3 \div 6 = 0.5$  bananas for every  $6 \div 6 = 1$  grape.
6. *Measurements – Units.* D. 1 liter = 1,000 milliliters, so a bottle filled with 4 liters of water contains  $4 \times 1,000 = 4,000 \text{ mL}$  of water. Since Joe drank half of the water, we divide the total amount in half. Therefore, we get  $4,000 \div 2 = 2,000 \text{ mL}$ .
7. *Measurements – Formulas.* C. The area of a square can be represented as length ( $L$ ) times width ( $W$ ). Since all four sides of a square are of equal length,  $L = W$ . This means that we can take the square root of the area of a square to find the length of a side. In this case,  $\sqrt{36} = 6$ . Since formula for the perimeter of a square can be represented as  $4L$  or  $4W$ . This makes the perimeter  $4 \times 6 = 24$  inches.

8. *Measurements – Units*. C. In the four years, Frederick will grow  $3 \text{ inches} \times 4 = 12 \text{ inches}$ . There are 12 inches in 1 foot.
9. *Algebraic Concepts – Solving Algebraic Equations*. C. Note that this is an expression, not an equation, which means we cannot cancel out values by dividing the entire expression. If we divide  $9x + y$  by 3, we could arrive at  $3x + \frac{y}{3}$ . However, this is NOT equal to the expression  $3(9x + y)$ . Simply plug in any two numbers for  $x$  and  $y$  to find out. If we distribute the 3 to each term in the parentheses, we end up with the value in C. Again, we can substitute in any values for  $x$  and  $y$  to see that this is true.
10. *Measurements – Formulas*. C. The volume of a pyramid is equal to one-third the area of the base times the height (or,  $V = \frac{lw h}{3}$ ). Plug in the values we are given:  $18 = \frac{lw(6)}{3}$ . Simplify by multiplying both sides by 3, then dividing by 6 to arrive at  $9 = lw$ . This is the area of the pyramid's base.
11. *Numbers & Operations – Percents*. C. A surefire way to calculate percentage change between an old value and a new value is to use the formula  $\frac{\text{new} - \text{old}}{\text{old}}$ . In this case, this becomes  $\frac{75 - 50}{50} = \frac{25}{50} = \frac{1}{2}$ , or 50%. 
12. *Algebraic Concepts – Solving Algebraic Equations*. B. To simplify, first subtract 8 from both sides, and simplify:  $63 - 8 = 5x$ , which becomes  $55 = 5x$ . Then, divide both sides by 5 to give us  $11 = x$ .
13. *Numbers & Operations – Integers*. A. Subtracting a larger number from a smaller number results in a negative number.  $9,515 - 5,283 = 4,232$ . Remember to carry all values and to flip the sign to  $-4,232$ .
14. *Numbers & Operations – Decimals*. C. Make sure the decimal points are aligned with each other and that values are carried appropriately. To be sure, write as  $1.500 - 0.240 + 10.261$ . 
15. *Algebraic Concepts – Ratios, Proportions, & Scale Factors*. C. We are given a ratio of Bartholomew's height to the length of the shadow that he casts. Set up a proportion to solve this question reliably. In this case, the numerator will be the height of an object, and the denominator will be the length of that object's shadow:  $\frac{6 \text{ ft (person)}}{20 \text{ ft (person)}} = \frac{x \text{ ft (flagpole)}}{60 \text{ ft (flagpole)}}$ . Cross multiply and solve for  $x$ :  $(6)(60) = 20x$  simplifies to  $(6)(3) = x$ , and then  $18 = x$ . 
16. *Algebraic Concepts – Solving Algebraic Equations*. C. Though we could solve for  $x$  in the equation and substitute that value into the expression, we need only recognize that if we multiplied  $3x - 2$  by 5, we would get  $15x - 10$  (this is accomplished by distributing, or multiplying each term inside parenthesis by the number outside of it). If this is the case, the 4 on the other side of the equation would also need to be multiplied by 5, which would give us 20.
17. *Algebraic Concepts – Ratios, Proportions, & Scale Factors*. D. The scale factor of  $\frac{3}{2}$  means that the dimensions of rectangle  $PQRS$  must each be multiplied by  $\frac{3}{2}$ , which is the same as multiplying by 1.5. This gives us  $4 \times 1.5 = 6$  and  $8 \times 1.5 = 12$ . The area of the rectangle similar to  $PQRS$  is  $6 \times 12 = 72$ . Note that we cannot simply multiply the area of rectangle  $PQRS$  by 1.5.
18. *Numbers & Operations – Factors, Multiples, Primes*. C. The safest way to solve this is to break each number into its prime factors. We do this by finding the smallest prime numbers that multiply together to give us the each number.  $36 = 2 \times 2 \times 3 \times 3$  (or  $2^2 \times 3^2$ ).  $15 = 3 \times 5$  (or  $3^1 \times 5^1$ ). Then, multiply together the prime number values with the greatest exponents:  $2^2 \times 3^2 \times 5^1 = 4 \times 9 \times 5$ . This equals 180. Note that though the prime number 3 shows up three times (twice when

