


Upper Level ISEE®

1500+ Practice Questions

Answer Key &

Explanations

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Diagnostic Practice Test (Form A)

Verbal Reasoning

1. B. Something precise is usually sharp, well-defined, or exact. "The artist made precise cuts to the paper so that each piece was the same size." In this case, substituting the other answer choices into the sentence results in a sentence that doesn't make sense.
2. D. "Random" generally means without aim, method, or direction. "Sometimes, it's possible to pick out a pattern in what might seem like a series of random events." Spontaneous is the best choice, because the other three are all synonyms for "planned" or "on purpose."
3. B. "Hamper" has a negative connotation as a verb and means to interfere or restrict the movement of something. It can also mean to disrupt or delay. "Bad traffic hampered our trip." Even if we didn't know this, we can see that the other answer choices do not have negative connotations.
4. A. "Abolish" has a negative connotation and means "destroy." Abolish is frequently used in legal contexts. i.e. to abolish slavery or abolish a law. Note that "establish" and "promote" have positive connotations, leaving us to choose between "eliminate" and "prevent."
5. A. "Haughty" has a negative connotation and is an adjective meaning "overly proud" and usually also means to have contempt for others. "The haughty aristocrats looked down on working class people."
6. C. "Chastise" comes from the same root word as "chasten." Both mean to criticize. Of the choices, "punish" has a similar meaning.
7. D. If something is sparse, it may be hard to find, and so it is lacking or meager. "In the winter, leaves on trees are sparse." Therefore "meager," which usually means "lacking," is the closest synonym.
8. A. "Relinquish" means "to give up." "When the general retired, he relinquished control of his troops." This means that the general gave up control, or abandoned control, of his troops. The other choices don't result in a sentence that makes sense.
9. D. "Rebuff" has a negative connotation, unlike all but the last answer choice. To rebuff something often means to reject it rudely, or spurn it. "She rebuffed all suggestions she didn't like."
10. B. "Augment" comes from a root word meaning "to increase" and generally means "to make larger or better." So, "minimize" and "reduce" are antonyms and "amplify" is the only synonym with a similar meaning.
11. D. "Wanton" most often means "hard to control," or describes doing something unprovoked. "The men who broke into the shop were guilty of wanton destruction of property." This is the opposite of something necessary.
12. A. "Venerable" has a positive connotation and is an adjective meaning "to love or charm." Something venerable is deserving of veneration, or deep respect and admiration. "She held the venerable teacher in great esteem." Notice that the other answer choices are all negative in connotation.
13. B. "Impudent" has a negative connotation. The prefix *im-* indicates a lack of something. This rules out "careful" and "gracious," which mean that things are "full of care" or "full of grace," respectively. Being reckless and disrespectful mean that someone lacks caution or respect, respectively. Impudent means a lack of shame or modesty, or a lack of respect.
14. C. "Pompous" has a negative connotation. "Pomp" comes from the Greek and means a ceremony or procession. Something pompous, such as decorations or speech, is excessively elevated or ornate. Someone who is pompous celebrates themselves, or is vain. The other answer choices are neutral to positive in connotation.
15. C. "Usurp" has a negative connotation and means "to take possession of." To usurp is to take or seize by force or without permission. "The army attempted to usurp new territory when they invaded."
16. A. A "tirade" is a speech with harsh language, often criticizing something. "What was supposed to be a brief speech turned into a tirade about his dissatisfaction over the state of politics in his country." In this context, "denunciation," which means publicly condemning something, fits.
17. C. "Onerous" has a negative connotation. If something is onerous, it is extremely difficult, imposing, burdensome, or arduous. "The American colonists accused the British government of imposing onerous taxes."

18. D. “Slovenly” means “untidy or sloppy.” It sounds a little like “sloth,” which implies laziness as well. So “messy” is the correct answer.
19. C. “Erudition” means “a state of great knowledge.” It is made up of the root word “rudis,” which means “crude” or “untrained” (think “rudimentary”). The prefix “e,” means “not” or “un.” Therefore, someone who is “not crude” or “not untrained” is sophisticated, knowledgeable, or literate about something. “The college professor exhibited articulateness and erudition in his lecture.” Thus, “knowledge” is the only synonym with a similar meaning.
20. A. The sentence describes a lightning flash that appears and disappears quickly, so the best adjective to describe the blaze would be a word that means to disappear quickly. “Ephemeral” means “to last a short time.” The other choices mean the opposite of this.
21. C. Edward doesn’t study or think through decisions, so “knowledge” can be eliminated as an answer right away. The missing word would therefore mean the opposite of knowledge or purposeful thought, such as a gut feeling or intuition.
22. D. There are a number of clues in the sentence. The musician is young but can play music adult musicians find difficult. Therefore, the musician must be gifted, or a prodigy.
23. D. Ask yourself, “What could be removed to simplify a text?” Removing unnecessary, or superfluous, words could help a reader better find the main idea.
24. D. The words “touching” and “deeply moving” in the first part of the sentence imply that the man in the sentence will act to do something to help plants and animals. His feelings are brought to the surface. The word “kindled” literally means “started a fire,” but figuratively means to stir up. Watching the documentary stirred up feelings of love for plants and animals. The other options have negative connotations.
25. B. To determine the type of clothing the designer specializes in, look for clues in the second part of the sentence: “customized” and “made-to-order.” “Bespoke” is a synonym for custom-made.
26. B. Clues in this sentence indicate that the missing word is a synonym for “secret” or “without public oversight.” The best answer is “clandestine,” which means conducted in secrecy.
27. B. Using context clues in the sentence, you can determine that the crime being described is especially terrible, so the answer will have a negative connotation. This is only true of “heinous.” The other choices all have positive connotations and imply the crime would be forbidden instead of punished.
28. B. If Sally still remembered years later, her rote memorization and practice left a permanent mark, so the word for the blank is a synonym for permanent. “Impermanent” can therefore be ruled out as an answer. “Indelible” means “lasting or memorable.”
29. D. If the government banned immigration, it must have been gripped by strong fear of immigrants. The word in the blank would have a negative connotation, but “equality,” “fraternity,” and “sorority” have positive connotations. “Xenophobia,” however, means “fear of outsiders.” The word “xeno-” means “foreign.”
30. D. The sentence implies there is conflict between the roommates, so they are held by metaphorical a thread that is probably not very strong. The only synonym with a similar meaning for “weak” of the choices is “tenuous,” which means “thin” or “lacking strength.”
31. D. The clue in this sentence is “avoid detection.” The word in the blank must be a synonym for “stealth.” The only synonym with a similar meaning of the choices is “furtiveness,” which means “a state of being quiet or secret.”
32. C. We’re looking for adjectives that describe judgment and decision-making that result in low performance. Both of these skills are impacted negatively by lack of sleep, so the missing words should be synonyms and both should be negative. All of the choices show antonyms except in the case of “impair” and “hinder” which both show the negative impact of lack of sleep.
33. C. The words “instead” and “actually” in the context of the sentence imply that something negative was expected but something positive happened instead. So, we’re looking for a negative verb that describes what is expected to happen to the opposition (“stifling”) and what happens instead (“provoking”).

34. D. Clues in the sentence indicate that the lecturer is boring. He could prattle or ramble, which would lose his audience's attention, but the topic is also boring, so the best fit is that the lecturer rambles and the topic is mundane.
35. D. Light reflected off the pearl such that the diver has to shield his eyes, so it must be bright. Any of the choices for the first blank could work in the sentence except for "leaden," but only "brilliant" describes the quality of the light reflecting off the pearl.
36. C. "Belied" means "giving a false impression." So his personality would give the impression that the gangster did not have a taste for physical violence. Thus, "threatening" and "menacing" can be ruled out. Since the man is a gangster feared by the public, he is likely not "reputable." Of the remaining choices, "notorious" which means "widely and unfavorably known," fits in the blank.
37. D. The sentence shows a cause and effect relationship. Because of the boxer's behavior, he frequently gets into fights. So, he is quarrelsome and aggressive, or pugnacious and truculent.
38. D. If the smells were delicious, they had a positive effect on the man's appetite. "Hone," "quell," and "taper" all imply a diminishing appetite. The choice of "whet...roused" also makes the most sense because the man's growing hunger would cause him to get out of bed.
39. C. For the first blank, we're looking for a verb that describes an activity a thief would do, such as pilfering or sneaking. For the second blank, we're looking for an adjective that shows the thief behaving in a way that is calm and collected. Therefore, "trepidation" can be eliminated, because it is an antonym for calm and collected, and "pilfering...nonchalance" is the best choice.
40. D. The old man lost interest in many of his passions, which is the key clue for the first blank. He'd be neither "lively" or "active" (since he now leads an "uneventful life") so those choices can be eliminated. If his life was uneventful, it is not "fascinating." "Stodgy" means "dull" and "sedentary" means "inactive."

Quantitative Reasoning

- Vocabulary.** C. $100 + 101 + 102 + 103 = 406$, so if the sum of all integers from 100 to 1,000 is equal to x , then the sum of all integers from 104 to 1,000 is $x - 406$.
- Scientific Notation.** B. To put this number into scientific notation, we only need to shift the decimal point to the right once. So, the expression in scientific notation is 5.13×10^{-1} .
- Speed.** D. If Hubert was 200 meters ahead, in order to calculate how long it took Ingrid to catch up, we would need to know how much faster Ingrid was jogging than Hubert, or the difference between their speeds. Hubert's or Ingrid's speeds aren't enough information to solve the question by themselves, because knowing one without the other is not enough information to determine how much faster Ingrid jogged.
- Nets.** C. The correct net has 3 adjacent sides that show all 3 symbols. Net C is the only net for which this is true. Note that in choices A and D, the cross shape and the diamond shape will end up on opposite sides of the cube, not next to each other.
- Imaginary Operations.** B. If $n \cdot 2 = 7$, $m = 2$, and $7 = 3(2) + \frac{n}{2}$. Combine like terms and isolate n to find its value: $1 = \frac{n}{2}$, so $1 \times 2 = n$ and $n = 2$.
- Factors.** A. The greatest common factor is the greatest factor that divides two numbers. First, find the factors of the given numbers. $5a^2b^4 = 5 \times a \times a \times b \times b \times b \times b$ and $a^3b = a \times a \times a \times b$. $8a^4b^3 = 8 \times a \times a \times a \times a \times b \times b \times b$. All three numbers contain $a \times a \times b$, so the simplified greatest common factor is a^2b .
- Distributing.** D. It may help to convert one side of the equation so both have the same format. For example, apply FOIL to rewrite $(x - 11)^2$. $(x - 11)^2 = x^2 - 22x + 121$. Therefore, $n = -22x$.
- Consecutive Integers.** A. If the odd integers are consecutive, then $x + (x + 2) + (x + 4) = 69$, which can be simplified as $3x + 6$.
- Function Notation.** D. When a number is raised to an even power, as in x^4 , the result will always be positive, but not necessarily odd. When a number is raised to an odd power, as in x^3 , the result is not

necessarily always positive, nor necessarily odd. However, when we have a composite function, we must evaluate the entire function. x^3 could be either negative or positive. If it is negative, and multiplied by itself 4 times, then the result is positive; if it is positive, and multiplied by itself 4 times, then the result will also be positive.

10. *Proportions*. A. Since both figures are square, each figure has 4 equal sides. So, side PM has the same length as side MN and side QR has the same length as side ST . Therefore, $\frac{5}{2}x = 10$ and $x = 10 \times \frac{2}{5} = 4$.

The length of side ST is x inches, or 4 inches.

11. *Probability – Conditional*. B. The probability of an event can be represented by the number of ways an event can occur over the total number of possible outcomes. Damon picked and removed one yellow pencil, so the probability of him choosing a black pencil is equal to the number of black pencils (10) over the total number of pencils remaining (29), or $\frac{10}{29}$.
12. *Creating Expressions & Equations*. A. James has a n nickels and he has 8 more dimes than nickels. We could say that the number of dimes $d = n + 8$. We're told the total value of the coins is \$1.55. Since each nickel is worth \$0.05 and each dime is worth \$0.10, we know that $n(0.05) + (n + 8)(0.1) = 1.55$. We can solve for n by simplifying and combining like terms: $0.05n + 0.1n + 0.8 = 1.55$, or $0.15n = 0.75$. This means $n = 5$. Therefore, James has 5 nickels.
13. *Averages*. C. The mean is the sum of all values in a set divided by the number of values in that set, or the sum of the values over the number of values. If the last test counts as 2 tests, then we can use the following equation to find the solution: $\frac{69 + 77 + 88 + 2x}{5} = 80$ where x represents the score on the last test. This simplifies to $234 + 2x = 400$. Combine like terms for $2x = 166$, and finally $x = 83$.
14. *Roots of Numbers*. C. This expression can be rewritten as $\sqrt{6 \times 3} = \sqrt{18} = 3\sqrt{2}$.
15. *Exponents*. A. $64 = 4^3$. Since $4^9 = (4^3)^3$, then 4^9 expressed in terms of 64 can be expressed as 64^3 .
16. *Permutations & Combinations*. A. Corinne wants to donate 3 out of 4 bears. In the combination formula, $n = 4$ and $r = 3$. $\frac{4!}{3!(4-3)!} = \frac{24}{6(1)} = 4$.
17. *Circle Graphs*. C. If there were 45,000 votes in all and 9,000 came from New York, then one fifth of the votes were from New York. One fifth of 360° is $360 \div 5 = 72^\circ$.
18. *Angle Sums*. C. The sum of the 4 interior angles of a rectangle is 360° . Applying the given formula, where n represents the number of sides of the polygon: $180(4 - 2) = 180(2) = 360$.
19. *Arithmetic*. A. Find the value of the numerator and denominator first. $5^2 + 5^1 = 25 + 5$, or simply 30. $3^3 + 3^1 = 27 + 3$, or simply 30. So, the value of the expression is $\frac{30}{30}$, or 1.
20. *Algebraic Relationships*. B. If $\frac{a}{b} = c$, then multiplying both sides by b yields $a = cb$. Dividing both sides by c yields and $b = \frac{a}{c}$.
21. *Sequences*. B. Each term in this sequence is twice the previous term. To save time, instead of finding an expression to represent this sequence, we need simply multiply each resulting term by 2 four more times: $1 \times 2 = 2$, then $2 \times 2 = 4$, then $4 \times 2 = 8$, and finally $8 \times 2 = 16$.
22. *Probability*. B. The probability the girl will spin an odd number on the spinner is $\frac{2}{4}$ or $\frac{1}{2}$. The probability the girl will roll an even number on the die is $\frac{3}{6}$ or $\frac{1}{2}$. The probability that she will do both is equal to $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, which is less than $\frac{1}{2}$.

23. *Negative Exponents*. C. A negative exponent shows an inverse relationship such that $x^{-2} = \frac{1}{x^2}$. So,
- $$\frac{d^3 e^{-5} f}{3 p q^{-2} r^{-8}} = \frac{d^3 f q^2 r^8}{3 e^5 p}.$$
- In Column B, dividing the numerator and denominator by 2 yields $\frac{d^3 f q^2 r^8}{3 e^5 p}$. Therefore, the amounts in both columns are equal.
24. *Percents*. A. This question can be represented by the equation $0.8x = 48$, where x is the original price of the pair of pants. (If the sale takes 20% off, then the discounted price is 80% of the original, or 0.8 times the original price.) $x = 60$, so the original prices was \$60, which is greater than \$58.
25. *Slope*. B. The slope of a line is equal to the rise over the run. This line has points at $(0, -3)$ and $(-2, 1)$, so the slope is equal to $\frac{-3-1}{0-(-2)} = \frac{-4}{2} = -\frac{2}{1}$. The slope of a line that is perpendicular to another line is simply the negative reciprocal. In this case, the slope of the perpendicular line is there equal to $\frac{1}{2}$. So, the amount in in Column B is greater.
26. *Area & Perimeter*. B. If the perimeter is equal to 36, then the maximum possible area is a square in which each side is equal to 9, in which case the area is 81 square inches, and the minimum is a rectangle with one side equal to 1 and one side equal to 17, in which case the area is 17 square inches. This smaller area is less than 36 square inches. Therefore, since $36 > 17$, B is the correct choice.
27. *Distributing*. B. Applying FOIL to the equation in Column A yields $x^2 + 8x + 15$, so Column B is greater than Column A by 1 no matter what the value of x .
28. *Area & Perimeter*. B. The sidewalk is 5 feet wide, so removing 10 feet from the length and width gives the dimensions of the lot without the sidewalk, or 95 by 45. $95 \times 45 = 4,285$, which is less than 5,000.
29. *Averages*. A. If the teacher adds 2 points to every test, each grade increases by the same amount, so the range of grades will remain the same. The median, however, will increase. That change is greater than no change.
30. *Exponents*. B. $64 = 2^6$, so $64^2 = 2^{12}$. Thus, Column B is greater.
31. *Frequency Charts & Graphs*. B. The median is the midpoint value of all given values. According to the graph, 8 people donated at the \$20 level, 6 people donated at the \$50 level, 3 people donated at the \$100 level and one person each donated \$200 and \$500, so laying out all of the values yields: 20, 20, 20, 20, 20, 20, 20, 20, 50, 50, 50, 50, 50, 50, 50, 100, 100, 100, 200, 500. The midpoint value is \$50. So, Column B is greater.
32. *Geometry with Variables*. C. A cube has an equal length, width, and height, so each side of cube M is equal to $\sqrt[3]{64x^3}$ or $4x$. The volume of a cube with a side length 2 times $4x$ has side lengths of $8x$, which means it would have the same volume as a cube with a side length of $8x$, so both cubes have the same volume.
33. *Algebraic Relationships*. B. Find the value of $2xy$. Foil the first equation: $(x - y)^2 = x^2 - 2xy + y^2 = 60$. Notice that we can rewrite this equation as $x^2 + y^2 - 2xy = 60$. Since $x^2 + y^2 = 30$, we can substitute 30 into the equation. This gives us $30 - 2xy = 60$, which simplifies to $-2xy = 30$, and $2xy = -30$.
34. *Probability – Conditional*. C. In both scenarios, the third flip is not affected by the first two flips. The probability of flipping heads or tails will always be $\frac{1}{2}$, so the value of both columns is equal.
35. *Histograms*. A. We know that the data set will be somewhere between the value of the mean of the low end of the ranges and the high end of the ranges. The lowest possible sum of the data is the sum of 5, 5, 10, 10, 15, 15, 15, 15, 15, 15, 20, 25, 25, 25, 25, 30, 30, 30, 30, 30, and 30, or 435. There are 22 points in this data set. The mean of this set is $435 \div 22 \approx 19.77$. The mean of the actual data shown in the histogram will be greater than or equal to 19.77 (with some unknown upper boundary), so the mean will be greater than 15. Thus, the value of Column A is greater.

36. *Proportions*. A. Since the given shapes are rectangles, $PQ = SR$ and $TU = WV$. If y represents the length of SR , then we know that $15 = 0.6y$. Solving for y shows that the length of SR is 25. Thus, the value in Column A is greater.
37. *Fractional Exponents*. C. We know that y is greater than 1. The expression in Column A can be rewritten as $\frac{1}{\sqrt{y}}$. The negative exponent in Column B indicates the number should be flipped over the fraction line, so this can also be rewritten as $\frac{1}{\sqrt{y}}$. So, the values of both columns are equal.

Reading Comprehension

1. C. *Main Idea*. The author uses the Easter Island statues to make a larger point about some historical phenomena being difficult for historians to understand, even after extensive research. The other choices are presented as supporting ideas that are used in context of the broader passage.
2. B. *Supporting Idea*. Lines 5–7 state that the statues are buried to their shoulders. Since only the heads of the statues are visible, many people assume the statues are just heads.
3. A. *Inference*. The lines describe “oral histories” and “stories.” These are not the same as documentation in “historical records.” It doesn’t make sense that the people who moved the moai didn’t know how they did it. There’s also no evidence that the information was once known but then lost.
4. A. *Vocabulary*. Lines 53–62 describe doubt about a possible explanation. In context, the passage describes how this is not “definitive,” meaning it is not “certain.” Of the answer choices, is “authoritative,” which means certain. Be careful not to choose “uncertain,” since the question asks for the word “definitive” and not for the phrase “no definitive.”
5. B. *Tone/Language/Style*. The author does not say anything with certainty in the last paragraph, so A and C can be eliminated as choices. Instead, the author describes how there’s still no definite answer – “no closer to discovering the truth.” This doesn’t mean that he is tolerant. Instead, it means that he is leaving room for mystery, or being cryptic.
6. D. *Organization/Logic*. The author presents the historical mystery of the moai, including several competing theories regarding them. The author does not, however, discuss the geological history of Easter Island in the passage.
7. A. *Main Idea*. Most of the passage describes the camp the narrator’s friends parents purchased. The passage does not describe a particular event, person, or process.
8. C. *Inference*. The narrator describes the owners as “enterprising and entrepreneurial,” which implies they have previous experience running their own businesses. There’s no mention of swimming, camping, or any other history of renovating properties.
9. B. *Supporting Idea*. In line 21–22, the author says the camp’s owners hired workers to comb the woods for mushrooms, of which chanterelles, hen-of-the-woods, and morels are types.
10. A. *Vocabulary*. Lines 47–49 say that the evaporator was “so beautiful” it “took my breath away.” In context, then, the intricately carved doors likely relate to that beauty. “Intricately” usually means something has a lot of small details, so the carvings on the door could also be described as “elaborate.”
11. C. *Supporting Idea*. Rereading the whole paragraph indicates that the metamorphosis, or change, the narrator is referring to is the process of turning sap into syrup.
12. A. *Tone/Language/Style*. The text shows that the narrator appreciates the camp and the things found there, and the passage concludes with the narrator excited to get to work. So, the best match for the tone of the passage is “admiring,” or appreciative.
13. B. *Main Idea*. Kennedy begins the speech by welcoming the listener to the White House. He goes on to compliment the people he is addressing, including the Nobel prize winners from the Western Hemisphere. Therefore, the speech is primarily meant to honor distinguished guests at the White House.