factoring 36 and once when factoring 15), we only use the two times it shows up in 36, and don't count it again.

19. *Numbers & Operations – Fractions*. C. We're given two mixed fractions. We can simply add the fractions together, then add together the whole numbers. In this case, we have  $\frac{1}{6} + \frac{2}{3}$ , which we can add together by creating a common denominator. Since 6 is a multiple of 3, we can multiply  $\frac{2}{3}$  by 2 to arrive at  $\frac{4}{6}$ . Now, we can add  $\frac{1}{6} + \frac{4}{6} = \frac{5}{6}$ . Then, add the whole numbers  $2 + 4 = 6$ . Thus, the answer is  $6\frac{5}{6}$ .

20. *Algebraic Concepts – Ratios, Proportions, & Scale Factors*. C. If the farmer sells 1 goat, then he only has  $5 - 1 = 4$  goats remaining. Set up a proportion to solve this question reliably.

$\frac{5 \text{ goats}}{1 \text{ field}} = \frac{4 \text{ goats}}{x \text{ field}}$ . Cross multiply and solve for  $x$ :  $(4)(1) = 5x$  simplifies to  $4 = 5x$ , and  $x = \frac{4}{5}$ .

21. *Numbers & Operations – Decimals*. C. Since the answer choices are given in fraction form, we know that we will need to convert between decimals and fractions. The first decimal is  $\frac{1}{3}$  and the second can be determined by remembering that 0.8 can be read as "eight tenths." This lets us know that  $\frac{8}{10} = \frac{4}{5}$ . Find the common denominator for both fractions, which in this case is 15:

$\frac{1}{3} = \frac{5}{15}$  and  $\frac{4}{5} = \frac{12}{15}$ . We can now add the fractions together to arrive at  $\frac{5}{15} + \frac{12}{15} = \frac{17}{15}$ . This is an

improper fraction, which when converted to a mixed number, gives us  $1\frac{2}{15}$ .

22. *Numbers & Operations – Integers*. C. When a negative number is multiplied by another negative number, the product becomes positive. When a negative number is multiplied by a positive number, the product is negative. In this case, the exponents 2, 3, and 4, tell us how many times to multiply a number  $x$  by itself. A negative number multiplied by itself an even number of times (for example, 2 and 4 times), results in a positive number. A negative number multiplied by itself an odd number of times (for example, 3 times), results in a negative number.  $x^3$  is "more negative" than  $x$  since  $x$  is less than  $-1$ , so  $x^3$  is further to the left on the number line. Use an integer if working with a variable is difficult: if  $x = -2$ , then  $x^2 = (-2)^2 = 4$ , and  $x^3 = (-2)^3 = -8$ , and  $x^4 = (-2)^4 = 16$ . Since  $-8 < -2$ , then  $x^3 < x$ .