14. B. *Tone/Language/Style*. Kennedy welcomes people to the White House in a positive way, so the mood of the first lines of the speech can best be described as hospitable, which means generous or cordial and is usually used in the context of receiving guests. The other choices, particularly “contentious” and “remorseful” are very negative.
15. D. *Organization/Logic*. Kennedy points out that the nationality of the Nobel prize’s recipient does not matter, implying that the prize is given because of merit, not country of origin. There’s no direct comparison between Nobel and Jefferson.
16. A. *Supporting Idea*. In this paragraph, Kennedy explains that the Nobel winners have built on previous knowledge, and also that other men and women in the world can build on the knowledge of the prize winners. The paragraph never mentions how prize recipients are given access to records that others aren’t, or that there is something particular to certain nationalities.
17. A. *Inference*. Kennedy states in lines 48–50 that the Western Hemisphere creates “an atmosphere which has permitted the happy pursuit of knowledge, and of peace.” This implies that people who live in the Western Hemisphere are encouraged, or given the freedom, to pursue knowledge in a way they may not be in other parts of the world.
18. B. *Vocabulary*. Kennedy refers to young people having the same drive desire for knowledge and peace as the Nobel prize winner. In this context, “drive” means an effort toward a goal, or motivation. Satisfaction or happiness can be a one of these goals, but it is not the actual drive or motivation itself.
19. B. *Main Idea*. The passage argues that technology is changing the workplace and can offer good solutions to some workplace problems. The author describes different styles of communication to support the larger argument that technology can help workplaces work more efficiently. But the article does not discuss absolutes; the author does not argue that traditional office spaces are all bad or that technology can solve all problems.
20. D. *Vocabulary*. Context clues indicate that technology offers cheaper, faster, and more convenient ways to accomplish what is done in the office. In other words, technology is replacing the office, not hiding it (conceal), or making it more prevalent (establish) or permanent (ingrain).
21. B. *Supporting Idea*. The author of the passage mentions exchanging new ideas (line 64), communicating with other people (lines 58–59), and working together on projects (lines 61–63) as advantages to technology in workplaces, but does not mention building a unified culture.
22. C. *Inference*. The main advantages of technology in the work place, according to the passage, mostly relate to increasing efficiency and decreasing cost. The author repeatedly emphasizes that technology makes working faster, cheaper, and easier. However, technology can’t replace some functions of a centralized office, so it cannot completely replace it.
23. B. *Tone/Language/Style*. A panacea is a solution or cure for all problems. By arguing that technology is not that, the author is saying that technology, while good and helpful for many workplace procedures, can’t solve all problems.
24. C. *Organization/Logic*. The author presents arguments to support the opinion that technology benefits the work place; gives pros and cons to technology in the workplace, which are different points of view; and explains the advantages and disadvantages of working in an office. The article speaks broadly about the workplace and doesn’t describe different industries.
25. D. *Main Idea*. The passage is a description of what caused the European Renaissance, a significant era in history. The passage lists factors that led to the Renaissance, which includes financial causes, but there were other factors as well.
26. C. *Tone/Language/Style*. If something swept across the content, it moved quickly and spread around a great area. This emphasizes the widespread impact of the Renaissance.
27. A. *Vocabulary*. In context, the author describes the economy as paramount; earlier the author states that Italy became stable because of commerce and trade. From this we can infer that the economy was more important than anything else, or “central” to Italian republics.
28. A. *Organization/Logic*. In the third paragraph, the author describes the economic importance of republics (or the impact of commerce), explains how the increase of wealth and trade led to more

goods, and gives specific examples of Florence and Venice. The author does not, however, blame religious decline on economic growth.

29. D. *Supporting Idea*. The paragraph explains that wealthy Italian merchants used their money to advance the art. These merchants patronized da Vinci, Michelangelo, and others, allowing them to pursue their dreams and visions.
30. A. *Inference*. The author emphasizes economic, political, and social causes for the Renaissance. In lines 10–12, the author states that a confluence of social and political events led to the Renaissance and then explains the economic factors in the second paragraph. Without these changes, the Renaissance might not have taken place.
31. B. *Main Idea*. Although there are some specific details about tornadoes, the passage is more a general overview of tornadoes than about specifics. The passage also is not a criticism or argument about tornado-strength measuring scales.
32. D. *Vocabulary*. “Concurrently” describes the way a column of air is in contact with both the earth and a cloud. Thus, “concurrently” means “simultaneously,” or that the column of air touches both at the same time.
33. C. *Inference*. The paragraph describes a range of storms, from weaker formations that still have fast wind speeds to more violent storms. We can infer that tornadoes are unpredictable because their appearance, strength, and longevity vary so much.
34. A. *Organization/Logic*. The paragraph is a list of the different shapes tornadoes can form, so the organization is best described as a compilation of physical tornado attributes.
35. D. *Supporting Idea*. The author explains in lines 56–59 that Tornado Alley is large, mostly flat, and without major landmasses to block airflow, so the unobstructed meeting of cold and warm air masses is the reason tornadoes occur more frequently in North America.
36. D. *Tone/Language/Style*. “Tornado Alley” is a nickname for the part of the US where tornadoes most frequently form. The author is not being humorous here but emphasizing that tornadoes happen often in this part of the country.

Mathematics Achievement

1. *Sequences*. C. All of the terms in the sequence are multiples of 4. One approach would be to apply each given pattern and see which works. 8 is the second term, and is equal to $4(2)^1$. The third term is 16 which is equal to $4(2)^2$. (Remember to use the correct order of operations and apply the exponent before the multiplication.) Therefore, the exponent is $n - 1$, so the n th term will have a value of $4(2)^{n-1}$.
2. *Converting Units*. A. Cody runs 6 miles per hour, meaning he runs $6 \times 1,760$ yards per hour. There are 60 minutes in an hour and 60 seconds in a minute, so to find the total yard per second, find the number of yard run per hour over the number of seconds in an hour, or $\frac{6 \times 1760}{60 \times 60}$.
3. *Exponents*. B. The exponent indicates the number of times a number is multiplied. So, 6^2 in expanded form is 6×6 .
4. *Factoring*. D. Some choices can be eliminated without multiplying based on what we know from the given question. In factored form, $x^4 - 81$ would have outer and inner terms that cancel each other out, which is not the case for choice A or C. Choice B would only give us x^2 , so it should be eliminated. D looks like the best answer. Multiply to check: $(x - 3)(x + 3) = x^2 - 9$; $(x^2 - 9)(x^2 + 9) = x^4 - 81$.
5. *Box-and-Whisker Plots*. C. The median divides the data into halves, and is represented in the box-and-whisker plot by the line that divides the box. The line appears to be between 60 and 65, so 62 is the only answer.
6. *Absolute Value Inequalities*. A. Isolate x . If $2|x + 4| - 6 < 8$, then $2|x + 4| < 14$ and $|x + 4| < 7$. So, $-7 < x + 4 < 7$ and $-11 < x < 3$, which is the span shown in graph A.
7. *Vocabulary*. D. A complex number is a number that can be expressed as $a + bi$ in which a and b are real numbers and i is an imaginary number. An imaginary number can be expressed as $i^2 = -1$. So, $i^4 = 1$. 1 is a whole number. Note that no number, when multiplied by itself 4 times, can be negative.

8. *Scientific Notation*. B. Write each number in standard form and add: $6,020,000 + 42,900 = 6,062,900$, or 6.0629×10^6 . The number of times a decimal has to be moved determines what power to raise 10 to when writing in scientific notation.
9. *Roots of Numbers*. A. Like terms can be combined. $\sqrt{54} = \sqrt{9 \times 6} = 3\sqrt{6}$. $\sqrt{24} = \sqrt{4 \times 6} = 2\sqrt{6}$. So, $\sqrt{54} - \sqrt{24} = 3\sqrt{6} - 2\sqrt{6} = \sqrt{6}$.
10. *Multiples*. D. To find the least common multiple of variables, simply take the highest exponent of that term. The greatest power of c is 5, and the greatest power of d is 3. The least common multiple of integers cannot be smaller than the given numbers (here, 2, 3, and 5). Only choices B and D have the variables c^5 and d^3 . Only choice D also has a coefficient of 30.
11. *Imaginary Numbers*. D. The solution set for $0 = x^2 + 400$ is the set of values that make the equation true. $x^2 = -400$, so $x = \sqrt{-400}$. Negative numbers within a radical are imaginary, so we can factor out $\sqrt{-1}$, which is equal to i . $i\sqrt{400} = 20i$. Remember that the value of x could be positive or negative, the solution set for the equation is actually $\pm 20i$.
12. *Appropriate Units*. B. Measuring the volume of water in a lake implies a measure of liquid capacity. Grams measure mass. Miles measure distance. Square feet measure area. But kiloliters are a liquid capacity measure, so this is the best answer.
13. *Roots of Variables*. D. Square factors can be removed from the radical. $\sqrt{p^8} = p^4$ and $\sqrt{r^3} = r\sqrt{r}$, so the value of the expression is $p^4 r \sqrt{qr}$.
14. *Histograms*. C. There are 23 total parks, so the park with the median area would be the 12th park if all of the parks were placed in order of area from least to greatest. The 12th park falls in the 600–699 range, so 602 is a the best choice.
15. *Frequency Charts & Graphs*. A. Be careful to interpret the graph correctly; the y-axis shows the number of values in the set. There were, for example, 3 teams for which 10 kids tried out for the team. The graph shows a total of $3 + 5 + 3 + 2 = 13$ teams. We're told that the median number of kids who tried out for a team is 20, which means this is the value in the. We can list out the values in the set: {10, 10, 10, 20, 20, 20, 20, 20, 30, 30, 30, 50, 50}. We can see that in order for the set to have a median of 20, we must only have a set that looks like this, with two 40's: {10, 10, 10, 20, 20, 20, 20, 20, 30, 30, 30, 40, 40, 50, 50}.
16. *Survey Samples*. A. Since students are unlikely to attend a movie without their parents, the parents of school-age children are the best sample for the information the movie studio is looking for. The other given may or may not contain people with knowledge of whether children will go see the movie.
17. *Area & Perimeter*. B. There are 9 squares. If the total area is 36 cm^2 , then the area of each square can be represented by x in the equation $36 = 9 \times x$. Divide both sides by 9 to find that the area of each square is 4 cm^2 . The length of each side is equal to the square root of the area, or 2 cm.
18. *Geometry with Variables*. A. The perimeter is equal to the sum of the lengths of each side. We have two sides of length a , and two sides of length $a - b$. However, if we include the length of b on each side, we have $a - b + b$, which leaves us with a . Therefore, the perimeter is $4a$.
19. *Circle Graphs*. C. A circle has 360° . The 12 students who chose green represent 120° of the circle graph, or one-third of the circle. If we know that 12 students represents one-third of the total number of students, we can solve algebraically. If x represents the total number of students, then we know that $12 = \frac{1}{3}x$. Multiply both sides by 3 for $x = 36$. Simply subtract to find the number of students who voted for orange: $36 - 12 - 9 - 8 - 5 = 2$.
20. *Solving for Zero*. B. Dividing both sides of the equation by $x^2 - 9$ yields $(x - 1)(x + 2) = 0$. The values that make this equation true are $x = 1$ and $x = -2$.

21. *Formulas*. C. Put the given values into the equation and solve for h to find the answer.

$$50\pi = \frac{1}{3}(25\pi)h \text{ So, } 2 = \frac{1}{3}h \text{ and } h = 6.$$

22. *Permutations and Combinations*. B. There are 5 books and Bob can choose 3 of them. This is a combination without repetition, since a book can't be chosen again once it has already been chosen.

We can use the formula $\frac{n!}{r!(n-r)!}$, where n is the number of things to choose from and r is the

number we want to choose. $\frac{5!}{3!(5-3)!} = \frac{120}{12} = 10$, so there are 10 possible combinations.

23. *Simplifying Expressions*. D. Like terms can be combined. In this case, first apply the subtraction so that the expression is $2m^2n + 3mn^2 - 4mn^2 + 5m^2n$, then combine the terms to get $7m^2n - mn^2$.

24. *Equation of a Line*. A. First, find two points on the line: $(-4, -2)$ and $(0, -3)$. Use these points to find the slope: $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - (-2)}{0 - (-4)} = -\frac{1}{4}$. The slope of the perpendicular line is the opposite reciprocal,

or 4. Simply plug in the value of x given (which is -4) into the two choices that have a slope of 4. Only choice A results in a true equation where $y = -2$.

25. *Distance on a Coordinate Grid*. B. Graphing the rectangle yields a rectangle with side lengths 5 and 12. Use the Pythagorean Theorem to find the length of the diagonal: $5^2 + 12^2 = c^2$ or $\sqrt{169} = c$. So, the length of c , or the diagonal, is 13.

26. *Slope*. A. The equation for slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$, so $-3 = \frac{0 - y}{5 - 3}$, and $-3 = \frac{-y}{2}$. Multiplying both sides by 2 yields $-6 = -y$ and $y = 6$.

27. *Polygons on a Coordinate Grid*. D. A right triangle has a 90° angle, so two adjacent sides of the triangle must have perpendicular slopes, or slopes whose product is equal to -1 (one slope is the negative reciprocal of the other). Slope is equal to rise over run. The given points are at $(1, 4)$ and

$(5, 2)$. The slope of that line is equal to $\frac{4 - 2}{1 - 5} = \frac{2}{-4} = -\frac{1}{2}$. So, the perpendicular line must have a

slope of 2. For point $(-2, -2)$ and point $(1, 4)$, the slope is $\frac{4 - (-2)}{1 - (-2)} = \frac{6}{3} = 2$. So, $(-2, 2)$ is the third vertex in this right triangle.

28. *Trigonometry*. A. In this triangle, $\sin 35^\circ$ is equal to the ratio of the opposite side (2) over the hypotenuse. So, $\sin 35^\circ = \frac{2}{h}$. So, the hypotenuse is equal to $\frac{2}{\sin 35^\circ}$.

29. *Roots of Numbers*. B. This expression can be rewritten as $6\sqrt{10} + 5\sqrt{3} - 2\sqrt{3} = 6\sqrt{10} + 3\sqrt{3}$.

30. *Probability*. B. To find the probability of two events taking place, multiply the probability of each event. In this case, the product $\frac{1}{6}$, and we know the probability of the first event is $\frac{1}{2}$. This means

that $\frac{1}{2} \times \text{probability of second event} = \frac{1}{6}$. Solving for the probability of the second event gives us $\frac{1}{3}$.

31. *Stem-and-leaf Plots*. B. The median is the midpoint in the data. The midpoint in this stem-and-leaf plot is the second 9 in the 125 row, which represents 1,259.

32. *Imaginary Numbers*. A. $i^2 = -1$, so $i^4 = (-1)^2 = 1$.

33. *Area & Perimeter*. D. Find the area of the shaded section by finding the area of the whole circle and subtracting the area of the unshaded circle. $A = \pi r^2$, so the area of the whole circle is $A = \pi(3 + 5)^2$ or 64π . The area of the unshaded circle is $A = \pi(3)^2$ or 9π . So, the shaded area is equal to $64\pi - 9\pi = 55\pi$.
34. *Matrices*. C. In a matrix, the 3 must be multiplied to all 4 values, so $\begin{bmatrix} 3 \times 4 & 3 \times -2 \\ 3 \times 9 & 3 \times c \end{bmatrix} = \begin{bmatrix} 12 & -6 \\ 27 & 3c \end{bmatrix}$.
35. *Angle Sums*. B. Adjacent angles in a rhombus add up to 180° , so the measure of angle Y is equal to $180^\circ - 102^\circ = 78^\circ$.
36. *Averages*. C. We can represent this question as $90 = \frac{4x + 5x + 6x}{3}$, which simplifies to $270 = 15x$, and $x = 18$.
37. *Probability – Conditional*. A. The probability of rolling any given number is $\frac{1}{6}$, so the probability of rolling a 1 and then rolling a 2 is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.
38. *Speed*. D. If Inez walks 500 meters in 15 minutes, she can walk 2 kilometers in an hour ($15 \times 4 = 60$ minutes = 1 hour; 1 km = 1,000 m). If Juan walks 1 kilometer per hour slower, he walks 1 kilometer per hour, so it will take him 3 hours to walk 3 kilometers.
39. *Percents*. B. If Bruce and Betty recycled a total 1,000 bottles, and Bruce recycled 100 more than Betty, then Bruce recycled 550 bottles. We can solve this algebraically by letting x represent the number of bottles Betty recycled, and $x + 100$ represent the number of bottles Bruce recycled. Since together they recycled a total of 1,000 bottles, we know that $x + x + 100 = 1000$, and $2x = 900$. Thus, x (Betty's number of bottles) is 450, and Bruce's is 550. As a percentage of total bottles Bruce recycled is equal to $550 \div 1,000 = 55\%$.
40. *Proportions*. B. Noon to midnight is 12 hours. We can represent this proportion as $\frac{2}{500} = \frac{12}{x}$. Cross multiply to solve: $2x = 6,000$, so $x = 3,000$.
41. *Creating Expressions & Equations*. D. If c is the total number of copies, remember to subtract the first copy when writing the equation: $1.75 + 0.95(c - 1)$.
42. *Area & Perimeter*. C. The figure is made up of 9 squares with 16 sides. If the perimeter is 32, then each square has a side length equal to the perimeter divided by the total number of sides, or $32 \div 16 = 2$. Each square has an area of 2^2 , or 4 cm^2 , and there are 9 squares, so the total area is equal to $4 \times 9 = 36 \text{ cm}^2$.
43. *Consecutive Integers*. C. If the average of four consecutive odd numbers is 12, then 12 is the integer between the two middle integers, so this set is 9, 11, 13, 15. (Test: $9 + 11 + 13 + 15 = 48$; $48 \div 4 = 12$.) The smallest integer is 9.
44. *Box-and-Whisker Plots*. D. The median is the middle number or the line in the middle of the box in the plot; it divides the data set into smaller and larger numbers. The third quartile is the median of the larger numbers, or the rightmost side of the box in the plot. On this graph, that point is at 900.
45. *Percent Change*. B. Today, Jeff read 50 more pages than the previous day. $50 \div 100 = 0.5$, or a 50% increase.
46. *Algebraic Relationships*. B. Apply the Pythagorean theorem. $(x + 3)^2 + (x - 3)^2 = 8^2$. Apply FOIL to the first part of the equation: $(x + 3)^2 + (x - 3)^2 = x^2 + 6x + 9 + x^2 - 6x + 9$. This simplifies to $2x^2 + 18 = 64$. So, $2x^2 = 46$.
47. *Distributing*. D. Start with the easier math in the second two binomials, since we can see that the binomials are the same except for the signs. $(x - 3)(x + 3) = x^2 - 9$. Now, substitute into the original expression and multiply by the first expression: $(x^2 + 9)(x^2 - 9) = x^4 - 81$.

Verbal Reasoning

Synonyms

Connotation

1. D. “Cautious” has a neutral connotation and is an adjective that means careful. “Wary” means watchful or careful and is the only synonym with a similar meaning for cautious. “Bothered” has a negative connotation while “righteous” has a positive connotation. Something unexpected is neutral also, but in this case, we must think to use the word in a sentence: “The burglar was cautious not to make a noise, and tiptoed around the house.” We can see that “unexpected” doesn’t make sense.
2. A. “Obvious” has a negative connotation and is an adjective meaning “easily seen.” “Marsha didn’t see the book, even though it was in an obvious spot on the table.” “Blatant” has a similar connotation and also means “easily seen.”
3. D. “Discredit” has a negative connotation and is a verb. The prefix *dis-* tells us the meaning will likely mean the opposite of “credit,” so “discredit” means “to take away from a good reputation.” The only choice with a negative connotation is “malign,” which means “to give a false report.”
4. B. “Glee” has a positive connotation and is a noun meaning “joy.” “Failure” can be eliminated, since it is a word with a negative connotation. “Delight,” which means “great pleasure,” is the closest in meaning to “joy.”
5. A. “Suspect” has a negative connotation and, as an adjective, means “doubtful.” All choices but “dubious” have positive connotations. Try using the word in a sentence: “The police officer was right to assume that the thief’s alibi was suspect.”
6. B. “Fiasco” has a negative connotation and is a noun that means, “a complete failure.” “The project was a complete fiasco; everything that could have gone wrong did.” “Triumph” can be eliminated because it has a positive connotation. The only other word that works in the sentence is “disaster.”
7. B. “Devious” has a negative connotation and is an adjective meaning “underhanded.” All choices but “dishonest” have positive connotations, so “dishonest” is the best answer.
8. A. “Travesty” has a negative connotation and is a noun meaning “an absurd imitation.” “Facsimile” has a similar definition, but a neutral connotation, so “distortion” is a better answer. A travesty can be a distortion of the original, not a copy or replica of it.
9. B. “Naïve” usually has a negative connotation and is an adjective meaning “simple” or “inexperienced.” Someone who is naïve doesn’t have enough experience to, for example, tell truth from lies, so they might be considered “gullible” or “credulous.”
10. C. “Amicable” has a positive connotation and means “characterized by goodwill” or “friendly.” “Detestable” and “insolent” can also be quickly eliminated because they have negative connotations.
11. D. “Outlandish” has a neutral connotation and is an adjective meaning “out of the ordinary.” The other choices are all synonyms of each other, but “unorthodox” means “unusual,” which is the closest synonym to outlandish.
12. D. “Meek” has a neutral connotation and is an adjective meaning “weak” or “cowardly.” “The meek kitten let me pick her up and carry her around.” We can substitute the word “submissive” in this sentence without changing its meaning.
13. B. “Abundant” has a positive connotation and is an adjective meaning “bountiful” or “in a great amount.” “Restricted” can be eliminated because it has a negative connotation., and the other two choices are neutral. “Profuse” means “a great deal” or abundant.
14. C. “Renovation” has a neutral connotation and is a noun meaning “restoration or improvement.” It’s often used in the context of home improvement. So, “improvement” is the only synonym with a similar meaning.
15. D. “Hinder” has a negative connotation and is a verb meaning “to hold back or impede.” The only choice with a negative connotation is “prevent,” which can also mean to impede, or stop something from happening.
16. B. “Contempt” has a negative connotation and is a noun meaning “hatred” or “disrespect.” We can eliminate the other answer choices because they all have positive connotations.

17. A. "Prosperity" has a positive connotation and is a noun meaning "great wealth or success." "The wealthy CEO bought a huge new home to celebrate his prosperity." "Abundant" could be substituted in the sentence without changing its meaning. The other choices are either negative (entropy) or neutral (proposal) in connotation.
18. B. "Procrastinate" has a negative connotation and is a verb meaning "to delay or put off." The other answer choices all have positive connotations and can be ruled out.
19. C. "Squander" has a negative connotation and is a verb meaning "to waste." "The football player was a fool to squander his scholarship opportunity." This is the opposite in connotation of the other words, which are positive in connotation.
20. B. "Gullible" has a neutral to negative connotation and is an adjective meaning "easily duped." "My little sister is so gullible, she believed me when I told her dragons are real." "Innocent" can have a similar meaning, because it implies someone doesn't have enough experience to know better.
21. A. "Exemplary" has a positive connotation and is an adjective meaning "serving as an example." "The police officer won an award for exemplary work on the job." "Admirable" is the only option that could be replaced in the sentence. The other answer choices are more neutral in connotation.
22. B. "Harass" has a negative connotation and is a verb that means "to annoy." The other answer choices all have positive connotations and can be ruled out. "Pester" also means "to annoy" and is the only synonym with a similar meaning.
23. D. "Frantic" has a negative connotation and is an adjective meaning "out of control." "Tame" and "serene" can be ruled out because they have positive connotations. "Wild" is a better synonym than "misunderstood," the latter of which has a neutral connotation.
24. D. "Affluence" has a positive connotation and is a noun meaning "wealth." The other answer choices all have negative connotations.
25. A. "Livid" has a negative connotation and is an adjective meaning "angry." "Enranced" and "gleeful" have positive connotations and can be ruled out. "Impartial" is neutral and is not a good synonym, so "distressed" is the best choice, since it is the only other word with a negative connotation.
26. C. "Admonish" has a negative connotation and is a verb meaning "to scold." "The teacher admonished her class for goofing around instead of studying." The other answer choices all have positive connotations, which we can use to rule them out.
27. D. "Adverse" has a negative connotation and is an adjective meaning "harmful" or "against one's interests." The other answer choices all have positive connotations and are antonyms. "Deleterious" also means "harmful."
28. C. "Terse" has a negative connotation and is an adjective meaning "brusque" or is a manner of speaking in which one uses few words. "The angry clerk gave a customer a terse, one-word answer in reply to her complaint." This is the opposite in meaning of "verbose," which also has a negative connotation, but means "wordy." "Cordial" has a positive connotation, and can be ruled out immediately.
29. A. "Reconciliation" has a positive connotation and is a noun meaning "an act of restoring a friendship or agreement." "After three days of fighting, the brother and sister finally agreed on something and began their reconciliation." "Agreement" has the closest meaning to the word in the sentence of the choices. "Discord" has a negative connotation, and "conversion" has a neutral one. We can't replace "approval" in the sentence.
30. A. "Devout" has a positive connotation and is an adjective meaning "very sincere." "Mrs. Rodriguez is a devout Catholic; she attends mass every Sunday." "Pious" also means very religious and is closest in meaning to "devout."
31. C. "Prudent" has a positive connotation and is an adjective meaning "thoughtful." Choices The other choices all have negative connotations, so they can be ruled out. "Sensible" has the closest meaning.
32. D. "Futile" has a negative connotation and means "without purpose." The other choices can be eliminated because they all have positive connotations. We might say that doing something with no result is something done in vain, or without purpose.

33. A. "Trite" has a negative connotation and is an adjective meaning "hackneyed" or "unoriginal." "The movie was trite and full of clichés." "Banal" also means "unoriginal" and could replace the word "trite" in the sentence. "Short" is neutral in connotation, while "original" and "exclusive" are positive in connotation.
34. C. "Provincial" has a neutral connotation and is an adjective meaning "concerning the country." It can have a negative connotation, depending on the context. "She is too provincial to handle life in the big city." "Patriotic" and "victorious" have positive connotations while "foreign" is neutral. The best word here is "simple," which can also have a negative connotation depending on the sentence.
35. D. "Disparage" has a negative connotation and is a verb meaning "to regard as having little worth." "He tends to disparage students he doesn't think are as smart as he is." Based on connotation, "adore" and "appraise" can be eliminated because they both have positive connotations. "Despair" is not a good fit in the sentence, but "vilify" (think "villain") means "to say negative statements about," which is close in meaning to "disparage."
36. D. "Palatial" has a positive connotation and is an adjective meaning "like a palace." "The wealthiest man in town lives in a palatial mansion." "Beautiful" is similar, but "grand" is more specific. Many things can be beautiful, but a palace could also be large in scope, or "grand."
37. A. "Derision" has a negative connotation and is a noun meaning "ridicule or scorn." "He is a snob and treats people he looks down on with derision." The only word that works in the sentence is "contempt." The other words have positive or neutral connotations.
38. A. "Infamy" has a negative connotation and is a noun meaning "the state of being known for something bad." Franklin Roosevelt famously said that December 7, 1941, the day of the attack on Pearl Harbor, was "A date which will live in infamy." The other choices all have positive connotations, but "disrepute" has a negative connotation and means "the state of having a poor reputation."
39. D. "Ominous" has a negative connotation and is an adjective meaning "foreboding." The other choices all have positive connotations, but "threatening" has a negative connotation and implies the possibility of future harm.
40. B. "Insatiable" has a negative connotation and means "incapable of being satisfied." "He ate three bowls of soup and asked for a fourth; he had an insatiable appetite." "Greedy" is closest in meaning to "insatiable" in the sentence, and has a similarly negative connotation.
41. A. "Ingenuity" has a positive connotation and is a noun meaning "cleverness." "The programmer's proposed improvements to the new smart phones showed great ingenuity." "Imagination" is the only choice that works in the sentence.
42. B. "Quarrelsome" has a negative connotation and means "likely to fight or argue." "The boy was quarrelsome and always getting into scrapes with his classmates." "Argumentative" is the choice closest in meaning.
43. B. "Quandary" has a negative connotation and is a noun meaning "a state of confusion." The other choices ("explanation," "reason," "setting") all have a neutral connotation, which can help you rule them out. Like a quandary, a "problem" is an unanswered question.
44. A. "Preposterous" has a negative connotation and is an adjective meaning "absurd." "Analogous," "lucid," and "reasonable" each have a neutral or positive connotation. "Though the idea that the earth is flat was widely accepted just a few hundred years ago, we know today that such an idea is preposterous."
45. D. "Exasperate" has a negative connotation and is a verb meaning "to cause annoyance or anger." "The child exasperated me with her constant questions." "Aerate" and "breathe" both have neutral connotations, and "desire" has a positive connotation.
46. D. "Appall" has a negative connotation and is a verb meaning "to shock or dismay." "The judge was appalled by the criminal's lack of remorse." "Apprehend" generally means to obtain or grasp something (a person, for example, or even an idea), although it might be confused with "apprehensive," meaning to be concerned for the future. "Fortify" and "obey" are more neutral or positive terms.

47. A. “Onerous” has a negative connotation and is an adjective meaning “burdensome.” “The hike to the top of Mount Everest is onerous, taking months of planning and many days of actual hiking.” “Exhausting” is the only word that works as a substitute in the sentence.
48. D. “Desecrate” has a negative connotation and is a verb meaning “to violate the sanctity of.” “Enshrine” has a positive connotation and an antonym for desecrate. “Ooze” and “specify” have neutral connotations.
49. D. “Surreptitious” has a negative connotation and is an adjective meaning “stealthy” or “secretively.” The word is often used to describe something that is negative. For instance, bribery: “The customer used a surreptitious gesture to pass money to the host without anyone seeing.”
50. C. “Travail” has a negative connotation and is a noun meaning “strenuous exertion” or “a difficult piece of work.” “Avail” and “countervail” are similar sounding words with very different meanings. (“Avail” has a positive connotation and is a verb meaning “to be an advantage.” Countervail has a negative connotation and is a verb meaning “to counteract or compensate for.”) “Success” has a positive connotation. Thus, “struggle” is the only synonym with a similar meaning.
51. A. “Roil” has a negative connotation and is a verb meaning “to stir up.” “The storm roiled the water in the lake.”
52. A. “Bedlam” has a negative connotation and is a noun meaning “chaos.” “After the car accident, the scene on the street was bedlam.” “Insanity” is a similar word with a similar connotation (the word bedlam comes from the name of a mental hospital in London) but the term generally refers to the scene of chaos, not an individual person.
53. D. “Slovenly” has a negative connotation and is an adverb meaning “messy or sloppy.” “Effective,” “heavenly,” and “lovingly” all have positive connotations and can be ruled out as synonyms.
54. B. “Heedless” has a negative connotation and is an adjective meaning “thoughtless or inconsiderate.” “Effortless,” “measured” and “steady” all have positive connotations and can be ruled out as possible synonyms. “Impetuous” has a negative connotation and is an adjective meaning “impulsive or thoughtless.”
55. A. “Trepidation” has a negative connotation and is a noun meaning “fear or unease.” “Courage” is an antonym. “He sat down with trepidation as he got ready to take the difficult exam.”
56. C. “Consternation” has a negative connotation and is a noun meaning “dismay.” “Transition” has a neutral connotation. “Comfort” and “composure” both have positive connotations.
57. C. “Listless” has a negative connotation and is an adjective meaning “lacking interest or energy.” “Enthused” and “giddy” both have positive connotations and are antonyms of “listless.” “Thoughtless” has a negative connotation but a different meaning. “Languid” is an adjective meaning “sluggish or droopy.”
58. D. “Pallid” has a negative connotation and is an adjective meaning “colorless or pale.” “Bright” and “colorful” have positive connotations. Along with “dark,” both choices are antonyms of “pallid.”
59. A. “Precocious” generally has a positive connotation and means “mature for one’s age.” “The precocious child was reading long chapter books while most of her classmates were still reading picture books.” “Oblivious” and “slow” have negative connotations. “Precious” has a similar connotation, but “mature” is a better substitute for “precocious” in the sentence.
60. C. “Serendipitous” has a positive connotation and is an adjective meaning “agreeable without being planned.” “She hadn’t expected to find a dress she liked at the store; it was serendipitous when the perfect dress was right near the door.” “Foolish” and “witless” have negative connotations. “Fortuitous” is the best substitution for the sentence.
61. A. “Infernal” has a negative connotation and is an adjective meaning “bad or unpleasant.” “Enjoyable,” “understandable,” and “welcome” all have positive connotations and can be ruled out as possible synonyms.

Root Words

1. D. “Hypothesis” includes the word “thesis,” which is a statement or theory about something. In other words, a thesis is a guess of some – an idea about how something works or why something is.

- “Hypothesis” is related to this word. It is a supposition or a guess, not a fact or necessarily a prediction.
2. D. “Apprentice” comes from a root word *apprendere* meaning “to learn” or “to understand.” An apprentice is someone who learns a trade from a more experienced teacher. Some process of elimination could be used here. “Instructor” or “master” describe a possible teacher, and an “artisan” is an expert at his or her craft, so “novice” is the best synonym for “apprentice,” and is the only word to not be a synonym of the other choices.
 3. C. “Adhere” comes from the same root as “adhesive” and means “to stick.” In the context of behavior, it can mean “to go along with.” “The players must adhere to the rules of the game, or they risk being disqualified.” “Obey” is the best substitution for “adhere” in the sentence.
 4. C. “Adept” has a root that means “to attain knowledge.” The prefix “ad-” lets us know that the word has to do with “moving toward” something, in this case, something positive. To be adept is to be skilled at something, or proficient. To move toward inability is to move toward “infirmness.”
 5. C. “Credible” is derived from the root word “cred” meaning “to trust or believe.” Something credible is believable or trustworthy. “The news report came from a credible source.” “Plausible” means “reasonable” or “able to be believed” and is the only synonym with a similar meaning for credible. This is different from simply having an idea or opinion of something.
 6. C. “Lavish” generally means ornate and expensive. It comes from a root word that means “to wash,” but you could say that something lavish is awash in luxury or expensive decorations (or even awash with money). “Frugal” and “meager” are antonyms and can be ruled out. “Opulent” is the only synonym with a similar meaning.
 7. A. “Advocate” comes from a root word “*voc*,” which has to do with voice or speaking. As a noun, it means a person who speaks favorably on someone else’s behalf. A “champion” is someone who fights on someone else’s behalf, or who might speak out or advocate for someone else.
 8. D. “Subtle” contains the root word “sub,” which means “below” or “lower than.” If something is subtle, it is often beneath one’s ability to notice it. “Only closer inspection revealed that curtain was not a solid color, but instead contained a subtle pattern.”
 9. C. “Acclaim” comes from a root meaning “to shout” (think of how someone might shout a “claim” of some kind). As a verb, acclaim means “to applause.” “Laud” also means “to praise or applause” (think “applaud”).
 10. C. A “congregation” is a gathering of people. The prefix “con-” means “with” or “together,” which gives us a clue that the synonym has to do with the coming together of something – in this case, people. “The church had a large congregation on Sundays.”
 11. D. “Amiable” contains the root word “ami-,” which means “friendly,” “lovable,” or “pleasant.” “The new kid in school was amiable and made friends easily.” Someone who is amiable will also be nice or pleasant.
 12. C. “Benign” contains a the root word “ben-,” which means “good” or “well” (think “beneficial” or “benefit). “The old man looked scary sometimes, but was benign.” In this context, “kind” is the only synonym with a similar meaning.
 13. A. “Conformist” contains a couple of root words. The first is “con-,” which means “with” or “together.” The suffix “-ist” refers to a person (think “dentist”). A conformist is someone who tries to blend in or be like (“with”) everyone else. “The boy from my class is a conformist, only dressing like others in school.” “Follower” could be substituted in the sentence for “conformist.” A leader or radical would not be concerned with being like everyone else, necessarily, like a follower.
 14. A. “Malicious” comes from the word “mal,” which means “bad” (think “malignant” or “maleficent,” like from *Snow White*). Someone who is malicious has bad intentions. “It is malicious for a man to leave his dog outside in bad weather.” “Cruel” is the only synonym with a similar meaning.
 15. A. “Innovate” means “to do something in a new way.” It comes from the root word “nov,” meaning “new” (think “novel”). “Recreate” literally means to create something again; it could also mean to create something in a new way.

16. B. “Beneficence” contains the root word “ben-,” which means “good” or “well” (think “benign” or “benefit”). “The aristocrat showed great beneficence by donating to charity.” You can also use connotation to eliminate the other choices because “avarice,” “jealousy,” and “stinginess” each have a negative connotation and “beneficence” has a positive connotation.
17. C. “Fallible” derives from a verb meaning “to deceive,” which is the same root as “fault.” To be fallible is to have faults or be imperfect. “Flawless,” “impeccable,” and “unspoiled” are all antonyms, and all mean similar things.
18. C. In science, a “volatile” substance is an unstable one, that can change suddenly (often exploding). Its origin can be traced back to the word “volare, which literally means to fly. “Volatile” has come to mean something is prone to sudden change, and is unstable.
19. B. “Ponderous” comes from the word “pondus,” meaning “weight,” from which we also get the word pound. “Ponderous” generally means to be large and unwieldy. “The ponderous piece of furniture was hard to move.” “Buoyant” and “nimble” are antonyms. “Cumbersome” means “unwieldy” or “burdensome.” To ponder something is to think about it because it is a difficult, complex, or weighty idea.
20. D. “Orator” is related to the word “oral,” which has to do with the mouth. The words “dentist” and “speaker” have to do with the mouth. “Orator” comes from the word “oratio,” which means speech. “Entertainer” is similar, but since the root word refers specifically to a speech, “speaker” is a better synonym.
21. C. “Belligerent” includes the word “bell-,” which means “war.” Someone who is “belligerent” can be described as “warlike” or “hostile.” Other words include “bellicose” and “rebel.”
22. B. “Superfluous” word “super” which means “over.” So, we know the word has something to do with having something “over” or “in excess of.” The word “fluid” describes something that shifts and flows, like water or another liquid. This describes something different than the quantity of something. Thus, “excessive” is the only synonym with a similar meaning.
23. A. “Servile” comes from a root word meaning “slave”; it’s the same root as the word “servant.” To be servile is to be like a slave or to serve in a menial position. “Docile,” which means “easily managed” is the only synonym with a similar meaning.
24. A. “Incessant” comes from a root word meaning “cease,” but the prefix “in-” indicates a negative, so the word means “not ceasing.” “Finite,” “interrupted,” and “yielding” all indicate stops or endings and are therefore antonyms. Thus, “eternal,” or “never ending,” is the only synonym with a similar meaning.
25. B. “Empathy” contains the root word “em-,” which means “into” or “with.” To have empathy for someone is to feel great compassion for them. One way to do this is to put oneself “into” another’s shoes – to understand what it is like to be that person. “Empower” is another word with the root word “em-,” and means “to put power into.” “Apathy” and “indifference” are antonyms which mean “to not care about.”
26. A. “Perturb” comes from the same root word as “disturb” and generally means to annoy. “My brother’s constant chattering perturbed me.” In this context, “bother” is the best fit for the sentence.
27. D. “Integrity” comes from a root word meaning “entire” (think of the word “integer,” in math, which means “whole number”). Although integrity can be a trait of someone who adheres to a strict ethical or moral code, it more generally means a state of completeness. “The American Civil War showed how fragile the integrity of the Union really was.” Thus, of the given choices, “unity” has the closest similar meaning.
28. A. “Incandescent” comes from the root word “cand” meaning “to glow” (think of old incandescent light bulbs; also a “candle,” which gives off light). “Luminous” similarly comes from a related root word (“lumin,” meaning “light”; think of the “lumen” which is a measurement of brightness).
29. C. “Recapitulate” comes from a root word that refers to gathering or organizing information. The prefix *re-* means to do something again. Thus, “recapitulate” generally means “to restate” or “to summarize.” (The term “capitulate” usually means to yield or surrender, but its older meaning is “to draw up terms.”)

30. C. “Exacerbate” comes from the root word “acer,” which means “harsh” or “sharp.” This same root is found in words like “acerbic,” “acid,” and “acrimonious.” The prefix *ex-* means “out of”; exacerbate often means “to make something bad worse.” “The rain exacerbated my already bad mood.” “Intensify” is the best answer.
31. B. “Philanthropist” comes from the root word “phil-,” meaning “love” and “anthropos” meaning “people” (an anthropologist studies people). A philanthropist is someone with a great love for people, usually someone who contributes a great deal to charity. These people make donations (give) to charities.
32. D. “Sinuous” comes from the root word “sinus,” which means “curve” (the math terms “sine” and “cosine” come from the same root; graphs of sines and cosines show curves or waves). One’s sinus cavity (the area behind the nose and above the throat) is made up of curving pathways. Sinuous, then, generally means “snakelike.” So, “winding” is the only synonym with a similar meaning.
33. D. “Pervade” comes from a root word meaning “to go” (think of the word “invade”). The prefix “per-” means “through” (like “permeate,” where something seeps through everything very thoroughly). So, “pervade” literally means “to go through,” or “to suffuse every part of something.” Therefore “saturate” is the only synonym with a similar meaning.
34. B. “Immutable” comes from the root word *mutare*, which means “to change” (think of “mutation”). The prefix “im-” is just like the prefix “in-” and means “not.” So, something that is “not changeable” is “immutable.” Thus, “enduring,” or “lasting,” the only synonym with a similar meaning.
35. C. “Disparity” comes from the root word “par,” meaning “equal” (a par score in golf means “equal to the standard score”). The prefix “dis-” shows a negative, meaning “not.” So, “disparity” means “a state of being unequal.” Thus, “imbalance” is the only synonym with a similar meaning.
36. A. “Solace” comes from a root word that means “to console.” In fact, the same root is found in “console” and “consolation.” As a verb, “solace” generally means “to comfort.” So, “calm” is the only synonym with a similar meaning, and the other words are synonyms of each other (generally meaning to excite or antagonize, not to comfort).
37. A. “Protracted” comes from a root word meaning “to draw” and the prefix “pro-,” which means “forward.” So, the word means “drawn forward” or “prolonged.” “Indefinite” which generally means “without a defined end” (“in-” means “not,” and “definite” means “certain”) is the only synonym with a similar meaning.
38. D. “Lucid” comes from a root word meaning “light” or “to shine a light on.” If something is illuminated, it is easy to see, or clear. “After a prolonged illness, her speech was more lucid once she began to feel better.” In this case, “articulate,” or “expressing clearly,” is the only synonym with a similar meaning.
39. B. “Innocuous” comes from the same root word as “noxious,” which means “harmful,” but the prefix “in-” implies a negative, so “innocuous” means “not harmful,” or “harmless.”
40. D. “Dexterous” comes from the root word “dexter,” which means “on the right side.” Since most people are right-handed, the word came to mean “skillful.” Think of words like “ambidextrous,” which means skilled with both hands, or “dexterity,” which means “skillful with the hands.”
41. A. “Assiduous” comes from a root word meaning “to sit beside” and generally means “to show great care.” Thus, “careless” and “insidious” are antonyms, but “attentive” is the only synonym with a similar meaning.
42. D. “Portend” comes from root words meaning “to stretch forward” and means “to give an omen or anticipatory sign of.” “Economic indicators portend of a possible dip in the stock market.” Thus, “prophesize” is the only synonym with a similar meaning.
43. C. “Vitriolic” comes from a root word meaning “glassy.” In science, “vitriol” is another word for sulfuric acid. Vitriolic generally means “caustic” or “harsh.” “The Senate candidate’s criticism for the current administration was angry and vitriolic.” So, “spiteful” is only synonym with a similar meaning.
44. B. “Salubrious” comes from a root word meaning “healthy.” It generally means something that promotes health or well-being. It is related to the word “salutation” or “*salut*,” which is the Spanish

version of the word “cheers” or “to your health.” “Detrimental” is an antonym, but “healthy” is only synonym with a similar meaning. Something “sacred” is important, and someone “loquacious” speaks well.

45. D. “Complacent” comes from a root word meaning “to please” and is related to the word “placate.” The prefix “com-” means “with.” So, together, the words mean “to be pleased with everything.” The word usually means “self-satisfied”—to be pleased with oneself—or “unconcerned.” “After a long period of prosperity, the business owner had grown complacent.” “Satisfied” is the only synonym with a similar meaning.
46. B. “Demagogue” comes from root words meaning “to lead people.” It means “a leader who champions the cause of the common people,” although it often has a negative connotation, implying the leader uses subterfuge or manipulation to gain power. The prefix “dem-” means “related to people,” as it does in “democracy.” Similarly, a “populist” (think “population”) is a leader who claims to advocate for the common people.
47. D. One way to think of “saccharine” is in terms of the artificial sweetener saccharin. The word generally means “overly sweet.”
48. A. “Anachronistic” comes from the root word “chrono” meaning “time.” “Ana-” is a prefix meaning “against.” So, together, something “anachronistic” is “out of or against time.” The word refers to chronological errors. “My neighbors still have a rotary phone, which seems anachronistic.” Of the choices, “antiquated” is the closest synonym.
49. D. “Tenacious” comes from the root word “ten” meaning “to hold” (think of “detention,” where one is “held” against one’s will). “Tenacious” generally means “persistent,” or holding on for a long time. “The tenacious reporter kept asking the politician questions, even though he wasn’t giving straight answers.” “Resolute,” which means “steady” or “resolved,” is the only synonym with a similar meaning.
50. C. “Desultory” comes from a root word meaning “to leap” (in ancient times, it meant literally to leap from horse to horse). It generally means “disconnected” or “lacking a plan.” “Intermittent” means something that disappears and reappears continuously, like an intermittent rain storm. Both words imply a certain amount of randomness or lack of pattern, and so are the only synonyms with a similar meanings.
51. D. “Prodigal” comes from a root word similar to “prodigious” which means “a great amount.” “Prodigal” has come to mean lavish or reckless way of life, wastefully spending “a great amount.”
52. C. “Espouse” comes from a root word meaning “betroth,” the same root as “spouse.” While it literally means “to marry,” more generally it has come to mean “to become attached to or take up as a cause.” “After a long period of study, the chemist began to espouse new beliefs about the helpful qualities of the drug.” Thus, “contradict” and “disagree” are antonyms and “endorse” is the only synonym with a similar meaning.
53. D. “Parsimony” comes from a root word meaning “to be sparing,” or to use restraint. It means “the quality of being careful with money.” So, “charity” is an antonym. “The volunteers were shocked by the parsimony of some of the town’s wealthiest citizens, and failed to make their fundraising goal.” “Miserliness,” or extreme stinginess, is the only synonym with a similar meaning.
54. B. “Risible” comes from a root word meaning “to laugh” (think of the word “deride,” which means “to make fun of”). To be “risible” is to be likely to laugh or cause laughter. “The risible clown brought peals of laughter from the audience.” “Funny” is therefore the only synonym with a similar meaning.
55. D. “Aspersions” comes from a root word meaning “to scatter.” One could say that to cast aspersions is to scatter with criticism. Generally, an aspersion is a false or misleading claim meant to harm someone’s reputation. Thus, “praise” is an antonym and “slander,” a false charge or misrepresentation intended to cause harm, is the only synonym with a similar meaning.
56. C. “Jocular” comes from the same root word as “joke” and means “playful” or “joking.” Thus, “humorous” is the only synonym with a similar meaning (and “boring,” “dangerous,” and “serious” are all antonyms).

57. D. “Torpor” comes from a root meaning “numb” or “sluggish.” “Torpor” is generally a state of inactivity. The only synonym with a similar meaning, then, is “weariness.”
58. A. “Elucidate” comes from the same root word as “lucid” and means “to make clear.” This word is “luc,” which means “light.” When one learns, or when something is clear, it is like there is a light shining on it. “The teacher’s simple explanation elucidated the difficult concept for us.” “Clarify” is therefore the only synonym with a similar meaning.
59. C. “Acuity” comes from the root word “acer,” or sharp, the same root word in “acute.” Acuity is a perceptive sharpness. “The gifted student aced a number of tests, demonstrating a great mental acuity.” In this context, “intelligence” is the only synonym with a similar meaning.
60. D. “Dolorous” comes from a root word meaning “pain or grief.” “The songwriter composed a series of dolorous ballads after the death of her husband.” “Miserable” is the only synonym with a similar meaning.
61. B. “Implacable” comes from a root word meaning “to please” (the same root as “placate”). The prefix “im-” implies a negative, so someone who is implacable cannot be pleased or forced to change. “The girl was implacable, screaming at everyone who came to her party.” “Merciless,” or “showing no mercy” is the only synonym with a similar meaning in this context.