23. *Numbers & Operations – Percents*. B. We can represent the information about 2014 as the total number of trees minus the number of trees cut down. Algebraically, this is  $200 - (200)(25\%)$ , or  $200 - 50 = 150$ . Then, during 2015, 30% of the 150 remaining trees were cut down. This can be represented as  $150 \times (1 - 0.3)$  or  $150 \times 0.7$ , which equals 105.

24. *Measurements – Formulas*. B. Use the Pythagorean theorem on right triangles:  $c^2 = a^2 + b^2$ , where  $c$  is the hypotenuse, and  $a$  and  $b$  are the legs of the triangle. We can use the values given and plug them into the formula:  $c^2 = 5^2 + 5^2$ , which simplifies to  $c^2 = 50$ . If we take the square root of both sides,  $c = \sqrt{50}$ . We know that the square root of 50 will be close to the square root of 49, which is 7.

25. *Algebraic Concepts – Solving Algebraic Equations*. B. First, eliminate the square root by squaring both sides. This gives us  $n + 2 = m^2$ . Then, subtract 2 from both sides for  $n = m^2 - 2$ .

26. *Data & Probability – Probability*. A. If there are 3 quarters for every 5 coins in her pocket, then Maria's pocket must contain 2 nickels for every 5 coins. This means  $q = 3$  and  $n = 2$ . Substitute:

$$\frac{n}{q} = \frac{2}{3}$$

27. *Measurements – Units*. D. We are told that a man weighs 3 times as much on Neptune as he does on Mars. If an object weighs 7.5 kg on Neptune, then it weights  $\frac{1}{3}$  as much on Mars.  $7.5 \text{ kg} \div 3 = 2.5 \text{ kg}$ . This is not given as an answer, but the gram equivalent of 2,500 grams is.
28. *Numbers & Operations – Decimals*. D. We can see that the value of  $y$  is 1.25 times the value of  $x$ . We can set up an equation accordingly:  $y = 1.25x$ . Therefore, if  $x = 3$ , then  $y = 1.25(3) = 3.75$ .
29. *Numbers & Operations – Integers*. B. Visualizing a thermometer can help. If the temperature started at  $-8^{\circ}\text{C}$ , then an increase of  $8^{\circ}\text{C}$  would bring the temperature to  $0^{\circ}\text{C}$ , and an increase of  $6^{\circ}\text{C}$  from there would bring the temperature to  $6^{\circ}\text{C}$ . In all, this is an increase of  $14^{\circ}\text{C}$ . Or, we can represent this as  $-8 + 14$ , which is also  $14 - 8 = 6$ .
30. *Algebraic Concepts – Solving Algebraic Equations*. A. The expanded form of the expression can be found in choice C, since  $y^2 = y \times y$  and  $2y = y + y$ . Another way of writing the expression is found in choice D. A simplified form of the expression can be found in choice B. This is because  $\frac{y \times y}{y + y} = \frac{y \times y}{y(1 + 1)} = \frac{y}{2}$ , since we can cancel out a common term  $y$  in both the numerator and denominator. Again, we can solve this question by substituting in any value for  $y$ .
31. *Data & Probability – Probability*. A. At first, we have 7 black hair-ties out of a total of 15 hair-ties. If Lamilla loses 3 black hair-ties and 2 other hair-ties, then she is left with 4 black hair-ties out of a total of 10 remaining hair-ties. This means her chance of picking a black hair-tie is now  $4 \div 10 = \frac{2}{5}$ .
32. *Measurements – Formulas*. C. Start with the formula:  $a = h \frac{(b_1 + b_2)}{2}$  and plug in the known values. This gives us  $36 = 4 \frac{(6 + b_2)}{2}$ , which simplifies to  $36 = 2(6 + b_2)$ , and eventually  $18 = 6 + b_2$ . This eventually gives us  $12 = b_2$ .
33. *Geometry – Geometric Objects*. D. Since we are given the length of a side of the square, we know that the total area of the square is  $4 \times 4 = 16 \text{ in}^2$ . Since the circle is inscribed in the square, we know that the circle touches the square at exactly 4 points (one on each side of the square). This means that the length of 4 is equal to the diameter of the circle. Since the diameter is two-times the radius, we know that the radius is  $4 = 2r$ , and  $r = 2$ . Therefore, the area of the circle is  $\pi(2)^2 = 4\pi$ . Thus, the area of the shaded area is the total area of the square (16) minus the area of the circle ( $4\pi$ ), or  $16 - 4\pi$ . Note that because this is an expression, not an equation, it is not appropriate to divide both terms by 4 (unless expressing the value as  $4(4 - \pi)$ ).
34. *Numbers & Operations – Percents*. B. 10% of \$50 can be expressed as  $0.1 \times \$50 = \$5$ . If the DVD was on sale for 10% off on Monday, this means it cost  $\$50 - \$5 = \$45$ . A faster way to do this is to realize that 10% off of something really just means 90% of that thing. In this case, we could multiply  $0.9 \times \$50 = \$45$  to save a step. If the DVD was marked back up 10% from \$45, we need to increase the price by  $0.1 \times \$45 = \$4.50$ . This means  $\$45 + \$4.50 = \$49.50$ . A faster way to do this is to realize that increasing something by 10% is the same as multiplying that thing by 1.1. In this case, we could multiply  $1.1 \times \$45 = \$49.50$ .
35. *Algebraic Concepts – Functions & Patterns*. D. Notice that the \$10 includes the cost of one topping, and that additional toppings (second, third, etc., toppings) cost \$1.25 per topping. If there are 3 toppings, and the first topping was already included in the price, then we don't count this as a \$1.25 topping. Therefore, there would be  $3 - 1 = 2$  toppings that cost \$1.25. Therefore, the expression for the cost of a pizza with  $x$  toppings is  $10 + 1.25(x - 1)$ .
36. *Algebraic Concepts – Ratios, Proportions, & Scale Factors*. B. Set up a proportion to solve this question reliably.  $\frac{150 \text{ words}}{9 \text{ minutes}} = \frac{500 \text{ words}}{x \text{ minutes}}$ . Cross multiply and solve for  $x$ :  $(500)(9) = 150x$  simplifies to  $4,500 = 150x$ . When we divide both sides by 150, we are left with  $x = 30$ .