Vocabulary

1. D. “Spontaneity” is a noun that generally relates to a momentary impulse. “The kids showed great spontaneity by acting in unpredictable ways.” The closest synonym is “randomness,” which implies the same sort of lack of pattern or predictability.
2. C. “Compromise” is a verb, and is when two parties agree but both sides concede something. “The siblings couldn’t agree on a movie they wanted to see at the theater, so they compromised and watched a movie at home.” Thus, “concede” is the only synonym with a similar meaning.
3. D. “Deceptive” comes from the same root as “deception,” “deceit,” and “deceive.” Someone who is deceptive has the potential to deceive or trick. “Misleading,” or “giving the wrong impression,” is therefore only synonym with a similar meaning.
4. B. “Consume” is a verb that generally means “to eat.” “The hungry children consumed their dinner quickly.” In this context, “devour” is the best word to replace “consume” in the sentence, since it is generally used in the same context.
5. A. If something is “possible” it is within one’s ability to do it. “She didn’t have time to go to the gym, but a quick run was possible.” “Feasible” also means “capable of being done,” even though that thing may be complicated (intricate), difficult (strenuous), or unreasonable.
6. C. “Lethargic” is an adjective meaning “sluggish” or “lacking in energy.” “Energetic” and “vigorous” are antonyms. “Lazy” is the only synonym with a similar meaning and has a similar connotation.
7. D. “Passion” has a positive connotation and is a noun referring to intense emotions and often refers to support for a cause. “She had great passion for helping homeless kids.” “Ennui” has a negative connotation meaning “boredom.” “Fashion” and “style” are neutral. “Zeal” has a positive connotation and means “eagerness or strong support for something.”
8. C. “Blunder” is a verb meaning “to stumble or make a mistake.” “The teacher blundered by not properly explaining the concept before she gave a pop quiz.” “Mistake” is the only synonym meaning a mistake. To flounder is to struggle, but not necessarily make a mistake.
9. D. “Incompatible” comes from a root word meaning “sympathetic.” If two things are compatible, they fit together or get along together well. The prefix “in-” means “not,” so “incompatible” means “incapable of going together or getting along.” Two things that are incompatible would be dissimilar by definition, so of the choices, “opposite” is the only synonym that works.
10. A. “Greed” is a selfish desire for something. The answer choices are probably less familiar words. “Avarice” comes from a root word that means “crave” and is a strong desire for wealth. It is often used a synonym for “greed.” “Aversion” means something that something prefers to avoid, while “aviary” and “aviation” have to do with flying.
11. D. “Rampage” has a negative connotation and could be a verb or a noun. As a verb it means “to rush about,” often violently. As a noun, it means “a path of violence or destruction.” A “riot” is public

- violence or disorder, and as a verb similarly means “creating public violence or unrest.” So, “riot” is the only synonym for “rampage.” A “campaign” has a more neutral and less violent meaning.
12. B. “Divergent” is an adjective meaning “moving away from a common point” or “moving away from a standard.” “The scientific study’s results were divergent from what was expected.” In this context, “contradictory” is the best substitution for the sentence.
13. A. “Careless” is probably a familiar word; it literally means “without a care.” “Negligent” comes from the same root as “neglect,” and also means “careless.” “Precise,” “thorough,” and “thoughtful” are all antonyms.
14. B. A “convergence” is a place where several things come together or meet. “The runners took different paths but would all finish the race at the convergence of all the paths.” In this context, “intersection” has the closest similar meaning.
15. D. An “anecdote” is a brief story. “The teacher told an anecdote about a group of dogs to help explain the math concept.” So, “story” is the only synonym with a similar meaning.
16. A. “Holistic” is an adjective meaning “relating to the whole system.” Think of the similar sounding “whole.” “The doctor took a holistic approach to medicine, looking for ways to improve overall health instead of trying to treat particular symptoms.” Thus, “complete” is the only synonym with a similar meaning.
17. A. “Transient” comes from a root word meaning “to go” or “to pass by”; it is the same root word as “transit.” It is an adjective meaning “staying briefly.” “Lasting,” “permanent,” and “ineffaceable” (which means “unable to be destroyed”) are antonyms. “Fleeting,” which means “passing swiftly” is the only synonym with a similar meaning to “transient.”
18. B. “Diligent” comes from a root word meaning “love or affection” but means to do something with steady effort. One is more likely to put steady effort into something one loves. “The painter was diligent about her craft.” “Industrious” means “regularly or steadily occupied” and is the only synonym with a similar meaning for “diligent.”
19. C. “Deference” means “respect for a superior or elder.” It comes from the same root word as “defer,” which means “to submit to someone else’s wishes.” “The student showed great deference for the teacher.” “Obedience,” then, is the only synonym with a similar meaning.
20. B. “Digression” usually means a straying from the main topic of a conversation. Think of the phrase, “But I digress.” It can also mean straying from the main path. Therefore, “detour” is the only synonym with a similar meaning.
21. D. “Guile” is a noun meaning “duplicity or cunning.” “The snake showed great guile while luring a mouse into its lair.” In this context “trickery” has the closest meaning.
22. B. As an adjective, “cavalier” means “indifferent or disdainful toward important matters.” “The unrepentant defendant showed a cavalier attitude toward the law during his trial.” In this context, “nonchalant,” or “indifferent,” shares the most similar definition.
23. B. Someone who is “resolute” shows strong determination (think of a New Year’s “resolution” and the word “resolved.” “Tenacious” means “persistent.”
24. C. “Peripheral” comes from the same root as “periphery,” which is the perimeter around something. So, “peripheral” relates to the perimeter, or edges. It often refers to “peripheral vision,” or seeing something out of the corner of your eye, or “being on the periphery,” meaning something that is right on the edge. Therefore, “fringe” has the closest meaning; in this context, being “on the fringe” is being on the edge of something.
25. A. An “epithet” is a characterizing word or phrase. Although it can have a negative connotation (like “slur”), it usually refers to words used to describe something succinctly (“The man” or “the gray cat.”), or a “description.”
26. D. “Absolution” comes from the same root word as “absolve,” which means “to forgive.” Thus, “absolution” is forgiveness, usually for having done something sinful or wrong. “Pardon,” which often means forgiving someone for a crime (and can be a noun or a verb), has the closest meaning.

27. D. "Dissipate" means "to break up." "When the police arrived, the crowd dissipated and everyone went home." In this context, "vanish" has the closest similar meaning. "Condense" means to concentrate, or grow thicker, while "reappear" has the opposite meaning of "vanish."
28. B. "Raucous" generally means "loud and/or disorderly" or "noisy." "It was impossible to hear over the raucous crowd at the game."
29. D. "Incorrigible" means "unmanageable" or "unable to change or be corrected." "The incorrigible little girl refused cover her mouth when she coughed." If something can't be changed or fixed, it might have a "hopeless" quality; the other choices do not have a similar meaning.
30. C. "Pristine" means "unspoiled" or "in original condition." "The vintage action figure was still in its box, in pristine condition." Of the choices, "pure" has the closest meaning to "unspoiled."
31. C. "Camaraderie" is a "spirit of friendship or goodwill." It comes from the same root word as "comrade." "Trust" is the only one of the choices with a related meaning.
32. B. "Potable" means "drinkable." It comes from a root word meaning "to drink." "When elevated lead levels were found in the water, people began to wonder if it was potable." In this context, "clean" is the only word that makes sense in the sentence.
33. C. "Ascetic" usually relates to self-denial or living very simply. "The ascetic monk lived in a barren room without luxury." The monk, then, might also be considered "humble."
34. D. A "bastion" is a fortified position, either literal (such as an army fort) or figurative (such as a group or place where strong standards or opinions are held firm). "Stronghold" works as a synonym in both contexts.
35. D. "Expedient" means "useful for achieving a goal." "Executives held a meeting to determine which methods would be most expedient for increasing profits." Something "practical" is also useful.
36. A. "Shirk" means "to avoid or evade." "He tried to shirk his chores by going to bed before the dishes were done." So, "avoid" is the only choice that makes sense.
37. B. "Grapple" means "struggle." It could mean to literally grapple, or wrestle, or to figuratively grapple, such as with a difficult concept. "He grappled with the math problem until he figured it out." To understand something is not necessarily to struggle with it; one could understand something very quickly and easily, for example.
38. A. "Rapacious" means "covetous" or "ravenous." It's usually used in the context of always wanting more: more money, more food. "The rapacious billionaire supported laws that would lower his taxes."
39. C. "Prosaic" generally means "everyday, ordinary." It comes from the same root word as "prose," which can be a contrast to "poetry," which some consider to be more beautiful or artistic. So something "prosaic" is plain, or "mundane."
40. D. To "extricate" something is to remove it. "In the game Jenga, the trick is to extricate one wooden piece without making the whole tower topple over." The prefix "ex-" means "out from/of," which is must like the action of removing something.
41. C. "Tremulous" comes from the same root word as "tremble," so we can think of something tremulous as trembling. Being "nervous" could cause trembling, or for one to feel tremulous.
42. D. When used as a verb, "pique" means "to cause curiosity or interest." "The trailer piqued my interest and made me want to see the movie." "Intrigue" is a good substitute in the sentence.
43. C. "Redolent" typically means "suggestive." "The store burned scented candles to make it redolent of evergreen trees around the holidays." Thus, "evocative" comes from the same root word as "evoke," which means "bringing thoughts or feelings to mind," which has a similar meaning to "suggestive."
44. B. "Indolence" means "laziness." "Cats may show great indolence by sleeping away the daylight hours, but they can be more lively at night." "Idleness" means "inactivity," which has a similar meaning to "laziness."
45. B. "Dissonance" means "disagreement," although not always a literal disagreement. "The weather report included rain, but there were no clouds in the sky, creating dissonance between what we heard on TV and what we could see with our own eyes." "Conflict" makes the most sense in this context.
46. D. "Euphonious" means "pleasing to the ear." It comes from the same root word as "phone," which means "sound." It also includes the prefix "euph," which is related to "euphoria," a strongly positive

word meaning “intense happiness or excitement.” “Melodious” literally means “producing a melody,” but often also means “having a pleasing melody.”

47. A. “Hackneyed” means “lacking originality.” One might call an artist without much talent a “hack.” “Creative,” “imagined,” and “unique” are all antonyms; “common” is the only synonym.
48. D. “Rancorous” has a negative connotation and refers to something malevolent or characterized by ill-will. It comes from the same root word as “rancid” and “rancor.” “Content” and “grateful” can be ruled out because they have positive connotations. “Resentful,” or “harboring ill will” is the only choice that makes sense.
49. B. “Censure” is usually a negative judgment. “The senator was censured for misconduct.” In this context, “criticism” is the only choice with a similar meaning.
50. B. “Prescience” is anticipation or intuition about a pending event. It comes from the same root word as “science,” which means knowledge. The prefix “pre-” means before, so we can think of “prescience” as knowledge before something happens. “The economist showed prescience when she predicted a fall in the stock market, which occurred three days later.” “Intuition” means “insight” or “the ability to know something without rational thought.”
51. D. “Sagacity” is “wisdom.” “The man had gained patience and sagacity by the time he reached old age.” The word is related to “sage,” which refers to a person with great wisdom.
52. C. “Evince” mean “to show evidence of” (and sounds like “evidence”). “The teenager evinced interest in history when she checked a number of historical biographies out of the library.” “Express” means “to say” and, of the choices, is the only word with a similar meaning.
53. C. “Umbrage” is a feeling of resentment, usually at an imagined slight. It comes from a root word meaning “shade” (and is the same root as “umbrella”), meaning someone who takes umbrage at something might imagine they’ve been insulted or cast in shadows.
54. B. “Caprice” is a sudden change. “His caprice made group projects difficult, since he would often change his mind about every decision.” In this context, “instability” is the only choice with a similar meaning as “often change his mind.” The word is related to “capricious,” which is used to describe a person whose mind changes constantly.
55. C. “Assuage” is a verb meaning “to lessen the intensity.” “Being forgiven by her father did little to assuage her guilt.” “Relieve” also means “to lessen the intensity.”
56. A. “Intercede” means “to intervene.” It comes from a root word meaning “go”; the prefix “inter-” means between. So, literally, to intercede is to go between. “The plaintiff and defendant might have fought in open court had the judge not interceded.”
57. C. “Obsolescent” means becoming obsolete, or no longer useful. “Grandma’s house was full of obsolescent items like typewriter parts and VHS tapes.”
58. A. “Capacious” comes from the same root word as “capable” (or “capacity”) and means “able to hold a great amount.” “The capacious tote bag could hold all my groceries.” In this context, “ample,” which means “generous in size,” is a good substitute in the sentence.
59. B. “Garrulous” means “talkative.” “The garrulous boy would not stop chattering.”
60. A. “Fastidious” means “difficult to please.” Someone who is fastidious might have high standards for how something should look or be done. “The fastidious man was constantly cleaning his office, frustrated that it never lived up to his standards for cleanliness.” In this case, “exacting” is the only choice with a similar meaning.
61. D. “Obstreperous” is an adjective meaning “unruly and noisy.” “Our obstreperous neighbors threw a party that kept everyone awake all night.” Thus, “rowdy” is a good synonym.

Association

1. D. The word “novel” can mean a book of fiction, but you can also think of the word “novelty,” which is usually something new and interesting. Both come from a root word “nov-” meaning “new.” Something new is also “unfamiliar.” Other words include “nova,” “novation,” and “innovate/innovation.”

2. B. When you see the word “liberate,” you might also think of words like “liberty.” To “liberate” is literally “to make free” and often refers to someone being freed from something. Thus, “release” has a similar meaning.
3. D. “Fluent” sounds like the word “fluid” and implies a certain ease or smoothness. People often talk of being fluent in a language. This means both that they can speak the language well, and also that they are well-versed in it.
4. D. We often call people who seem stuck up or overly confident “arrogant.” “The arrogant quarterback, who had all but guaranteed that his team would win, was humbled by the loss.” “Proud” has a similar meaning.
5. B. “Corruption” usually refers illegal activity or of something being compromised. “After the police officer bought a shiny new car that he never should have been able to afford, his colleagues accused him of corruption.” Corruption implies doing something deceitful, so “dishonesty” is a synonym.
6. B. “Intrepid” is a positive adjective often used to describe people. “The intrepid reporter traveled to the dangerous city to finish her report.” It generally means “fearless.”
7. B. “Inconsequential” contains “consequence.” Since the prefix “in-” means “not,” we can conclude that “inconsequential” means “without consequence.” Usually, it means something that is insignificant. “Paltry” means “inferior” or “trivial,” which are synonyms of “insignificant.”
8. B. “Longevity” contains the word “long,” so we can infer that the word relates to length in some way. Generally, it refers to something lasting a long time. “I was impressed by the longevity of the cyclists who raced for miles.” Thus, “endurance,” or the ability to last a long time, is a synonym.
9. A. “Conditional” means something has conditions, or depends on something else. “His scholarship was conditional on his keeping a high GPA.” In this context, “dependent” can replace “conditional” in the sentence.
10. B. If something is “atrocious,” it is terrible. “The children’s behavior was so atrocious, the teacher threatened to cancel the field trip.” “Horrible” is a synonym.
11. A. When we hear the word “inevitable,” we generally think of something that will happen no matter what, or that is unavoidable. The sun rising and setting each day is inevitable. We can also say that something “destined” to happen is “inevitable.”
12. B. “Esteem” may make you think of “self-esteem” or positive regard for oneself. “Esteem” is a positive thought for someone else. Think of “my esteemed colleague.” So, “reverence” is a synonym.
13. B. An “enigma” is a mystery. We might think of the Enigma machine, which created coded messages during World War II (featured in the movie *The Imitation Game*). So, “riddle” is a synonym.
14. D. “Tentative” sounds a little like “hesitant.” Something tentative is not permanent. “We had tentative plans to have dinner on Friday, unless Dad has to go out of town.” “Temporary” is a good synonym.
15. B. “Denounce” comes from the same root as “announce,” but has a negative connotation, which we can infer from the prefix “de-”. To “denounce” is announce negatively or to criticize. Thus, “condemn,” which means “to declare wrong or evil,” is a synonym.
16. B. A “strut” is a way of walking, usually with confidence. We can think of phrase like “strut your stuff.” Thus, “parade” is a good synonym.
17. C. To “scrutinize” something is to look at it with scrutiny, or close attention. Thus, “inspect” is a good synonym.
18. B. “Reclusive” shares a root word with “recluse,” or someone who keeps to him or herself. “My reclusive neighbor rarely leaves the house, not even to get his newspaper.” Someone who is reclusive hides himself, so “hidden” is a good synonym.
19. C. In religious tradition, an “epiphany” is the appearance of a divine being. In modern English, it often means an idea that appears as if out of nowhere. “After days of mulling over the problem, the scientist had an epiphany and knew just how to solve it.” So, “idea” is a synonym.
20. B. “Emulate” means “imitate.” You might have heard of “emulators” or computer programs that mimic another system, such as a program that allows you to run old console games on a PC. As a verb, “mirror” means to copy or imitate (as if looking in a mirror).

21. D. “Cadence” frequently refers to music or speech; it is a rhythmic sequence of sounds. “The conductor clapped his hands on each beat, helping the orchestra establish the cadence.” “Rhythm” is a good substitute in the sentence.
22. A. “Exasperation” comes from the root word “asper” meaning “rough.” The prefix “ex-” can mean “outside of” (think “extra” or “extreme”). A feeling of exasperation is something extremely irritating, or not smooth. So, “annoyance” is a good synonym.
23. B. When you read “sedate,” you might think of “sedation,” or putting someone to sleep. Similarly, something “sedate” is quiet and calm. “Composed” can also mean calm.
24. D. “Endear” contains the word “dear,” which you can think of when remembering the definition. To endear is to become admired or beloved (or dear). “His skill endeared him to his bosses.” In this context, “recommend” works as a synonym, because if something is endeared to something else, it is recommended positively.
25. A. When you see “discord,” you see the prefix “dis-” and the word “cord.” A cord is something that hold things together, and the prefix “dis-” implies doing the opposite. So, one way to think of discord, which means “quarrel or disagreement,” is to break the cord keeping two people in agreement. Of the choices, “tranquility” and “uniformity” are antonyms, and “record” has an entirely different meaning, but “pandemonium,” or “chaos,” has a similar meaning.
26. B. “Motley” may make you think of the phrase “motley crew,” which usually refers to a group of people who are different from each other. “Motley” simply means “made up of different people or things.” So “diverse” is a synonym. “Motley” is similar to “mottled,” which describes spots or patterns of color.
27. D. “Erratic” comes from a similar root word as “error.” Something “erratic” has no fixed path, or might contain errors or deviations from the norm. “The results of the experiment were erratic, showing no consistency.” So, “turbulent” is a good synonym.
28. B. “Ecstasy” comes from the same root as “ecstatic”; it means “extreme happiness.” “After trying the expertly prepared dish, the diner was overcome with ecstasy.” “Happiness” is a good substitute in the sentence.
29. B. If something is “material,” it is important, like material evidence in a police investigation. The prefix “im-” means “not,” so “immaterial” means “not important.” Thus, “irrelevant,” which means “unimportant or unrelated,” is a good synonym.
30. D. If you bump into someone, you may “jostle,” or gently shake, them. So, “push” is a good synonym.
31. B. “Dwindle” contains the word “wind,” so you can think of “dwindling” as “winding down.” “As the party wound down, the amount of food left dwindled to nothing.” So, “disappear” is a good synonym in this context.
32. A. A “prolific” writer is one who writes many books. It comes from the same root as “proliferate” or “proliferation,” which refers to increasing or accumulating. So, “abundant,” or occurring in large amounts, is a good synonym.
33. C. You can break “forthright” into its two words: “forth” and “right.” It can mean “directly forward” or “without hesitation.” Someone who is forthright is “honest” and “straightforward.”
34. B. “Substantiate” comes from the same root word as “substance.” To substantiate is to support with evidence or substance. In other words, “substantiate” means “prove.”
35. A. “Revere” comes from the same root word as “reverence.” “The guitarist was excited to meet several famous musicians he revered.” To “revere” is to “honor.”
36. B. If you think of the movie studio when you see “paramount,” you can consider the studio’s logo, which includes a picture of a mountain. Something “paramount” is “superior” or “on top.” In other words, it is “important.”
37. C. “Coddle” sounds a little like “cuddle,” which can help us remember the definition. To “coddle” is to treat with care and kindness. So, “indulge,” which can mean “to treat with care and generosity” in some contexts, is a good synonym.

38. D. You may have heard “immaculate” in religious contexts, where it means untainted or pure. We generally mean it to mean “unblemished” or “extremely clean.” “The vinyl record had been saved in its original packaging; although it was fifty years old, it was still in immaculate condition.” In this context, “perfect” is a good synonym.
39. D. “Asylum” can refer to a mental hospital, but it more broadly refers to a place of protection. Refugees fleeing their home countries can apply to the United States for asylum, or a safe place to stay. If you think of refugees applying for asylum, you can see that the word “asylum” means “refuge.”
40. B. “Seethe” can both mean to boil or burn, as with a negative emotion; one might seethe with anger or jealousy. “Smolder” has a similar meaning; it can mean the last dregs of a fire burning and smoking or to smolder with an emotion like anger or jealousy.
41. B. “Flippant” is similar to the word “flip,” which can mean “rude.” Of the choices, “polite” and “deferential” are antonyms and “elusive” has an entirely different meaning, but “disrespectful” is closest to “rude”.
42. A. The word “lobbyist” contains the word “lobby.” Think of someone waiting in the lobby of a capitol building to talk to a senator. A lobbyist is someone who advocates for a certain cause, or an “advocate.”
43. C. The prefix “dis-” in “dissent” indicates the word will have a negative connotation. As a verb, “dissent” means “to disagree.” So, “oppose” is the correct answer. Just because someone opposes something does not mean he or she insults it.
44. A. “Provocative” comes from the same root as “provoke,” which means “to irritate.” Provocative has a few different meanings, but one of them is “irritating.”
45. D. To “coerce” is to persuade or pressure someone to do something. “The burglar held up the bank teller at gunpoint and coerced her to hand over a lot of money.” So, “force” is the correct answer.
46. C. To be “contrite” is to be apologetic or remorseful. “The children, who apologized repeatedly, seemed to be genuinely contrite.” So, “repentant,” which means “expressing regret,” is a good synonym.
47. B. You might think of a “drone” as a flying machine, but it can also refer to the sound that machine makes. “The drone of the refrigerator was the only sound in the otherwise silent kitchen.” So, “drone” can mean “buzz.” It just so happens that many drones (the flying machines) make a buzz, or a droning noise.
48. C. “Dogmatic” comes from the same root as “dogma,” which is a strongly held belief. Someone who is dogmatic has very strong opinions. They might be considered “rigid,” or unyielding in their beliefs.
49. C. We often think of “extenuating circumstances,” or something that excuses or makes light of a situation. “He had an essay due, but told his teacher there were extenuating circumstances.” “Mitigating” also means “excusing.”
50. B. “Discern” means “to tell the difference between.” Someone with “discerning” tastes might be able to tell the difference between good music and terrible music. So, “detect” works as a synonym.
51. C. Often used in a religious context, a “heretic” is someone who disagrees with established belief. The word can be more generally someone who disagrees or dissents. So, “dissenter” is the correct answer.
52. B. Sometimes when someone breaks a bone and has a cast put on an arm or leg, when the cast is removed, the arm or leg looks smaller than it did. This is because the muscles “atrophy,” or shrink, from not being used. So, that is one way to think of the definition of the word. “Decay” is correct because it also means to decrease in size or waste away.
53. B. “Tangent” has a mathematical meaning, but it also refers to a divergence from the main path, both figuratively and literally. “The professor went off on a tangent about twentieth century playwrights in the middle of his lecture on Shakespeare.” Thus, “departure” is the correct answer.
54. A. To be “magnanimous” is to be “generous.” “We were impressed by buy the businessman’s magnanimous offer to help us build a new playground.”

55. A. “Florid” comes from the same root word as “floral,” and while it can mean “flowery” (flowery language, for example, or decorated with flowers), it can also mean “tinged with red,” like a rose. So “flushed” is the correct answer.
56. A. To “abscond” with something is to steal it. The root word “ab-” means “away from.” So, to steal something would be to take it “away from” where it is supposed to be. “The dog absconded with a piece of chicken while no one was looking.” “Pilfer” also means “to steal.”
57. C. “Fervent” comes from the same root word as “fever,” which implies something hot or passionate. “Fervent” generally means “passionate.” “It was my fervent desire to work toward ending hunger in our community.”
58. B. A “mendacity” is a lie. “The writer was accused of fabricating his memoirs because a number of critics found mendacities and inconsistencies in the book.”
59. A. “Constituent” comes from the same root word as “constitute,” which means “to be a part of.” People who live in a particular electoral district are often referred to as an elected representative’s constituents. So, “component” is a good synonym, because a component makes up part of a larger whole the same way a constituent does.
60. B. Something “ostentatious” is over the top, usually in a showy, tacky way. “After winning the lottery, the couple decorated their house in an ugly, ostentatious way.” So “gaudy” is a good synonym.

Sentence Completion

Single Blank

1. C. Context clues indicate the blank is a synonym for “bright” or “reflective,” since Tabetha’s ring nearly blinds the crowd. Thus, “dull” and “unassuming” can be ruled out. “Permanent” refers to something’s ability to last – not to how bright something is. “Scintillating” can mean “brilliant” and is the best fit for the blank.
2. A. If the executive cared only for money, he’s likely greedy. Therefore “avarice,” or greed, is the best fit for the blank. If he was generous, he would have likely cared more about his employees and less about money.
3. A. The word “although” indicates the word in the blank will likely be an antonym for “even-tempered” and “slow to anger.” “Bellicose” means “quick to start fights,” and is a good fit for the blank. The other words are all positive, like “even-tempered” and “slow to anger.”
4. D. If the dog will not abide by commands and cannot be trained, the dog is the opposite of “obedient” or “loyal.” “Guilty” isn’t a good fit either, because the dog is acting disobedient, not like he did something wrong. “Recalcitrant” means “defiant of authority,” or disobedient.
5. A. According to the sentence, the student’s paper was riddled with error, so the arguments were not “judicious,” “reasonable,” or “valid,” but rather “fallacious,” which means “false” or “misleading.” We can see that “fallacious” is the only word with a negative connotation that matches “riddled with errors.”
6. C. The sentence presents evidence supporting a claim before making that claim. In this case, the scientist is describing global warming, which is reasonable, or “plausible” given the evidence.
7. B. The younger brother dresses like his older brother, or copies him. If this pleases their parents, they think the older brother is worth copying. The word with the closest meaning to copying is “emulating” which means “to imitate or strive to be like.”
8. C. In the context of the sentence, the blank must be a word that is a synonym of “hurtful” and “slandorous.” The only similar word among the other choices is “malicious,” which means “cruel.” The other words are all positive, and not in keeping with the negative description of the playwright’s jokes.
9. A. The poker player betrays nothing about his hand by his facial expressions, so his face must be difficult to interpret or understand. “Inscrutable” means “unable to be interpreted.”
10. A. If the movie theater is usually open past midnight, then it would be abnormal or unusual for it to be closed early, and would also be a surprise to the group of friends. So, “normal,” “uninteresting,” and “unsurprising” can be ruled out as answers. “Aberrant” means “unusual or abnormal.”

11. C. In the context of the sentence, the blank must be a synonym for “realistically” and “logically.” “Pragmatic,” which means “practical,” is the best word for the blank.
12. C. The artist dressed in a way that clashes and that other artists avoid, so her fashion sense is not “conservative” or “conventional.” “Eccentric” means “deviating from the usual pattern or style.”
13. A. If the question had two possible answers, it was likely poorly worded, and thus the opposite of “clear” or “apparent.” “Lengthy” wouldn’t necessarily mean confusing. “Ambiguous” means unclear and is the best fit for the blank.
14. C. The criminal employed deceit, according to the sentence. “Obfuscate” means “to hide or confuse” and so best shows what the criminal was attempting to do with the evidence.
15. B. The mother feels her son “led a life of leisure,” implying that she thinks he’s lazy. “Indolent” means “habitually lazy,” and is the best fit for the blank. If we don’t know this word, we can rule at least rule out the others based on their meanings.
16. C. If the customers find that the salesman “did not always full disclose shortcomings of his products,” then they do not think he is “genuine” or “honest.” “Perfidious” means “deceitful” or “disloyal,” and is the best match for the blank.
17. A. The sentence explains that the baby girl doesn’t like to be without her blanket. “Exclusion” and “repulsion” are antonyms for this sentiment. “Affinity” is “a liking for something,” and is the best fit for the blank.
18. D. In the context of the sentence, the word in the blank must be a synonym for “baseless” and “unsubstantiated,” or is a word with a negative connotation. “Vacuous” means “lacking in content” and is the only choice with a negative connotation.
19. D. The blank must be a word that makes sense with “between.” “Vacillate” means to move back and forth, and of the choices, makes the most sense in the context of the sentence.
20. A. The word in the blank must be a verb meaning “to say something negative about,” since the prosecution wants to make the defendant seem “untrustworthy” and “unreliable.” “Impugn” means “to attack with words,” and is the best fit for the sentence. One’s character is not “nullified” or “validated, but “lauded” or “impugned.” However, “laud” (think “applaud”) has a positive connotation.
21. B. “Theoretical” is the best clue in this sentence; the theoretical concepts were difficult to understand, meaning they were not “simple” or “easy.” One meaning of “abstract” is “difficult to understand.”
22. A. “Gradual” and “irrevocable” provide clues in this sentence. In the context of disease, “insidious” means gradual. “Virulent” is a possible answer, but usually means “to spread quickly,” and so is an antonym for the answer. Something “looming” hasn’t yet happened.” Something “obvious” may not necessarily be bad.
23. D. If Stephen’s mother always knows where is he, what he’s doing, and what he’s thinking, it’s implied she knows everything. The only word that works in context is “omniscience,” which means “knowing everything.”
24. A. If the party is continuing, the word in the blank should indicate that the party shows no signs of ending. Thus, “continuing” can be ruled out as an answer. “Abating” means “decreasing or ending.”
25. B. The sentence implies that John is obsessive about keeping his hands clean, since he washes them so frequently with good soap. Therefore, his behavior could be described as “fanatical,” which means “with excessive or extreme devotion.”
26. D. If the townsfolk were offended, then we know that the townsfolk did not like the traveler’s mannerisms, and would think his behavior to be negative. This helps us rule out “adherent” and “harmless.” The traveler’s behavior could be described as “uncouth,” which means “lacking in polish.” “Rash” means “impulsive” or “foolish,” which we can’t tell from the sentence alone.
27. B. If the dinner conversation became “heated,” there was likely shouting or an argument, which would be the opposite of “boring” or “vapid.” “Contentious” is the only synonym that makes logical sense., because it means “likely to cause arguments.”

28. C. If the guests felt the concierge cared, the description of his smile would have a positive connotation, which rules out “apathetic,” “dour,” and “forced.” “Effusive” means “with great enthusiasm.”
29. B. The young politician’s colleagues laugh at his optimism. “Young” and “newly elected” are clues that highlight the politician’s inexperience. “Naïve” means “inexperienced or innocent,” and is the best fit for the sentence. “Seasoned” means “experienced.”
30. A. Linda’s interest is in something few other people care about, so it would not be “widespread” or “ordinary.” If something is “arcane,” it is only known about by a few people.
31. C. If George is difficult to travel with, he is not “appealing” or “harmonious,” because we’re looking for a word with a negative connotation. The clue is “continually changing his mind,” the best match for which is “mercurial,” which means “frequently changing.” If we don’t know this, we can also rule out “unswerving,” which means “steadfast” or “unchanging.”
32. C. “Short-term” is the best clue in the sentence. “Myopic,” or “near-sighted” is the best match for the blank, as “accepting,” “enthusiastic,” and “nonchalant” don’t carry a meaning having to do with a “short-term” view.
33. B. The boulders “seem as if they could topple over,” so the blank is likely a synonym for “unstable.” The boulders sit “precariously,” or “lacking stability.” While the boulders might be irregular, this doesn’t have a parallel meaning as easy-to-topple.
34. C. “Few” and “scattered” are context clues to indicate that the landmasses are few and far between. “Sporadic,” or “occurring randomly,” is the only synonym that makes logical sense.
35. B. “Though” is a clue to indicate that the answer will be an antonym for “wild at heart” and “dangerous if provoked.” The only synonym that makes logical sense, therefore, is “docile,” which means quiet or tame. “Tempestuous” carries a meaning similar to “wild at heart,” and something tame or docile may still not be “permissive.”
36. B. If the boy accepted that he had done something wrong, he also accepted his punishment. Thus, “avoid” and “resist” can be ruled out as answers. “Acquiesce” also means “to accept.”
37. D. “Despite” is the key clue in this sentence. The harsh sentence therefore did not have an effect on the criminal, so he did not display even a small amount of remorse. The best synonym for “a small amount” is “vestige,” or “a tiny or trace amount.”
38. C. In the context of the sentence, the word in the blank is a synonym for “uninteresting” and “lackluster.” “Insidious” can mean tasteless, which would be a reason for the critic to write a scathing (or negative) review.
39. A. The word in the blank is likely a synonym of “logical” or else an adjective that would make the argument seem compelling, so “cogent,” which means “convincing,” is the best word for the blank. We can rule out “malevolent” immediately, since it is highly negative and the sentence describes the argument positively, or at least, effectively.
40. C. In the context of the sentence, the word in the blank is an antonym of “classic and timeless.” “Ephemeral” means “lasting a short time,” and so is the best word for the blank. “Customary” carries a meaning similar to “classic and timeless,” and logically contradicts the sentence given the word “whereas.”
41. A. In the context of the sentence, the word in the blank must be a synonym for “insulting” and “disrespectful.” “Derogatory” means “expressing a low opinion.” “Honest” and “traditional” carry a positive connotation, and can be ruled out immediately.
42. A. The general vows to fight despite overwhelming odds, so we can infer he will not surrender. The word in the blank is a synonym of “surrender.” “Capitulate” means “to stop or surrender.”
43. A. In the context of the sentence, Joyce stared “with disbelief and shock.” The root word of “incredulously” means believable, so the prefix *in-* indicates the word means the opposite of “believable.” Thus, Joyce stared “incredulously.”
44. A. The sentence implies that everyone has strong feelings about cilantro, so the word in the blank means the opposite of that. “Ambivalent” means “noncommittal” or “having contradictory opinions”

- about something. This fits logically into the sentence, since “few people are noncommittal about cilantro” is logically related to the fact that people either love or hate it.
45. B. A trip that is “fraught with hardships” is difficult, so the word in the blank would be a synonym of “difficult.” The best match is “arduous,” or “marked by great labor or effort.”
46. A. In the context of the sentence, the word in the blank is a synonym for “bold and risky.” The best match is “audacious,” which means “daring or bold.”
47. C. If the fumes prevented life from thriving, they were likely toxic, and the word in the blank would have a negative connotation. “Noxious” means “harmful or destructive to life.” The fumes may also be very strong smelling, but don’t let that lead you to “aromatic” or “pungent.” There are pleasant strong smells which are not “harmful or destructive to life.”
48. A. If the doctor’s smile put Rachel at ease, the word in the blank has a positive connotation. The only word with a positive connotation of the choices is “benevolent,” which means “marked by doing good or having goodwill.”
49. B. “Old” is the best clue in this sentence. The word in the blank refers to the old, difficult to understand words. “Esoteric” means “understood by a few.”
50. A. Harry’s dreams are difficult to remember – the details “slip away.” Therefore, they are “evanescent” or “tending to vanish.” The dreams may have been “pleasant” (or not) and may or may not have been important. What we know is that she can’t remember them because they are fleeting.
51. A. A vegetarian’s attitude toward meat is summarized by the word “repulsed.” The vegetarian is likely disgusted by meat, which is a synonym of “repulsed.”
52. D. The sentence implies that the internet has replaced print media, from which we can infer that print media is useless or old-fashioned. “Obsolete” means “no longer useful,” which is the best fit for the blank. “Indispensable” has the opposite meaning of “obsolete.”
53. C. Since there is “no tolerance for cheating,” we can infer that the teacher speaks forcefully, with no room for argument. “Equivocation” means “avoiding commitment”; the way the teacher speaks does not allow for arguing or disagreement.
54. A. We can infer from the sentence that the principal gives the student a detention. Since the principal does not excuse the student, the word in the blank must be a synonym for excuse. “Exemption” means “the act of excusing,” so it is the only synonym with a similar meaning.
55. B. The steam is “like ghosts,” meaning it fades and disappears. “Ethereal,” which means “lacking material substance.”
56. B. The blank must be a synonym for “coerced,” since the rest of the sentence describes how the plaintiff “was coerced” (forced) into signing it. “Duress” means “force or compulsion,” which is the only synonym with a similar meaning for “coerced.” The contract itself is under contention, but this doesn’t tell us under what circumstances it was signed.
57. A. In order to say more papers, the reporters must have been saying things that would attract attention, so “irrelevant” and “logical” can be ruled out. Incendiary can relate to starting fires, but incendiary language is meant to incite arguments, which would naturally attract the attention of newspaper readers.
58. D. “Without a care in the world” is the clue. Therefore, the experience must have been pleasant. The only synonym with a similar meaning for pleasant is “idyllic,” which means “pleasing or picturesque.” “Arboreal” describes things having to do with trees.
59. A. “Indemnify” is a term meaning “to secure against loss or damage.” This is the best fit for the sentence and explains that the owners wanted protection against liability (responsibility).
60. C. The horse was very fast (since it “overcame its competitors” and was “incredible”). Therefore, the only synonym that makes logical sense will be one having to do with speed. The only choice that fits is “alacrity,” which means “promptness or readiness.”
61. C. “Concessions” is the clue word in this sentence, which means “compromises.” The government wanted the protestors to calm down by making compromises. “Inflame,” “interrogate,” and “rouse” are all antonyms for a description of the action the government would take – these would all incite the

- protestors, not calm them—and can be ruled out. “Mollify” means “to soothe,” which is what happens when one tries to make compromises, or makes concessions.
62. D. In this case, the protestor refused to back down, so he must be very passionate, even to the point of not caring about going to jail. The only synonym with a similar meaning for passionate is “zealous,” which means “passion for a cause.” “Demure” has the opposite meaning of this. We aren’t told whether or not the protestor is happy (jovial) or evil (dastardly).
63. A. “Great fortune” is the best clue in this sentence, telling us that the alignment of the planets must be a positive event. The only word with positive connotation is “auspicious” or “showing future success is likely.” The other words can be ruled out since they have a negative connotation.
64. A. If nothing can sway the activist, she must be secure in her beliefs. The only synonym with a similar meaning for “secure” is “adamant,” which means “unshakable or insistence.” The sentence doesn’t describe whether or not she is condescending (talks down to others). “Malleable” and “yielding” have the opposite meaning of “secure” and “steadfast.”
65. C. In the context of the sentence, the word in the blank must be a synonym for “balance.” “Equilibrium” also means balance (think “equal”).
66. A. “Meeting” is a clue in this sentence; the word in the blank describes the spot where the two rivers come together. “Divergence” is a separation and can be ruled out as a choice. “Conspiracy” (secret plan) is not relevant to the sentence. A “delta” is the place where the river forms a triangle that empties into a larger body of water and does not describe where the rivers would meet. The “confluence” (the word “con-” means “with”) of two rivers is the meeting point.
67. C. The word that fits in the blank describes the kind of fear the students feel as they approach the principal’s office. The next part of the sentence says the fear grew and grew, so the word in the blank must be a synonym for growing. “Incipient” means “becoming more apparent,” and is the best fit for the sentence.
68. B. “Persevered” is the clue in the sentence; the spirit the Olympian demonstrates should relate to perseverance. In addition, we’re told that “despite” setbacks, she “persevered.” “Indomitable” means “incapable of being subdued or conquered,” and is the best choice for the blank.
69. D. If the activist shared his opinion with anyone who would listen, he must have voiced that opinion often. “Vociferous” has the same root word as “voice” and “vocal” and means “to speak out strongly.” “Capricious” has the opposite meaning of “staunch.”
70. A. The lawsuit destroyed the partners’ relationship, so the word in the blank is the opposite of “cooperative prosperity.” The word in the blank must have a negative connotation; the only choice for which that is true is “acrimony,” which means “anger and bitterness.”
71. D. The word in the blank should describe the behavior in the sentence: shifting uncomfortably and refusing the answer questions. In other words, the defendant “prevaricated,” which generally means “deviated from the truth by refusing to answer questions.”
72. C. We can infer from the sentence that Phillip is waiting for payments to pay for education, so he is waiting to receive those payments. “Disburse” means “to pay out,” and is the best match for the blank. Do not be confused by “dispersed,” which means “to scatter.”
73. A. “Unbearable noise” and “earplugs” are clues that the word in the blank should describe the noise created by the insects. “Cloud” and “swarm” describe what the insects might look like, not the sound. “Symphony” is a possible answer, but in the context of the sentence, the noise is unpleasant. “Cacophony” is a better answer, because it describes a harsh noise.
74. B. If the hillside is otherwise uniform, the word that best fits in the blank will describe the way the flowers break up the sameness. “Desultory” means “lacking a plan,” which would mean that the flowers are placed randomly on the hillside.
75. D. If lobbyists keep industries in the status quo, or the current situation, then they prevent creativity, innovation, and invention. “Cultivate,” “foster,” and “nurture” are all antonyms for prevent. The best choice is “stymie,” which means “to stand in the way of.”
76. B. If something is put on for appearances, it’s like an act, or a play. The closest synonym for “act” is “farce,” which is another word for a humorous play and can also mean “a ridiculous act.”

77. A. The “but” indicates that the graduate student’s experiences were contrary to her expectations. She thought the classes were easy, but instead they were “abstruse,” or “difficult to understand.”

Double Blank

1. A. The word “backfired” is the clue; she tried to do something and failed, which had a negative consequence. The only fit for the sentence is “ingratiate...detest.” To ingratiate is to gain favor. If she failed to gain the favor of the well-heeled, then they would naturally detest, or dislike, her overtures. “Promote” is similar to “ingratiate,” but “accept” does not work, since her overtures “backfired.”
2. C. The judge has either an abundance or lack of time; whichever it is, the corresponding arguments he likes are either long or short, respectively. The only choice that makes sense is that he likes “succinct,” or direct and to the point, arguments, because he has a “paucity,” or small amount, of time. Though “scarcity” also means “little of” (like “paucity”), “verbose” arguments would take a long time, so he wouldn’t like them.
3. A. The “though” at the beginning of the sentence is setting up a contrast, so we know that the correct answer will be antonyms of “quiet and reserved.” Thus, “boisterous...gregarious” is the only choice, because both words mean loud and social.
4. D. The “because” at the beginning of the sentence sets up a cause-and-effect relationship. If the billiards player is defeated, he probably is not happy. Therefore, ; he has a “penchant,” or inclination, toward showing off, so when he’s defeated, he feels “ignominy,” or deep humiliation.
5. A. If churches were places where people assembled, they’d need to be large places for people to get together. This is reinforced by the sentence after the comma, which tells us that they were “spacious” and where “people assembled.” “Diminutive” and “petite” both mean “small,” and can be ruled out. “Capacious” means “capable of holding a great amount,” and “congregate” means “gather,” so these are the only choices. “Voluminous” works as “capacious” does, but “disperse” is the opposite of having people assemble.
6. A. Clues in the sentence indicate the monks isolate themselves; they seek to avoid “outside distractions,” for example. As a noun, “cloister” is another word for monastery. As a verb, it means to seclude oneself as if at a cloister. “Sequestering” means “to set apart or seclude.” So these are the only choices for the sentence.
7. B. It may be helpful to work backwards with this sentence. The explorer does something with his last ounce of stamina to get to the top of the mountain; a mountain climb is a struggle, so he likely “summons” his stamina. So he possesses both “fortitude,” or strength in the face of adversity, and strength of will. We can rule out “frailties” since this contrasts with “strength of will,” and we can rule out “dismissed” and “misplaced” as actions he took with his last ounce of stamina.
8. A. The sentence describes how the only difference between the peppers are their color. If the shopper is confused, then he could be in any one of the words used for the second blank. If there was no difference, then the difference would *not* be appreciable (large). Note that the word “no” would contradict the words “insignificant,” “negligible,” and “trivial,” changing the meaning of the sentence to be that there was a large difference between the two peppers.
9. B. The word “despite” indicates a contrast. If the client was absolved, it was probably in spite of a great deal – “copiousness” – of evidence and the talent of the lawyer. “Sophistry” comes from the same root word as “sophisticated,” meaning the lawyer showed skill and sophistication. A “quantum” is a very small amount (quantum physics has to do with tiny things). If there was a lot of damning evidence, and the lawyer was inept, there would have been no absolution.
10. A. If the dog only played with its tennis ball, it liked it a great deal, or showed “affinity” or “preference” for it. (“Aversion” and “disdain” are antonyms and can be eliminated as choices.) So, the dog probably avoided, or “eschewed,” all other toys (it would not make sense for it to prefer the tennis ball and all other toys at the same time).
11. D. If the yoga instructor shouted obscenities, then he likely became angry despite his normally peaceful demeanor. The word “and” tells us that there should be a word that is similar to “peace,” which we find in “tranquility.” The opposite of “peace” happens when the phone rings, leading the instructor to be “irate.”

12. A. The word “though” at the beginning of the sentence sets up a contrast. The first blank is probably a synonym for “awareness,” since we’re told that “shoppers consider their shopping habits,” but the other blank likely means the opposite, since other shoppers do wasteful things. The only match is “conscientious,” or aware of one’s conscience, and “indulgences,” or something someone takes great pleasure in.
13. C. This sentence shows a contrast. Zachary is “disinclined to speak,” and thus quiet. Bill speaks too much and shares unnecessary details. So Zachary is “reticent,” or quiet and restrained, and Bill shares “superfluous,” or unnecessary, details.
14. B. The word “permanent” is a clue about the meaning of the first blank. “Declining” is a clue for the second blank. Thus, “lasting” is a good fit for the first blank, and “stagnant,” which means “not advancing” is a good word for the second blank. “Persisting” means something similar to “lasting,” but “avant-garde” means “new” and “progressive,” which would be nonsensical to associate with the school of Realism if it was not as progressive as Impressionism.
15. D. This sentence shows a contrast. The words “shocking reversal” and “now warn” indicate that vitamins probably have a negative effect, so we’re looking for a word with a negative connotation in the second blank. Thus, in the first blank, we’re also looking for a word with a negative connotation. The only choice that fits is “punitive...detrimental.” Though “injurious” is also negative and has a similar meaning as “detrimental,” it would not make sense to say that the experts previously believed that vitamins had “no advantageous health effects.” Instead, it makes sense to say that they had “no punitive health effects,” since this expands upon the idea of a “shocking reversal.”
16. C. According to the sentence, Galileo had a belief that contrasted with the beliefs of the day. Another word for belief is “hypothesis,” or a proposed explanation. This was “tantamount,” or equivalent, to heresy, because it contradicted the beliefs of the day. Though “synonymous” and “identical” are similar in meaning to “tantamount,” using “denial” or “refutation” does not describe his different beliefs.
17. C. If the city wants to renovate, the building is probably in ill-repair, so “austere” and “pristine” are antonyms for the word in the blank. For the second blank, if the city is renovating, they probably want to improve the neighborhood. The only combination that works is “derelict,” or abandoned, and “revitalize,” or give new life to.
18. A. The clue here is “memorial”; the town wants to build one to remember something tragic that happened in town. In other words they want to “commemorate” the battle by building a “somber” memorial. Though “observe” can have a similar meaning to “commemorate,” it does not make sense to have an “exuberant” memorial to observe a “tragic” battle.
19. A. The sentence implies that nature could be friend or foe, so “steadfast” and “unwavering” are antonyms to the word in the first blank. The word in the second blank is a synonym for “good,” in context. So the only choice is “fickle,” which means “lack of stability,” and “exorbitant,” which means “abundant.” “Vacuous” means “empty” or “meaningless.”
20. B. Since the sentence is about a lie, the word in the first blank should imply something untrue. The word in the second blank should mean “tell the difference,” since the lie is so elaborate, it’s hard to tell truth from fact. The only pair that works is “fabricated,” which means “made up,” and “discern” which means “to understand the difference.”
21. C. “Risk” is the best clue in the sentence. All of the first words could work but “debunks” would work. Doing something risky would require skill equal to the task, so “commensurate,” or “corresponding in size”; a “commensurate skill” would be one that is up to the task. It wouldn’t make sense to say that something taking considerable risk would require “negligible” or “minimal” skill, which both mean “very little.”
22. C. From the context of the sentence, we can tell we’re looking for a pair of words with positive connotations; the Internet is helpful to students. “Demoralizing” and “undermine” can thus be eliminated. “Supplant” means to replace, but the context of the sentence indicates the Internet supports (not replaces) classroom learning. So, “supplements” and “effective” are the only choices that make sense.