37. *Data & Probability – Probability*. C. Because we return the first card chosen to the deck, both instances where a card is chosen are independent of one another (the total number of cards is 52 in both cases). Since there are 4 aces and 52 cards, the probability that the first card is an ace is  $4 \div 52 = \frac{1}{13}$ . The card is returned to the deck, so the total is again 52 and there are 4 kings in the deck. So the chances that the second card is a king is also  $4 \div 52 = \frac{1}{13}$ . The probability of these independent events both happening is  $\frac{1}{13} \times \frac{1}{13}$ .
38. *Data & Probability – Mean, Median, Mode, & Range*. A. The table represents a set of values. The right-hand column tells us how many times the value appears in the set. For example, the value 1 appears 4 times in the set. We know that to find the mean, we must take the sum of values in the set, and divide by the number of values in the set. First, find the sum of values in the set. Since 1 appears 4 times in the set, we know that the sum of 1 values is  $1 \times 4 = 4$ . Similarly, we know that 2 appears 3 times, so we know the sum of 2 values is  $2 \times 3 = 6$ . We repeat this for 4 + 6 + 6 + 4 = 20. We know that there are a total of  $4 + 3 + 2 + 1 = 10$  values in the set, since this is the sum of the right-hand column. Dividing the two, we get  $20 \div 10 = 2$ .
39. *Data & Probability – Interpreting Data*. B. Use one of the orders in the table to determine the tax. If the designer orders 2 shirts, then the shirts cost  $2 \times 10 = 20$ . We know that the shipping cost is 10 per order, bringing the cost to  $20 + 10 = 30$ . Therefore, the tax must be the total cost of the order minus the cost of the shirts and the shipping cost:  $32 - 30 = 2$ . As a percentage of the shirt cost,  $2 \div 20 = 0.1$ , or 10%. We can check this answer with any other order size to verify.
40. *Algebraic Concepts – Functions & Patterns*. A. A penny has a value of 0.01 dollars, and a quarter has a value of 0.25 dollars. The value of  $x$  pennies is  $0.01x$  (the value of a coin times the number of coins there are). The number of quarters is  $x - 8$ , and the value of this is  $0.25(x - 8)$ . Distribute this and find that  $0.25x - 2$ . Together, the total values is  $0.01x + 0.25x - 2$ , or  $0.26x - 2$ .
41. *Measurements – Units*. B. When converting between area units (like square meters), divide by the square of the unit-conversion value. In this case, converting from centimeters to meters normally requires dividing by 100. To go from square centimeters to square meters, divide by  $100^2$  or 10,000. So:  $18,473 \div 10,000 = 1.8473 \text{ m}^2$ .
42. *Data & Probability – Mean, Median, Mode, & Range*. C. First, determine the set of numbers itself. The first six prime numbers are {2, 3, 5, 7, 11, 13} (REMEMBER: 1 is not a prime number!). The median would be the average of the third and fourth lowest prime number, since there are an even number of values in the set. This means we must find the average of 5 and 7, which is 6.
43. *Numbers & Operations – Percents*. B.  $\$60 - 20\%(\$60) = \$60 - \$12 = \$48$ . Alternatively:  $60 \times 0.8 = 48$ . This is the price at Store B. At Store C,  $\$48 + 10\%(\$48) = \$52.80$ . Alternatively,  $48 \times 1.1 = 52.8$ .
44. *Numbers & Operations – Decimals*. A. When we don't know the value of something, we represent that value with a variable. In this case, let  $x$  represent the number of tennis matches that Joe won. In terms of  $x$ , we know that Kevin won  $0.7x$  matches. Since Larry won 1.3 times as many matches as Kevin, then he won  $1.3(0.7)(x)$  matches. If we simplify this expression we arrive at  $1.3 \times 0.7x = 0.91x$ . So, Joe won  $x$ , Kevin won  $0.7x$ , and Larry won  $0.91x$  matches. Therefore, Joe won the most matches. We can also pick an easy-to-use number to represent the number of matches that Joe won. If we assume Joe won 100 matches, then Kevin won 70 matches, and Larry won  $70 \times 1.3 = 91$ .
45. *Numbers & Operations – Fractions*. C. If  $\frac{1}{4}$  of the marbles are green, and  $\frac{1}{3}$  are red, then together, the green and the red represent  $\frac{1}{4} + \frac{1}{3} = \frac{3}{12} + \frac{4}{12} = \frac{7}{12}$  of the jar. This means that  $\frac{5}{12}$  of