23. C. The first blank will be a synonym for “amiable” and the second will be a synonym for “animosity.” “Blithe” means “lighthearted” and “enmity” means “ill will” (often mutual). In this case, the connotations of the sentence help eliminate answer choices.
24. B. The second half of the sentence indicates the book is over the top (“fantastically out of control” and “larger than life”). So both words should indicate this, and “sedate,” “realistic,” “measured,” and “conservative” can be ruled out. “Melodramatic” and “exaggerated” both describe the state of something being excessive and over the top.
25. B. Jane’s skepticism of John indicates we’re looking for words with negative connotations. Thus, “compliment,” “truth,” and “sincere” can be eliminated, leaving “glib,” which means “informal” or “nonchalant,” often in a deceitful way. A “ruse” is a “subterfuge” or trick, which parallels to the idea of John having “ulterior motives.”
26. A. If the friends are cheerfully engaged, it’s a good party, so the word in the first blank has a positive connotation. “Inauspicious” (“unlikely to be successful”) and “portentous” (“foreshadowing something bad”) can both be eliminated. Conversation shows “no sign of stopping,” so the word in the second blank should be a synonym of “stopping” since the second part of the sentence begins with “rather.” Thus, only “convivial” (“full of life and energy”) and “subsiding” (or “decreasing”) work in the sentence.
27. C. “Even miles away” implies the sound is loud and can be heard from great distances. So words like “contracted,” “muffled,” and “dampened” can be eliminated, since the opposite is happening in the sentence. “Reverberate” means “reflect” or “echo,” and “amplified” means “to make greater or louder.”
28. D. For the first blank, it’s likely the celebrity landed in some kind of trouble, making “quagmire” the best word for the blank. “Quagmire” means “difficult position or predicament.” The second blank should be an adjective that indicates an ability to make money; “lucrative” means “producing wealth.”
29. D. Since frogs are mostly active at night, the word in the first blank relates to that. “Nocturnal” means “active at night.” All of the frogs croaking makes a great deal of noise, but the best clue is “growing”: “crescendo” means “increasing in volume.”
30. B. “Upper,” “wealth” and “privilege” are clues for what goes in the blanks. “Echelon” means “grade or class” and “milieu” means “social setting.”
31. B. The musician was talented, which the words in the blank should refer back to. “Callous,” “thundering,” and “languidly” can be eliminated for having negative connotations or not being related to sound. “Dulcet” means “pleasant to the ear” and “mellifluously” means “having a smooth, rich quality.”
32. B. Susan watches the waves, so she is likely fascinated by their “beauty and mystery.” “Drawn” and “gravitated” imply she wants to move toward the waves, which is not information given in the sentence. The second blank should describe the waves; “undulating” means “rolling in a wave-like pattern.”
33. D. It may be easier to find the answer for the second blank first, since there are a few possibilities for the first blank. The little girl regrets her actions. “Laments” means “strongly regret.” “Retrospect” means “thinking back on past events.”
34. C. The sentence tells how landlords are driving people (stores, restaurants, tenants) out of the city, so the word to describe landlords should not be positive. Therefore, we can rule out “frugal.” People might leave the city if their rent went up and they can’t afford it, so the word in the second blank should indicate an increase. Of the choices, “plummeting” means decreasing, but “escalating” means “increasing,” as does “mounting.” However, “spendthrift” means “cheap,” but not necessarily concerned with increasing rent or making more money, as “rapacious” does.
35. D. “More” is an indicator that café culture is becoming more common (which is echoed by having more cafes “opening around the city to meet the demand”). Thus, “appalling” and “causal” can be eliminated, because both “prevalent” and “ubiquitous” mean “occurring frequently.” The second

blank should show a relationship between these facts, so “corollary,” or something that naturally follows from something else, is the appropriate fit for the blank.

36. B. The sentence implies that Neil very much wanted to be an astronaut, so “captivated” and “inspired” are options for the first blank, as both show interest. Wanting to be among the stars is a strong desire, or “yearning.”
37. A. The first part of the sentence implies that the play goes deeper than the surface, so “superficial” can be eliminated, since it means “relating to the surface.” If the playwright seeks to teach a lesson, that could relate to either blank. In this case, the word for the first blank is didactic, or “designed to teach” and the word for the second blank is “covert” or “not openly shown,” since the play hid the message in the story (“cleverly concealed message on morality”).
38. A. If the professor is impressed by how several students already “grasp complex concepts,” the students are likely smart, so they could be “astute” or “shrewd,” since both mean “clever.” The students pursue solutions to age-old problems, and those solutions are “elusive,” or “hard to solve” (since otherwise they would have been solved long ago).

Reading Comprehension

Narrative Passages

Passage 1

1. D. *Main Idea*. The narrator compares and contrasts his experience with camping and his parents’, whom he describes as “camping experts.” He is bored, whereas his parents are enthusiastic. Therefore, the best answer is that the author is explaining the differences between him and his parents.
2. D. *Tone/Language/Style*. The author describes a failure, so we can infer that he found his first camping incident embarrassing. While he uses humor, the purpose of the paragraph is to show a time in the past when he had a negative camping experience he regrets; he goes on to explain another attempt at camping. Therefore “regret and embarrassment” is the best answer.
3. A. *Supporting Idea*. The author doesn’t refer to specific equipment; he runs back inside because he’s scared, not because his equipment doesn’t work. He also doesn’t mention his parents not allowing him to sleep outside. However, he does say that he went into the kitchen “just to satisfy my craving for light, warmth, and company.” This implied that he’s lonely camping by himself.
4. C. *Organization/Logic*. The author refers a few times to the technology and comforts he wishes he had while camping. This causes his parents to “look at him strangely,” but is a specific example provided in context of how the author and his parents think differently of camping, even though the author wants to show his parents how serious he is about camping. The rest of the paragraph lists things that the author misses.
5. A. *Vocabulary*. In context, the author uses the word “thumbed” to mean he used his thumb to look for games on his phone. Thus, “browsed” is the word closest in meaning.
6. B. *Inference*. The story describes the disconnect between the author and his parents regarding camping. The “strange looks” the parents give the author imply that the parents don’t understand their son, and are therefore unable to empathize with them.

Passage 2

1. D. *Main Idea*. The passage is mostly an explanation for the reason behind the bank holiday. Roosevelt is addressing average citizens who probably don’t understand banking very well, but the main idea he wants them to take away is that the bank holiday is intended to help solve a problem.
2. A. *Inference*. The “many proclamations from state capitols and from Washington, the legislation, the Treasury regulations, etc.” were what the public found confusing, so Roosevelt is saying he intends to clarify. He also mentions how things were “couched” in legal and banking terms, and that these should be “explained for the benefit of the average citizen.” Thus, we can conclude that the “average citizen” doesn’t understand these terms.
3. A. *Supporting Idea*. Roosevelt states that the banks “invest your money in many different forms,” which is why they don’t keep it all in a vault. He goes on to explain that this investment “keeps the

wheels of industry and agriculture turning,” so banks must be investing deposit money in industry and agriculture.

4. C. *Vocabulary*. In context, “fabric” must be some kind of structure; government intends to reconstruct “our financial and economic fabric.” Thus, “framework” is the only synonym with a similar meaning.
5. D. *Tone/Language/Style*. Roosevelt is explaining a problem and what the government intends to do about it. The passage is mostly free of emotion, so “angry and practical,” “wistful and yearning,” and “hysterical and pragmatic,” can be ruled out as answers. Instead, the tone is straightforward, or “realistic and matter-of-fact.”
6. A. *Organization/Logic*. Roosevelt describes a lack of currency causing the problem – the banks did not have enough money on-hand to give to customers who wanted to withdraw – so an excess of currency cannot have been one of the reasons listed.

Passage 3

1. A. *Main Idea*. The author isn’t really providing information so much as describing a specific experience. The author is clear about the fact that he did not fall, and doesn’t talk about how the glacier is dangerous. The best answer is that this passage is about a vivid memory.
2. A. *Inference*. The author says, “The temperature was significantly warmer than I expected” inside the glacier, so the author expected it to be colder.
3. D. *Supporting Idea*. There is nothing supernatural happening in the passage, so the choices about hallucinations and a crystal ball can be eliminated. The author also states in line 40 that the ice is free of air bubbles and blemishes, so the choice about imperfections can be eliminated. Rather, the ice takes hundreds of years to accumulate, so the author imagines he can see those years.
4. D. *Tone/Language/Style*. The author uses words like “astounding” and “savor” to describe her experience, which implies that he was in awe. “Reverent” means “expressing respect,” which is the best match for his tone.
5. A. *Vocabulary*. The author is not literally burning anything, but instead making an impression on his mind to remember later. To “etch” is literally to burn an image or pattern onto something, so this is the best match for “burn” in the context of the paragraph.
6. B. *Organization/Logic*. The author does not give advice for climbing glaciers, but instead describes the beauty of the Franz Joseph Glacier in particular. He calls it “unique,” which is a comparison to other glaciers. He also describes the impact of the glacier on his body and mind.

Passage 4

1. C. *Main Idea*. The passage mostly describes a memory of a childhood daydream of a beach vacation to Greece. The memory was a pleasant one, although the author does not attempt to persuade others to go to Greece. The best answer is that the author expresses a feeling of nostalgia and wonderment about a daydream.
2. B. *Tone/Style/Language*. “Did I fall asleep?” in line 18 is a clue that the experience felt otherworldly. Therefore, the tone can best be described as “ethereal,” which can mean “otherworldly.”
3. A. *Supporting Idea*. The author states, “My mother’s voice pulled me back to reality.” In other words, it was the author’s mother’s voice that ended the daydream.
4. C. *Vocabulary*. The experience of the daydream made the author feel shocked to find herself at her home. In other words, the daydream was vivid, so the best answer will be a synonym of “vivid” or “lifelike.” Thus, “realistic” is the best choice.
5. C. *Inference*. The author states in lines 35–36 that she now finds herself on a beach very similar to the one she imagined. The first paragraph of the passage explains that the daydream was inspired by seeing a travel poster for Greece. Thus, it can be inferred that the beach where the author is now vacationing is in Greece.
6. A. *Organization/Logic*. The author describes a present day vacation in the last paragraph (lines 35–40) and states in the conclusion that the past can influence the present. Most of the passage describes the daydream. The author does not, however, give instructions.

Passage 5

1. D. *Main Idea*. In this address, Kennedy describes the U.S. and its relationship with the rest of the world and thus summarizes a number of national policies. He does not offer particular judgment or address allies or enemies explicitly, so the other options can be eliminated as main ideas, since they at most are mentioned in reference as supporting details to a larger goal.
2. D. *Supporting Idea*. In line 43, Kennedy says “well shall join with them” in a show of unity opposing “aggression and subversion” in the Americas.
3. B. *Inference*. Kennedy is not talking about literal tigers, but rather referring to people who sought power in the past; it’s a warning against possible negative consequences of leaders who sought power instead of fostering freedom.
4. B. *Vocabulary*. Kennedy seeks to prevent the UN from becoming a forum for invective, suggesting that its meaning is opposite of working together. “Condemnation” is the only antonym for “working together”; it means “criticism or blame.” Notice that the other choices are all related to one another, and could be interpreted as synonyms for each other.
5. B. *Organization/Logic*. Kennedy spends a lot of the speech discussing peace and freedom, so he would not want to deny peace or cause destruction. The paragraph doesn’t mention science, but rather encourages diplomacy.
6. A. *Tone/Language/Style*. Kennedy doesn’t shy away from mentioning bad forces in the world, but he seems optimistic, so “candid yet steadfast” best describes the tone of the speech. There is no evidence of his being nervous or fearful.

Passage 6

1. C. *Main Idea*. The author describes an old backpack, then explains in line 35 that it changed the way he behaved, so his point of view about the backpack changed. The other choices are tangential to the main arc of the story.
2. C. *Organization/Logic*. The sentence is a description of the backpack with particular emphasis on how it is worn. This makes the backpack unique and special. The explanation of the author’s thinking changing over time takes place later in the passage.
3. D. *Vocabulary*. An heirloom is usually something with special value that is passed down. The author says the knapsack was a gift from his father, and thus is valuable to the author, or a treasure.
4. D. *Supporting Idea*. The author says in lines 21–25, “This practice of taking good care of things was unfamiliar to me, as I grew up in a world of convenience and plenty – a world in which everything is replaceable.” So, the author had never needed to take care of something in this way before.
5. D. *Organization/Logic*. The author discusses realizing his old habits were wasteful, because now he can carry things in the backpack. So, he realizes that disposable bags are unnecessary.
6. C. *Inference*. The author has changed his habits, and says in line 48, “I’ve made a conscious decision” to stop his wasteful daily routines. The author does mention the importance of polishing the leather knapsack, but we must read between the lines and beyond this superficial action to interpret the importance or significance of it. The author never mentions banning plastic bottles or bags, and thinks the opposite of prioritizing convenience.

Passage 7

1. D. *Main Idea*. Johnson spends the first part of the speech explaining America’s accomplishments, then gives a series of proposals aimed at making America better. So America has accomplished a great deal, but has more to accomplish.
2. B. *Supporting Idea*. If the economy is stable, it isn’t fluctuating, and we haven’t peaked at the height of prosperity, so those answers could be eliminated. Johnson mentions that the dollar is sound and prices are stable in the next paragraph (lines 4–10).
3. A. *Inference*. Johnson lists positive things America has done; he says “we want to grow and build and create” in line 18, so he is not saying the U.S. has abandoned creative pursuits.
4. A. *Vocabulary*. Johnson also uses the words “isolated” and “bleak,” so we can infer from context that when he says “blighted,” he means “neglected” as opposite to anything positive, like “pleasant” or “radiant.”

5. *A. Organization/Logic.* Johnson lists a series of proposals to improve the country, or goals. He uses some examples of the past, and describes opinions about what should be done in the future, in order to help make his list more substantial.
6. *C. Inference.* The proposals in the second half of the article are all things Johnson wishes to accomplish, so he is outlining a future course for America.

Passage 8

1. *B. Main Idea.* The passage is mostly about the author's travel experiences. He explains that he likes to look at rivers, which is one of a few reasons he prefers to travel by boat.
2. *A. Inference.* Boat travel has actually declined, which is the opposite meaning of the word "proliferation." The author points out in lines 45–48 that boat travel is relatively rare.
3. *D. Vocabulary.* The author is describing what rivers look like, and since he clearly loves rivers, "depraved," "polluted," and "unyielding" can be eliminated; "sinuous" generally means "snakelike" or "winding."
4. *A. Tone/Language/Style.* The author is contemplative about the nature of travel, though he concludes in line 49 that it's sad rivers are no longer used as a means of transportation.
5. *D. Supporting Idea.* In lines 37–41, the author describes the claustrophobic nature of plane travel (comparing it to being a sardine) as a way to show why he prefers boat travel. The author never makes the points mentioned in the other choices.
6. *B. Supporting Idea.* The author likes to look out the window and ignore the rest of what is happening inside the plane. He finds plane travel uncomfortable, but looking outside is nice because he sees the view. The other choices may be true, but there's no evidence of this in the passage.

Passage 9

1. *D. Main Idea.* In the passage, Brad has an experience at Shinjuku Station, the busiest train station in the world. He compares it to Times Square, but the passage is mostly about this one particular experience. The other choices factor in as supporting details only.
2. *B. Inference.* It's implied in the passage that Brad has experience navigating busy areas and has experience with travel, so it can be inferred he is a seasoned traveler. We only know that he had lived in New York for a decade, not that he was born there.
3. *A. Tone/Language/Style.* The words used in these lines ("bewildered," "disorienting") imply that Brad is having a difficult time navigating this environment and is growing increasingly worried.
4. *B. Vocabulary.* "Cacophonous" is used in the context of noise, or "din." It can be inferred from this context that it is noisy.
5. *A. Organization/Logic.* The author seeks to show busy the station is by using Times Square in New York City – a place often swarming with people and activity – as an example of somewhere similar.
6. *D. Supporting Idea.* Brad doesn't muse at the signs and ads, but rather they add to his confusion and make finding his friend more difficult.

Persuasive Passages

Passage 1

1. *D. Main Idea.* The passage gives both benefits and history of vegetable gardens, so this is what the passage is mostly about. The other choices are arguments made in the passage in support of this main idea.
2. *C. Organization/Logic.* The first paragraph does not include historical information, instructions for growing vegetables, or a balance of benefits and drawbacks, but it does show benefits for a vegetable garden.
3. *A. Supporting Idea.* Lines 47–50 explain that "victory gardens" helped alleviate the burden on the public food supply.
4. *A. Tone/Language/Style.* "Victory garden" is a term given to the gardens governments encouraged their citizens to make. This emphasizes their contribution to the war effort.
5. *C. Vocabulary.* In context, citizens plant wherever there is "arable" land, which implies the land is good for farming, or "fertile."

6. *C. Inference.* In line 64, the author expresses envy for people who built victory gardens, from which you can infer that the author wants to start her own garden.

Passage 2

1. *D. Main Idea.* The author doesn't discuss history or advocate for a particular solution. While the passage does give reasons why no-shows are a problem for restaurants, the purpose of the piece is not just to give examples. Instead, the author compares and contrasts different solutions to the problem of no-shows.
2. *C. Tone/Language/Style.* "Plagued" implies a vast problem; a plague is a contagious disease that spreads quickly. The use of the word in line 2 is a way to use exaggeration to prove a point about how frequent the problem of no-shows is for restaurants.
3. *B. Vocabulary.* The author says, "with strong proponents and opponents," from which we can infer that the word "contentious" relates to an argument or an issue that has people arguing for or against. So the word means "controversial."
4. *D. Organization/Logic.* The author gives several solutions to the problem of no-shows, many of which are controversial or unfair. So the "middle ground" is one that treats diners fairly by allowing them to pay a fixed price for a meal in advance, and also incentivizes diners to go to the restaurant so they don't lose their money, hence eliminating the problem of no-shows.
5. *B. Supporting Idea.* The author mentions selling reservations, requiring diners to pre-pay, and requiring a credit card for a reservation as ways to solve the problem of no-shows. However, allowing tables to sit empty is similar to having diners make reservations and not show, so this is not an argument the author makes.
6. *D. Inference.* Most of the passage is about solving the problem of no-show diners, and the author gives a possible solution in the last paragraph that he feels would be most fair to both restaurants and diners.

Passage 3

1. *C. Main Idea.* The passage lists several ways trees benefit the environment to prove that trees are an important part of the ecosystem. It is true that trees absorb carbon and sunlight, but this is only one way trees are beneficial.
2. *C. Vocabulary.* Trees regulate temperatures, according to the passage, meaning that they can provide an escape from the heat. The next sentence explains that placing trees around a building can reduce air-conditioning needs because the trees block and absorb the heat. So, "respite" means "shelter."
3. *B. Tone/Language/Style.* The author provides specific details to support his argument. Statistics from reliable sources can add credibility to a persuasive essay. The author never states that trees should never be cut down, and mentions the USDA Forest Service only as a way of building credibility.
4. *B. Organization/Logic.* The paragraph mentions the general biological process of trees absorbing carbon dioxide and using it in its cellular processes, which is a way trees contribute to the overall health of the atmosphere, and supports the author's opinion that trees are important. The author does not, however, mention the impact of rising temperatures in the paragraph.
5. *C. Supporting Idea.* The author describes the deep roots of the tree preventing landslides and erosion in lines 42–45, for example, which is a way trees help fight soil erosion.
6. *D. Inference.* The passage only lists a few reasons that trees are beneficial, but the implication is that there could be many more, given the author's passion for trees. The other choices can be eliminated based on the statements made by the author throughout the passage.

Passage 4

1. *D. Main Idea.* The passage is mostly about the benefits of race-walking, so the purpose is to educate. It includes a description of race-walking, but the focus of the passage is more on the benefits.
2. *B. Vocabulary.* It is stated in the passage that fitness experts often are strong advocates for race-walking. So "extol" means "promote" in context.
3. *D. Inference.* The author gives a number of reasons why race-walking is beneficial, including that it is good for the cardiovascular system, likely just as much as running and cycling, since it involves the same muscle groups.

4. *C. Supporting Idea.* The author says in line 32 that proper posture is what helps engage the abdominal muscles.
5. *D. Tone/Language/Style.* The author stays professional, explaining in mostly neutral terms that injuries from running and cycling are more frequent because race-walking strains the body less. The author doesn't use inflammatory adjectives to make her case.
6. *A. Organization/Logic.* In this section of the passage, the author makes an argument about race-walking being safe, and then supports it with facts.

Passage 5

1. *D. Main Idea.* The passage lists a number of reasons why the national parks should be preserved. The evidence includes reasons both environmental and economic. The author cites Costa Rica as an example, but the passage isn't entirely about it. Similarly, comparisons are made, but this is not the focus of the passage.
2. *B. Tone/Language/Style.* The passage is mostly about conserving a section of public park. Ferdinand Hayden argued for conserving a particular park, and the paragraph explains the reasoning behind the conservation movement (which is to prevent exploitation).
3. *C. Supporting Idea.* Parks are not private, so that answer can be eliminated. While they are inexpensive and restrict mining of resources, these are not reasons that support the argument about economic growth. Rather, the description of people spending an increasing amount of money explains how the national park system grows the economy.
4. *D. Vocabulary.* Preservationists bemoaned the lack of economic opportunity, meaning they were unhappy with it. Of the choices, "capered" and "inspired" don't fit. The preservationists don't "devastate" the loss (they may themselves be "devastated"), either; rather, they "lament" the lack of opportunity, or mourn it.
5. *C. Inference.* The author uses Costa Rica as an example of a country that has successfully increased tourism using national parks. Other countries could use that as a model.
6. *D. Organization/Logic.* In the passage, the author uses facts and evidence to support the main point about national parks being preserved. One of those arguments is that there are economic benefits to parks, so the argument that environmental protection has led to monetary gain supports the main idea about economic benefits of national parks.

Passage 6

1. *D. Main Idea.* The author gives a number of historical examples that show that focusing on one crop can have disastrous consequences. In other words, the author wants to warn the reader about "monoculture," which is cultivating a single crop, as defined in lines 4–5.
2. *A. Supporting Idea.* The author explains that the Lumper potatoes dominated because the Irish sold other food to pay the British, it could be grown sufficiently, and the Irish were blocked from other employment. The paragraph does not mention a preference for taste.
3. *D. Vocabulary.* There wasn't much land, as we're told in lines 31–32, so from context, we can infer that "paucity" is a lack or scarcity.
4. *D. Organization/Logic.* The main idea of the article is that focusing on only one crop can lead to bad consequences, so the most important detail of those given is that the Irish had come to rely almost solely on the Lumper potatoes.
5. *C. Inference.* The author is trying to make an argument against monoculture. The last paragraph is a call to action, asking the reader to get involved. The examples show how monoculture can pose a threat to the country generally.
6. *D. Language/Tone/Style.* The bulk of the passage explains why monoculture is dangerous, and the questions in the last paragraph support that opinion by imploring the reader to get more involved.

Passage 7

1. *D. Main Idea.* The passage explains that groups of consumers have different priorities, which affect how they approach shopping. The passage doesn't really pass judgment on consumers, but rather

makes a larger point about how prioritizing low price has affected small vs. large businesses. This prioritizing affects the shopping experience.

2. D. *Supporting Idea*. Paragraph 2 explains that many small businesses are unable to buy in bulk and therefore can't compete with bigger companies because they can't lower their prices as much and still turn a profit.
3. C. *Vocabulary*. In the context of the sentence, "volume" refers to quantity, not sound; the sentence describes the amount of products sold in order to generate a profit.
4. D. *Inference*. Since small business often can't compete with larger companies in terms of low price, small businesses need other advantages. This section of the passage explains that smaller businesses get to know their customers better (line 58), for example. Thus, small businesses can appeal to consumers who have priorities besides low price.
5. C. *Tone/Language/Style*. This section of the passage gives examples of what small businesses can do to stay competitive with larger companies; small businesses appeal to consumers who have priorities besides just low price.
6. B. *Organization/Logic*. The author does not ridicule or complain, nor does he call for a boycott, but instead, he makes a larger argument about placing too much emphasis on low price. In other words, he suggests the reader rethinks his or her own shopping preferences and consider what aspects of the shopping experience have value.

Passage 8

1. D. *Main Idea*. Although the opening paragraph describes a pleasant experience with a grass lawn, the author goes on to explain that maintaining such lawns has a negative impact on the environment, and thus have hidden costs. He does not adopt a stubborn or traditional stance (which would be to advocate for grass lawns). Instead, he does the opposite, suggesting tat people think outside the box.
2. C. *Tone/Language/Style*. The author describes childhood memories of growing up in the suburbs with a house that had a lawn, so the mood is nostalgic, or reflecting back on something pleasant that happened in the past.
3. C. *Vocabulary*. We can infer from the sentence, since the author is dismayed to learn about the real impact of lawns, that "deleterious" has a negative connotation, so "beneficial," "effective," and "important" can be ruled out. "Deleterious" means "harmful."
4. B. *Supporting Idea*. In the third paragraph, the author details the ways in which maintaining a lawn can be harmful, including negative impacts on the environment. The paragraph does not mention lawns being aesthetically pleasing, nor is this a reason lawns are wasteful.
5. D. *Organization/Logic*. The main idea of the passage is that maintaining lawns is harmful; this paragraph suggests alternative to maintaining lawns. The author does not try to convince the reader to continue to maintain a green lawn.
6. C. *Inference*. The author makes a compromise conclusion, arguing that some green lawns should be kept, but maybe people should think about alternatives. But generally speaking, he argues, we should try to fight nature less, or fewer people should cultivate lawns.

Descriptive Passages

Passage 1

1. D. *Main Idea*. The passage is a description of driving through the desert. The author doesn't explain why he is there, nor does he provide a contrast with the city. He does describe mythical creatures, but in terms of the rock formations he sees; he does not see literal giants or ogres.

2. D. *Tone/Language/Style*. The words “growled,” “roared,” and “devoured” are animalistic. He uses them in a few different contexts – the growl of the engine, the roar of the wind, the road devoured by the car – meaning that the whole experience is primal and visceral.
3. A. *Supporting Idea*. The author explains that the land around him is so flat that his view is unimpeded, so he thinks he can see the curvature of the earth.
4. A. *Organization/Logic*. Through these lines, the author is creating a sense of suspense, describing something unusual he sees to interest the reader and then later explaining what exactly he saw. In other words, he’s building tension and suspense.
5. B. *Vocabulary*. The author offers the vision of the rock formations as a contrast to the “hundreds of miles of nothingness,” so they must be great. In context, therefore, “sublime” means “magnificent.”
6. B. *Inference*. The author is happy to have reached his destination and says “finally” as if he had been planning to go there all along, which implies that his trip was deliberate.

Passage 2

1. A. *Main Idea*. The passage is mostly about why Anne likes to ride elevators, and though it mentions digital and analog displays, Anne’s enjoyment is not dependent on one or the other.
2. D. *Organization/Logic*. The paragraph states that, for Anne, pressing buttons is tactile in a way touching her smartphone is not, but it does not go so far as to state that Anne doesn’t like using her smartphone.
3. C. *Vocabulary*. The text shows a contrast between the feel of pressing buttons and the lack of it when using a screen like on a smartphone, so we can conclude that “tactile” has something to do with touch. Therefore, “physical” has a close similar meaning.
4. A. *Tone/Language/Style*. Words like “plaques,” “honoring,” and “inscribed” indicate importance. The author intends to show that the buttons are an important part of the elevator riding experience for Anne.
5. D. *Supporting Idea*. Touching the Braille lettering is part of the experience of touching the buttons for Anne and they enhance her experience. She wonders how the dots translate into numbers (line 32), and thus marvels at how people can interpret the bumps.
6. B. *Inference*. The paragraph describes Anne’s particular joy in watching someone get on the elevator, press a button, and then have the button refuse to light up. From this we can infer that the buttons are supposed to light up.

Passage 3

1. A. *Main Idea*. Most of the passage describes the ceiling in the Main Concourse of Grand Central Station and does not go into depth about the other features of the train station. Although there is some explanation of astronomy, it is to explain the errors in the ceiling.
2. B. *Tone/Language/Style*. The phrase “form and function” refers both to aesthetics and to practicality. The author is saying that Grand Central is both a beautiful train station and that it serves a practical purpose. The phrase implies a balance between the two, not that one or the other is more important.
3. A. *Vocabulary*. In the context of the sentence, we know that the station has famous heirlooms, so we can infer that the meaning of the phrase “steeped in” means there is a lot of history, or Grand Central is “filled with” history.
4. A. *Organization/Logic*. In line 49–51, the author states, “The most likely explanation is that a transcription error occurred...” Human error caused the mistakes in the ceiling.
5. C. *Supporting Idea*. The author mentions the color, scale, and depictions of constellations as reasons the ceiling is beautiful. However, the reference to the flap-board destination sign is in the first paragraph (line 15) and refers to historical heirlooms in the station, not the beauty of the ceiling.
6. B. *Inference*. The bulk of the information in the passage describes how beautiful the ceiling is, so it can be inferred that the author stops to admire the ceiling whenever he is in Grand Central Station.

Passage 4

1. D. *Main Idea*. The passage details the destruction caused by the hurricane and the effort required to rebuild, so the main purpose of the passage is to describe the impact of the hurricane on the community. Though there is an implicit warning, much of the passage also describes rebuilding.

2. *A. Organization/Logic.* The sentence includes a simile comparing the destruction caused by the hurricane to that of an angry child ripping his room apart. This comparison serves to illustrate what the damage was like to the reader by providing an image the reader can relate to.
3. *C. Tone/Language/Style.* The author is not referring to a literal minefield, but rather to the danger the beach poses because broken glass, metal, and plastic is everywhere.
4. *C. Vocabulary.* The damage to the beach is so extensive that the author believes beachgoers will continue to find “detritus” even after the cleanup is finished. From this context, we can infer that “detritus” refers to “remnants” of the damage.
5. *B. Supporting Idea.* Line 34 refers to “the community and its inhabitants,” implying that they were instrumental in the cleaning effort, not the government, other countries, or charities. The author also references metal detectors in line 38 as part of the process.
6. *A. Inference.* The author is clear about how destructive the hurricane was, and says in the last paragraph that, although the community rallied to rebuild, the beach will never be the same. We can infer that the threat of another hurricane to undo all this good work remains, and that “there will be a reminder of what has been lost.”

Passage 5

1. *A. Main Idea.* Most of the passage describes the open-air market. The author is not trying to persuade the reader, merely describing what the market is like.
2. *A. Vocabulary.* In context, the phrase “veritable gem” refers to the market, not a literal stone or object. “Veritable” is a synonym for “real” and is often used to modify a metaphor. Gems are relatively rare, so we can infer from the sentence that the market is “a rarity.”
3. *D. Tone/Language/Style.* The phrase “sun unrelenting gaze” is a good clue about why the detail about the awnings is included; without then, the sun beating down on customers would be unpleasant.
4. *C. Organization/Logic.* The author only chooses to include produce that starts with the letter *p*, from which we can infer that the author is not trying to provide a comprehensive list, but rather to give an example of the kinds of produce that can be found in the market. The author wants the reader to appreciate the range of products available.
5. *D. Supporting Idea.* The author lists all of the sources of smells in the market. Each of these products produces a scent, but the overall effect is pleasant. Toward the end of the paragraph, the author states, “no one scent overpowers another, and the multitude of smells coalesces and combines into something greater.”
6. *B. Organization/Logic.* In the last paragraph, the author describes things to do at the market that aren’t buying or selling, such as eating at a restaurant or having a cup of coffee. So, the author is not reinforcing the idea that the market is only for buying and selling.

Passage 6

1. *D. Main Idea.* The passage describes a lobby and the sculptures within it, and we can infer from the language the author uses that he believes it is beautiful. The author is not making a persuasive argument about marble nor advocating for all lobbies to include sculptures.
2. *A. Tone/Language/Style.* “White as milk” is a simile comparing the color of the marble to milk in order to help the reader picture what the marble looks like.
3. *C. Vocabulary.* The author uses the word “minimalist” to describe the lobby as well, meaning it is not too elaborate, or that there isn’t much in the lobby. “Austere” has a similar meaning, so “stark” is the correct answer.
4. *C. Inference.* The sculpture is described in lines 30–36 as very large, so we can infer that it was delivered in pieces because it too large to be moved in one piece.
5. *D. Supporting Idea.* In line 39, the author states the water trickling through cracks and wrinkles was “by design,” so the cracks and wrinkles were placed there intentionally. In line 41, the author states this prevents “unwanted splashing.”
6. *C. Inference.* The passage is primarily descriptive and does not try to make an argument beyond that the lobby is beautiful. The passage includes a number of details that support this main idea but does not describe obsidian as “obsolete,” nor does it compare a particular lobby with another.

Passage 7

1. D. *Main Idea*. The passage is mostly a description of a nice spring break and is not trying to make a persuasive argument beyond that the day is picturesque. The author describes birds and mentions flowers, but these are not the primary focus of the passage.
2. B. *Inference*. The author implies that April and May “keep with tradition” after stating that April is usually raining. (This is likely a reference to the saying, “April showers bring May flowers.”) The author does not state the tradition May follows but does explain that April is rainy, so we can infer that the author expects April to be rainy, but we can’t reasonably conclude anything beyond that.
3. A. *Vocabulary*. If today is “a perfect spring day,” then a “paragon of pleasantness” will be something positive (where the others are not positive). In other words, it’s an ideal.
4. B. *Supporting Idea*. The author mentions the sky (line 22), the humidity (line 16), and the temperature (line 14), but not the breeze.
5. C. *Tone/Language/Style*. The sound is not a literal shower; the author is using a metaphor to describe the sounds of the songbirds. This is meant to show the sounds are surrounding the author. There’s no indication of rain or that the birds are bathing.
6. B. *Organization/Logic*. The author is saying in the final paragraph that there may be another day like this soon, but maybe not, so he intends to savor this day (“When will there be another day like today?”). That implies the day is precious.

Passage 8

1. C. *Main Idea*. All of the details in the passage – what the theater looks like, for example – relate to what it’s like to wait for the show to start. The countdown (“30 minutes to go” and “15 minutes to go”) are clues that the big event is coming.
2. A. *Inference*. Based on the description – people are still finding their seats, John is impatient, the orchestra warms up at the end – we can infer that the narrator is waiting for the play’s opening.
3. D. *Supporting Idea*. The author says, “The sections of the theater were all labeled,” and then goes on to describe the directions. He’s impatient because the directions are all clearly labeled but people are still milling about the theater.
4. C. *Tone/Language/Style*. Since the start of the play isn’t delayed by people not finding their seats, the question indicates the narrator’s impatience for the play to start. The emphasis on “me” shows that the narrator is focused on himself.
5. B. *Vocabulary*. In the sentence, “the relief” refers back to “the oval crest above the stage.” The author says he can’t make out the particulars of it from his seat. From this we can infer that the relief is a design.
6. B. *Organization/Logic*. The countdown indicates a this is a chronological series of events. The other choices can also be eliminated. The passage is not really a list of facts, since the narrator offers opinions, but it’s not only opinions, so it’s not unsubstantiated. The passage mentions the Gilded Age only in the context of the design of the theater.

Expository PassagesPassage 1

1. A. *Main Idea*. The passage is a brief explanation of the history of anesthesia. The clues are in the second paragraph, which uses transition words to show how the history progressed (“In ancient times...”; “Today...”). The passage does not really explain the science or explain how to make homemade anesthetics. It is also not a persuasive passage.
2. D. *Tone/Language/Style*. If something is varied, it is changing or diverse. So, if the history of anesthesia is as long as it is varied, it likely changed a lot – or there were many kinds of anesthetics used – over a long period of time.
3. A. *Inference*. In lines 8–9, the author explains that patients “suffer little if any discomfort or pain during a procedure,” meaning the number of patients who suffer pain during anesthesia are few.
4. C. *Supporting Idea*. The author explains that ether caused patients to fall asleep and feel no pain while asleep (lines 26–30). The other choices are not supported by the text.

5. D. *Vocabulary*. In context, painful stimuli are something anesthetics decrease, so we can infer that they are painful feelings or sensations.
6. C. *Organization/Logic*. Modern anesthetics, according to the last paragraph, can be administered in a couple of ways, are reversible, and have allowed medicine to advance. The passage does not state that anesthetic dosages are hard to determine, even if that is factually true.

Passage 2

1. D. *Main Idea*. The passage is specifically about Archimedes. It details a number of Archimedes' important discoveries. In the first paragraph, the author writes that "even now...his discoveries are still central to our understanding of the physical world" (lines 4-6). In the concluding paragraph, the author states that his discoveries "are central to our everyday way of life" (lines 75-76).
2. D. *Supporting Idea*. The passage states that Archimedes' friend Heracleides wrote a biography that was later lost. Thus, we don't know much about Archimedes aside from other stories and hearsay.
3. C. *Vocabulary*. In the context of the passage, what we know of Archimedes is from stories and hearsay, so we can conclude that hearsay are rumors or other kinds of passed around speech.
4. D. *Tone/Language/Style*. Although this is an entertaining story, the author wants to show that stories about Archimedes show that he made real contributions to science.
5. B. *Inference*. Archimedes shouts "Eureka" after finding a way to solve a puzzle. From this we can infer that Eureka conveys satisfaction.
6. A. *Organization/Logic*. The author mentions a few of Archimedes' contemporaries, but does not directly compare him to other scientists. The passage does, however, describe some of Archimedes' inventions, explain how his discoveries affect us today, and highlight uncertainties about him.

Passage 3

1. C. *Main Idea*. The passage is mainly a description of the process for staining bacteria. Although there is some information about dyes and equipment, the primary purpose of the passage is to describe the process. The passage does state that staining is useful, but doesn't otherwise make an argument for why it's important.
2. B. *Supporting Idea*. The first paragraph explains that different dyes have different effects on the bacteria because of the positive and negative charges of the dyes and biomolecules. These are electrostatic charges, not magnetic, thermal, or gravitational charges.
3. D. *Organization/Logic*. These lines are unbiased and focus on the scientific facts of the effects of staining bacteria. The author does not provide historical information, opinions, or a chronological retelling of events.
4. C. *Tone/Language/Style*. The author compares a visual that most readers will be familiar with to the sterile loop so that readers can better picture what one looks like.
5. A. *Vocabulary*. We can infer from context that "expedite" relates to changing the drying time; it means to speed up. Thus, accelerate is the correct answer.
6. C. *Inference*. The author describes a careful process and uses words like "distilled" and "sterile." We can infer, then, that rubbing the stain will contaminate or damage it in some way.

Passage 4

1. C. *Main Idea*. The passage uses the Greek myths about Demeter to show how a story can explain a natural phenomenon, in this case, the change in the seasons and the life and death of flowers and plants.
2. A. *Inference*. In the larger context of the passage, the story about Demeter and Persephone shows that Demeter loved her daughter and that their reunion and separation has an effect on the flowers and plants, since Demeter is the goddess of the harvest. The writer mentions Eleusis, but that is not the primary purpose of the paragraph.
3. B. *Vocabulary*. Demeter teaches someone the secrets of agriculture, and he propagated the knowledge to all who would listen. In other words, he spread the knowledge or told other people. "Distributed" is the correct answer.
4. B. *Organization/Logic*. Lines 43-46 explain that Persephone ate the pomegranate that Hades offered her, and so she was not allowed to leave.

5. *A. Supporting Idea.* Demeter is unhappy when she is separated from Persephone, and so causes plants to wither and die. In other words, they are separated during the cold months of autumn and winter.
6. *C. Tone/Language/Style.* The last paragraph explains that, when Demeter and her daughter are together, flowers blossom, but when they are separated, flowers wither and die. This draws the connection between the story of Demeter and the seasons.

Passage 5

1. *C. Main Idea.* The passage describes how to paint a room, including the equipment you'll need and instructions for painting. Although the first paragraph makes an argument for why you should paint, the passage is otherwise not persuasive.
2. *D. Inference.* The author states in lines 39–40 that using an extension rod can save time because otherwise you will need to get a footstool or ladder and move it around.
3. *D. Supporting Idea.* The author states that the tape helps paint with precision, and also that it doesn't damage the wall or pull up paint when removed.
4. *C. Vocabulary.* In the context of the sentence, the nooks and crannies are around doorframes or windows, so we can infer these are not bright rooms or flat surfaces. Doorframes and windows usually don't have round edges, but they might have inaccessible spaces, so this is the correct answer.
5. *A. Organization/Logic.* The author gives the steps for painting in order, so this is a sequential order of events.
6. *C. Tone/Language/Style.* The description of painting is very practical and doesn't contain a great deal of emotion. The tone could best be described as pragmatic, which means practical.

Passage 6

1. *D. Main Idea.* The passage is about the life of John Glover, who had modest origins but went on to play a major role in American history. His life wasn't dull, nor was it easy, which can help us eliminate some of the choices. The author also does not try to teach the reader a lesson about the key to success or life in 18th-century society.
2. *C. Supporting Idea.* The author states that Glover is remembered because of his contribution to the war effort, meaning his role in the American War of Independence.
3. *D. Tone/Language/Style.* Glover's contribution to the war included his ship, and many consider this ship to be the first in the American Navy. This draws a parallel between the American naval ship, the *USS Glover*, named for him because of this contribution.
4. *A. Vocabulary.* In context, Glover helped guard the Hudson against British incursion, and also he "helped to contain troop movement in the region." From this, we can infer that incursion means "attack."
5. *D. Organization/Logic.* Although the passage offers an argument that Glover is worthy of remembering, the passage is mostly an accounting of historical facts in the order they occurred, or a chronological account.
6. *B. Tone/Language/Style.* The passage is mostly a straightforward account of historical fact without much emotion, so the author's tone can best be described as "impartial."

Passage 7

1. *A. Main Idea.* The passage lists and explains several types of surface mining, which shows that there are several different types. The passage doesn't conclude that surface mining is the best, and it is straightforward about the environmental harms, so the other choices are not supported by the text.
2. *D. Supporting Idea.* The passage states that "materials covering a mineral deposit are removed" (lines 15–16), which is harmful to the environment.
3. *D. Vocabulary.* In the first sentence of the paragraph, the author explains, "Efforts are sometimes made to refill the retired mine with dirt, clay, and vegetation." The next sentence refers to "this type of rehabilitation," so we can infer that "rehabilitation" means "restoration."
4. *B. Inference.* The first sentence in the passage explains that people have sought precious metals and minerals for thousands of years. We can infer from this that people continue to mine because they value these metals and minerals more than they value the environment, because people continue to mine despite environmental harms.

5. *C. Organization/Logic.* The passage is a list, with explanations, of several surface mining techniques. So, the organization is a description of different mining techniques, including their pros and cons. The object of the passage is not to give an opinion or argument.
6. *D. Tone/Language/Style.* In context, “scarred” refers to mountainsides damaged by mining. This is a strong visual that helps the reader “see” what the damage from mining looks like. This would not satisfy supporters of surface mining, or emphasize that the practice is harmless.

Passage 8

1. *B. Main Idea.* This is an expository passage, meaning it explains something but is not necessarily making an argument. This passage explains how Memorial Day came to be the last Monday in May.
2. *D. Tone/Language/Style.* This section of the passage describes to an event in which townspeople honored fallen American soldiers by hanging “flags at half-mast” and draping plants with “black.” The tone is reverent, which means respectful.
3. *A. Vocabulary.* In context, “fallen” refers to American soldiers who have died – flags at half-mast and black being draped on trees and shrubs are generally symbols associated with mourning. Thus, “fallen” means “deceased.”
4. *C. Supporting Idea.* The author explains in lines 73–74 that confusion and a resistance to break tradition were responsible for the slow adoption of the Uniform Monday Holiday Act. Communities had their own traditions that they were reluctant to change.
5. *C. Inference.* Because communities were reluctant to change the day Memorial Day was celebrated, we can conclude that the Uniform Monday Holiday Act was unpopular.
6. *B. Organization/Logic.* The author compares Memorial Day to Veterans Day in the first paragraph, provides historical examples of Memorial days in the second paragraph, and explains why Memorial Day is no longer celebrated on different days in the last paragraph. However, the author does not criticize Congress for including Memorial Day in the 1968 act.

Passage 9

1. *D. Main Idea.* This is an expository passage, meaning it explains something but is not necessarily making an argument. The passage does not make a strong argument regarding differences between the rugby union and league. The passage also explains that the two are not interchangeable; though there are differences, they remain distinct.
2. *C. Vocabulary.* The first paragraph explains that rugby union and rugby league have similar origins, but subtle differences have led them to be distinct sports. From this context, we can infer that “subtle” means small or minor.
3. *C. Supporting Idea.* In the third paragraph, the author explains that ball can be carried down the field, passed between players backwards or sideways, or kicked. The paragraph does not say the ball can be passed forward.
4. *A. Supporting Idea.* A conversion is worth two points in both games. However, in league play, a try is worth four points. So, if a team scores a try and a conversion, they will score a total of six points.
5. *A. Organization/Logic.* The passage lists the rules of rugby and explains how the two kinds of rugby are different and similar from each other. The best description of the organization is of a list of rules and explanations.
6. *D. Inference.* In lines 58–59, the author states that a player can only be tackled if he or she is in possession of the ball. So, we can infer that tackling a player who does not have the ball is against the rules and will have a negative consequence.