the jar is orange. This means that for every 12 marbles in the jar, 5 are orange. Since there can't be any fractional marbles, the actual number of marbles must be a multiple of 12. Only 24 is evenly divisible by 12.

46. *Algebraic Concepts – Functions & Patterns.* B. The speed of the blue car is given as  $B$ . If we multiply an amount of time by a speed, we get a distance (for example, if we drive 60 miles per hour, and drive for 1 hour, we have driven  $60 \times 1 = 60$  miles). In this case, we know that the blue car has driven  $20 \text{ minutes} \times B \text{ feet per minute} = 20B$  feet. We also know that the red car drives 4 times as fast, or  $20 \text{ minutes} \times 4B \text{ feet per minute} = 80B$  feet. Since, during the 20 minutes, both cars are now 2,000 feet apart, we know that the difference between  $80B$  and  $20B$  must be 2,000. This translates to an equation  $80B - 20B = 2,000$ .
47. *Data & Probability – Mean, Median, Mode, & Range.* C. We know that Lorna has taken 3 tests so far, and that she will take one more, that counts twice. Right now, the set of scores is  $\{72, 88, 94\}$ . After she takes her final exam, the set of scores will be  $\{72, 88, 94, x, x\}$ . Algebraically,  $\frac{(72+88+94+x+x)}{5} = 86$ . Simplify:  $254 + 2x = 430$ , and  $2x = 176$ , so  $x = 88$ .