Passage 10

1. *C. Main Idea.* The passage is a description of traditions associated with Japanese tea ceremonies. The use of the word “typically” in line 5 indicates the list of rituals that follow are part of a typical ceremony. Thus, the passage describes the rituals of a typical (not specific) tea ceremony.
2. *D. Supporting Idea.* The author explains in line 30–34 that crawling through a small space is a symbol of entering the calm of the teahouse amidst the chaos outside.

3. *C. Inference.* The author states in lines 40–41 that guests are seated in order of rank. Thus the last guest to enter the room is most likely of the lowest social rank.
4. *B. Vocabulary.* In context, the tea implements are the tools used to make and serve the tea. Thus, “tools” is the correct synonym.
5. *D. Organization/Logic.* As is true elsewhere in the passage, the last paragraph describes a series of rituals that are part of the ceremony. The last paragraph specifically describes the series of steps taken to end the ceremony.
6. *A. Tone/Language/Style.* The article is a description of rituals and the tone remains impartial. It can thus be described as “academic,” since it is a description and not biased (partial) or argumentative.

Quantitative Reasoning & Mathematics Achievement

Numbers & Operations

Arithmetic

1. *B.* Keep in mind the order of operations: PEMDAS (parentheses, exponents, multiplication, division, addition, subtraction). Thus, the parentheses should be dealt with first: $6 + 4 = 10$. Then multiply by 5: $10 \times 5 = 50$. Last, add 9: $50 + 9 = 59$.
2. *B.* Remember PEMDAS. Find the value of the exponent first: $2^2 = 4$. Then do the multiplication: $5 \times 4 = 20$. The value of the expression is now equal to $20 - 20 + 4 = 4$.
3. *A.* Remember PEMDAS. Find the value of the parentheses first: $2^2 + 4 = 4$. The value of inside the parentheses then becomes $4 + 4 = 8$. Multiply by 5: $8 \times 5 = 40$. Subtract this value from 20: $20 - 40 = -20$.
4. *B.* Remember PEMDAS. Find the value of the term in parentheses: $20 - 5 = 15$. Then find the value of the term with the exponent first. $2^2 = 4$. The value of the expression is now $15 \times 4 + 4$. Perform the multiplication first: $15 \times 4 = 60$. Finally, add 4: $60 + 4 = 64$.
5. *C.* Remember PEMDAS. Find the value in the parentheses first: $20 - 5 \times 2$. Multiply first: $5 \times 2 = 10$, and $20 - 10 = 10$. So, the value of the expression in parentheses is 10. Next, find the exponent: $10^2 = 100$. Now the value of the expression is $100 + 4 = 104$.
6. *D.* Remember PEMDAS. Find the value of the term with an exponent first: $2^2 = 4$. Then perform the division. $2 \div 4 = 0.5$. The value of the expression is now $2 - 0.5 + 2 = 3.5$.
7. *A.* The note suggests canceling out factors before multiplication. Since 5 is a common factor of 15 and 10 and 7 is a common factor of 14 and 21, the expression can be simplified to $\frac{3(2)}{2(3)}$. This expression can be further simplified to $\frac{6}{6} = 1$.
8. *B.* Keep in mind that you can use common factors to simplify the expression. 52 and 24 have a common factor of 4, so the expression can be simplified to $\frac{13(12)}{6(13)}$. We can multiply this out, or simply cancel the common factors of 6 and 12 (which is 6) and 13 and 13 (which is 13). This leaves us with $\frac{2}{1} = 2$.
9. *A.* Find the value of the terms with exponents first: $2^2 = 4$ and $2^3 = 8$. Thus, $4 + 8 = 12$. $2 + 4 = 6$, so the expression is equal to $\frac{6}{12} = \frac{1}{2} = 0.5$.
10. *C.* First, simplify inside the parenthesis to arrive at $\frac{(8)(7)}{(4)(14)}$. We can cancel out the common factors of 4 and 8 (which is 4) and 7 and 14 (which is 7). This gives us $\frac{(2)(1)}{(1)(2)} = \frac{2}{2} = 1$.

11. B. The numerator and denominator have a common factor of $(5 + 5^2)$, which cancels out. So, the value of the expression is simply $\frac{10}{5} = \frac{2}{1} = 2$.
12. B. To find the value of the expression in Column A, do the multiplication first. $3 \times 4 = 12$. $1 + 2 - 12 = -9$, which is less than 0, so the expression in Column B is greater.
13. B. To find the value of the expression in Column A, find the value of the expression in parentheses first: $5 \times 2 = 10$. Then apply the exponent: $10^2 = 100$. So, the expression is equal to $20 - 100 + 4 = -76$, which is less than 104. So, the value in Column B is greater.
14. C. Simplify inside the parenthesis of the expression in Column A to arrive at $\frac{(10)(6)}{(4)(16)}$. Then, simplify by dividing every term by 2, the common factor of all terms. This gives us $\frac{(5)(3)}{(2)(8)} = \frac{15}{16}$. This is equal to the value in Column B.
15. B. Find the value of each the numerator and the denominator of the expression in Column A first. $2^6 = 64$ and $6^2 = 36$. This gives us the numerator of $64 - 36 = 28$. Then, do the same for the denominator: $2^3 = 8$ and $3^2 = 9$. This gives us a denominator of $8 - 9 = -1$. So, the expression is equal to $\frac{28}{-1} = -28$, which is less than 1. So, the value of Column B is greater.

Factors

1. D. The integers greater than 30 and less than 40 are 31, 32, 33, 34, 35, 36, 37, 38, and 39. The question asks for the numbers that are the product of two different (distinct) prime numbers. REMEMBER: 1 IS NOT PRIME! The first several prime numbers are 2, 3, 5, 7, 11, 13, 17, 19. It is worth memorizing these numbers, or else knowing that prime numbers have only factors of 1 and themselves. Next, we look to see which numbers we can quickly rule out. We can rule out 31 and 37 because they are prime (since these numbers are prime, the product is by definition not made up of two different prime numbers, since 1 is not prime). 36 has factors of 3 and 6, so we can rule this out also. 32 has factors of 2 and 4, so we can rule it out as well. What we are left with are 33, 34, 35, 38, and 39. Testing these, we can see that $33 = 3 \times 11$ (both of which are prime), $34 = 2 \times 17$ (both of which are prime), $35 = 5 \times 7$ (both of which are prime), $38 = 2 \times 19$ (both of which are prime) and $39 = 3 \times 13$ (both of which are prime).
2. A. In terms of $5y$, $10y = 2(5y)$. So, the greatest common factor between $5y^2$ and $10y$ is $5y$.
3. A. Since we don't know the value of b^2 , the greatest common factor of $12b^2$ and 18 must also be the GCF of 12 and 18, which is 6. (The factors of 12 are 1, 2, 3, 4, 6, and 12. The factors of 18 are 1, 2, 3, 6, 9, and 18. The greatest factor they have in common is 6.)
4. B. You can break apart the expressions to find the GCF. The GCF of 9 and 15 is 3. The GCF of a^3 and a^4 is a^3 . (To find the common factor of like variables with exponents, the least exponent is the common factor. If you write out the multiplication of a^3 , you get $a \times a \times a$. If you write out the multiplication of a^4 , you get $a \times a \times a \times a$. Count the number of times a appears in both expressions (3). So, a^3 is the common factor.) So, the GCF of the expressions is $3a^3$.
5. A. The GCF of 2, 4, and 6 is 2. The GCF of x^4 , x^3 , and $6x^2$ is x^2 . Thus, the GCF of all 3 expressions is $2x^2$.
6. A. The GCF of 36, 18, and 30 is 6. The GCF of m^4 , m^8 , and m^6 is m^4 . So, the GCF of all 3 expressions is $6m^4$.
7. D. 14 is a factor of 28, but not a factor of 1 (in $1p^8$). There is no p in the term 14. So, there is no common factors in all three terms except for 1.
8. A. $5ab^2$ and a^2b have only a and b in common, so ab is the GCF.
9. D. 4 and 5 have no common factors save for 1, so the common factor will be found in the variables. xy^4 and x^2y^2 have a common factor of xy^2 .

10. A. a is not a factor of $40b^2$, so it is not part of the GCF. 40 and 25 have a GCF of 5. b^2 and b have a GCF of b . So, the GCF of both expressions is $5b$.
11. C. Breaking down the expressions to compare like terms can help us solve. The GCF of 81, 45, and 63 is 9. The GCF of c^5 , c^3 , and c^4 is c^3 . The GCF of d^3 , d^8 , d^2 is d^2 . So, the GCF of all three expressions is $9c^3d^2$.
12. D. 2, 3, and 5 have no common factors except 1. None of the variables appear in all 3 terms, so x , y , and z are not factors of the GCF of all three expressions. Therefore, the only common factor of all 3 expressions is 1.
13. A. If a and b are common factors of 10 and 15, respectively, the smallest value that ab must be a factor of is 150. This is greater than 30, so the value in Column A is greater.
14. D. If x and y are factors of c and d , respectively, then we can draw conclusions about how xy and cd relate to each other, but since we don't know the values of any of the variables, we don't have enough information to determine how the values of cx and dy relate to each other.
15. D. The factors of 6 are 1, 2, 3, and 6. The factors of 7 are 1 and 7. The value of km could be as low as $(1)(1) = 1$, or as high as $(6)(7) = 42$. This means that Column A is either equal to or less than Column B. This makes the answer D, as the relationship cannot be determined.
16. C. The prime factors of 14 are 2 and 7, so p could be 2 or 7. The prime factors of 20 are 2 and 5, so q could be 2 or 5. The largest value of pq is $7 \times 5 = 35$. This means the value of the columns are equal.

Multiples

1. D. To find the least common multiple, first find the prime factors of each number. $16y^3 = 2 \times 2 \times 2 \times 2 \times y \times y \times y$. $24y^4 = 2 \times 2 \times 2 \times 3 \times y \times y \times y \times y$. Multiply each factor the greatest number of times it occurs: 2 occurs 4 times in $16y^3$, 3 occurs once in $24y^4$, and y occurs 4 times in $24y^4$, so the LCM is equal to $2 \times 2 \times 2 \times 2 \times 3 \times y \times y \times y \times y = 2^4 \times 3 \times y^4 = 48y^4$.
2. D. You can consider the coefficient and the variables separately using prime factorization (breaking down a number into the prime numbers that are multiplied together to make it). $6 = 2 \times 3$; $10 = 2 \times 5$; $12 = 2 \times 2 \times 3$. The coefficient of the LCM is $2 \times 2 \times 3 \times 5 = 60$ (see previous question for why). The exponent of the variable is the greatest of those listed, or z^6 . So, the LCM of these three terms is $60z^6$.
3. A. Since $9a^6$ is a factor of $81a^{12}$ (in fact, it's the square root), the latter is the least common multiple.
4. A. Consider the variables and coefficients separately. $12 = 2 \times 2 \times 3$. $16 = 2 \times 2 \times 2 \times 2$. Remember that, to find the LCM, find the greatest number of times each prime factor occurs, then multiply the factor the number of times it occurs. So, the LCM is equal to $2^4 \times 3 \times x^3 \times y^3 = 48x^3y^3$.
5. A. Consider the variables and coefficients separately. $40 = 2 \times 2 \times 2 \times 5$. $24 = 2 \times 2 \times 2 \times 3$. So, the LCM is equal to $2^3 \times 5 \times 3 \times a^5 \times b^2 = 120a^5b^2$.
6. B. $6 = 2 \times 3$. $8 = 2 \times 2 \times 2$. $4 = 2 \times 2$. So, the LCM is equal to $2^3 \times 3 \times c^2 \times d^3 = 24c^2d^3$.
7. A. $8 = 2 \times 2 \times 2$. $10 = 2 \times 5$. $12 = 2 \times 2 \times 3$. So, the LCM is equal to $2^3 \times 5 \times 3 \times m^4 \times n^8 = 120m^4n^8$.
8. B. Since 7 is prime (its factors are 1 and itself), we only have to find the prime factors of the non-prime coefficients. $28 = 2 \times 2 \times 7$ and $14 = 2 \times 7$. So, the LCM = $2^2 \times 7 \times p^7 \times q = 28p^7q$.

Exponents

1. D. An exponent indicates the number of times a number is multiplied by itself. So, $3^4 = 3 \times 3 \times 3 \times 3 = 81$.
2. C. $5^3 = 5 \times 5 \times 5 = 125$.
3. C. $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$.
4. B. Expanded form would show the multiplication needed to find 4^3 . Since the exponent indicates the number of times a number is multiplied, the expanded form is $4 \times 4 \times 4$.
5. B. The exponent indicates the number of times a number is multiplied. So, 8^4 is $8 \times 8 \times 8 \times 8$ in expanded form.
6. C. The question is asking to find 36^2 in terms of 6, meaning the correct answer will be the number 6 with an exponent. 36 is 6 multiplied twice (6^2), so 36^2 would be equal to 6 multiplied 4 times, or 6^4 .

Notice that when we raise an exponential term to another power, we multiply the two exponents together. In this case, $36^2 = 6^2 \times 6^2 = (6^2)^2 = 6^4$.

7. C. 25 is equal to 5×5 . So, 25^4 is equal to (5×5) 4 times, or $5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5 \times 5$, or 5^8 . Another way to write this would be $(5^2)^4 = 5^8$ (where the exponents 2 and 4 multiply to give us the exponent 8).
8. B. $27 = 3 \times 3 \times 3$, so $27^3 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$, or 3^9 . Or, $(3^3)^3 = 3^9$ (since the exponents 3 and 3 multiply to give us the exponent 9).
9. C. Since factors cancel each other out, this expression can be simplified by subtracting the exponents from each other (this only works when the base is the same). This is the opposite operation as was performed in questions 7-9. The simplified expression is then $(5^1)(2^2) = 5 \times 4 = 20$.
10. D. This question shows an exponential relationship. The first chicken represents 1 in the expression. The 4 resulting chickens each laid 4 eggs, or 4×4 , or 4^2 . Then each of those chickens laid 4 more eggs, or $4^2 \times 4$, or 4^3 . So, the expression that shows how many chickens are now in the henhouse is $1 + 4 + 4^2 + 4^3$.
11. A. $2^6 = 2 \times 2 \times 2 \times 2 \times 2 \times 2$. $8 = 2 \times 2 \times 2$. So, 8 can be multiplied by itself twice (8^2) to be equal to 2^6 .
12. C. The question asks for the number of people who were given a piece of candy. Joey was not given a piece of candy. He gave candy to 2 people, so our expression must start with a 2. We can eliminate every other answer choice because all other expressions begin with 1.
13. A. A quick way to find the answer is to find the value of one number in terms of the other. 36^2 is equal to 6^4 in terms of 6. This is because $36 = 6^2$, so $36^2 = (6^2)^2 = 6^4$. Therefore, 6^6 is the greater value.
14. C. The expression in Column A simplifies to $\frac{6^3}{8^2} = \frac{216}{64} = \frac{27}{8}$. So, both columns are equal.
15. B. If the number of flies doubles each day, then the pattern of fruit flies multiplying would be $2 + 2^2 + 2^3 + 2^4 + 2^5 + 2^6 + 2^7 + 2^8 + 2^9 + 2^{10}$. So, the number of fruit flies on Day 10 is 2^{10} . Thus, the number in Column B is greater by 2.

Negative Exponents

1. B. A negative exponent indicates that the base value should be flipped to the other side of the fraction line, and the exponent changes signs. So, the value of 7^{-1} (which can be written as $\frac{7^{-1}}{1}$ is $\frac{1}{7^1}$, or simply $\frac{1}{7}$
2. D. Since a negative exponent indicates the number should be flipped to the other side of the fraction line, 6^{-2} is equal to $\frac{1}{6^2} = \frac{1}{36}$.
3. C. $3^{-4} = \frac{1}{3^4} = \frac{1}{81}$
4. D. A 0 in the denominator is not defined and cannot exist. 0^{-1} would be $\frac{1}{0^1}$, which does not exist.
5. D. The negative exponent indicates that the numbers should flip over the fraction line. So, this expression is equal to $(2)^2 = 4$.
6. D. The negative exponent indicates that the numbers should flip over the fraction line. So, this expression is equal to $\left(\frac{3}{2}\right)^3 = \frac{27}{8}$.
7. D. This expression is equal to $\left(\frac{4}{5}\right)^3 = \frac{64}{125}$.

8. B. Since a negative exponent would flip the number over the fraction line, this expression is equal to 125^{-1} . Since that is not an option, we should find an equivalent number. $125 = 5 \times 5 \times 5$, or 5^3 so the expression is also equal to 5^{-3} .
9. B. $32 = 2^5$, and a negative exponent would flip the number over the fraction line, so this expression is equal to 2^{-5} .
10. C. The negative exponent flips the number over the fraction line, so III is equivalent. For multiples of 10, the exponent is equal to the number of zeroes, so 1,000 also equals 10^3 , and I is also equivalent. Thus, only I and III are true; II is not because $100^2 = 10,000$.
11. D. The negative exponent flips the number over the fraction line, so III is equivalent. $16 = 2^4 = 4^2$. So, I, II, and III are equivalent.
12. D. The negative exponent flips the number over the fraction line, so this expression is equal to $\frac{1}{x^5}$.
13. C. The negative exponent flips the number over the fraction line, so this expression is equal to $\frac{1}{m^6}$.
14. B. Only the base with the negative exponent would flip over the fraction line, so this expression is equivalent to $\frac{x^2}{y^3}$.
15. A. Only the base with the negative exponent would flip over the fraction line, so this expression is equivalent to $\frac{12b^2}{a^6c^4}$.
16. B. Only the base with the negative exponent would flip over the fraction line, so this expression is equivalent to $\frac{1}{a^2c^4}$.
17. A. Only the base with the negative exponent would flip over the fraction line, so this expression is equivalent to $\frac{4}{m^6n^5}$.
18. B. Only the base with the negative exponent would flip over the fraction line, so this expression is equivalent to $\frac{p^4}{q^4r^2s}$.
19. C. Only the base with the negative exponent would flip over the fraction line, so this expression is equivalent to $\frac{w^3x^2y^2z}{6}$.
20. C. Only the base with the negative exponent would flip over the fraction line, so this expression is equivalent to $\frac{1}{w^3x^2y^2z}$.
21. B. Only the base with the negative exponent would flip over the fraction line, so this expression is equivalent to $\frac{2xy^2z}{3x^2}$.
22. D. Only the base with the negative exponent would flip over the fraction line, so this expression is equivalent to $\frac{9d^4f^9w^8}{3e^2uv^9}$. Since factors can cancel out, and $9 \div 3 = 3$, this expression is equal to $3\frac{d^4f^9w^8}{e^2uv^9}$.

23. A. The negative exponent indicates that the base number will flip over the fraction line. So, the number is a fraction but still a positive number. Since the number in Column B is negative, the number in Column A is greater.
24. B. In Column B, since the numerator is greater than the denominator, the number is greater than 1. In Column B, since the negative exponent indicates the numbers will be flipped over the fraction line, the numerator will be less than the denominator, and thus less than 1. Therefore, the value greater than 1 in Column B is greater.
25. D. We don't know the value of y . If y is a whole number, Column B will be greater, since Column A would then be a fraction. But if y is a fraction, Column A would be greater. Therefore, we don't have enough information to determine which is greater.
26. C. Remember that only the base with the negative exponent would flip over the fraction line. So, the expression in Column A is equal to $\frac{b^4 cz^8}{12a^6 x^2 y}$. So, the columns are equal.
27. A. Since the negative exponent flips the numerator and denominator over the fraction line, the fraction in Column A has a numerator greater than the denominator, meaning the fraction is greater than 1. Thus, the value in Column A is greater.
28. D. If the negative exponents are applied, the expression in Column A is equal to y^2 and the expression in Column B is equal to $\frac{1}{y^2}$. However, since we don't know whether y is a whole number or a fraction, we don't have enough information to determine which column is greater.

Fractional Exponents

1. B. A fractional exponent tells you to find the root. An exponent of $\frac{1}{2}$ tells you to find the square root.
 So $100^{\frac{1}{2}} = \sqrt{100} = 10$.
2. C. $8^{\frac{1}{3}}$ is the cubed root of 8, or $\sqrt[3]{8} = 2$. ($2^3 = 2 \times 2 \times 2 = 8$. The cubed root is the reverse operation. $8 = 2 \times 2 \times 2$, so the cubed root of 8 is 2.)
3. D. $125^{\frac{1}{3}} = \sqrt[3]{125} = 5$. ($125 = 5 \times 5 \times 5$)
4. B. The fractional exponent indicates we need to find the root. You can find the square roots of the numerator and denominator separately. So, $\left(\frac{1}{9}\right)^{\frac{1}{2}} = \sqrt{\frac{1}{9}} = \frac{\sqrt{1}}{\sqrt{9}} = \frac{1}{3}$.
5. B. It may be helpful to separate the numerator and denominator and find the square root of each. This expression can be rewritten as $\frac{\sqrt{81}}{\sqrt{49}} = \frac{9}{7}$.
6. A. You can find the cubed root of each the numerator and denominator. This expression can be rewritten as $\frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{3}{4}$.
7. B. The negative sign in the exponent indicates the number should be flipped over the fraction line. So, this expression can be rewritten as $\frac{1}{\sqrt{16}} = \frac{1}{4}$.
8. C. The negative sign in the exponent indicates the number should be flipped over the fraction line. So, this expression can be rewritten as $\frac{1}{\sqrt{100}} = \frac{1}{10}$.
9. B. This expression can be rewritten as $\frac{1}{\sqrt[3]{8}} = \frac{1}{2}$.
10. A. Since the negative exponent indicates a flip over the fraction line, this expression can be rewritten as $\sqrt{9} = 3$
11. A. Flip over the fraction line and find the square root. This expression can be rewritten as $\frac{\sqrt{49}}{\sqrt{81}} = \frac{7}{9}$.

12. A. Flip over the fraction line and find the cubed root. This expression can be rewritten as $\frac{\sqrt[3]{64}}{\sqrt[3]{27}} = \frac{4}{3}$.
13. B. When comparing these expressions, we know that both are negative and both will require us to find the square roots. The square root of 9 is 3 and the square root of 16 is 4; $-\frac{1}{3} < -\frac{1}{4}$, so the amount in Column B is greater.
14. B. The negative exponent in Column B indicates the number should be flipped over the fraction line. So, the value of Column A is $\frac{1}{\sqrt[3]{8}} = \frac{1}{2}$ and the value of Column B is $\sqrt[3]{8} = 2$. So, the value in Column B is greater.
15. A. Fractional exponents indicate we must find the root. So, the value of Column A is $\sqrt{9} = 3$. The value of Column B is $\frac{1}{\sqrt{9}} = \frac{1}{3}$. Thus, the value in Column A is greater.
16. A. An easy way to consider these values is to see that the value in Column A will be a whole number, and the value in Column B will be a fraction less than one. So, the value in Column A will be greater.
17. D. Because x is a variable and we don't know if it is a whole number, fraction, or negative number, we don't have enough information to determine whether Column A or Column B is greater.
18. D. We know that y is a positive number, but that positive number could still be a fraction, so we do not have enough information to determine which value is greater.
19. A. We know that z is greater than 1, so we can determine that the square root of z is greater than the square root of $\frac{1}{z}$ because the former will be greater than 1 and the latter will be less than 1.
20. B. These expressions can be rewritten as $\frac{2}{3} < \frac{3}{2}$ (the value in Column A is less than 1 and the value in Column B is greater than 1). So, the value in Column B is greater.

Roots of Numbers

1. B. This is a simple square root question because 4 is a perfect square (a number that has an integer as its root). $\sqrt{4} = 2$ $2 \times 2 = 4$.
2. A. 121 is a perfect square. $\sqrt{121} = 11$. ($11 \times 11 = 121$)
3. B. 225 is a perfect square. $\sqrt{4} = 2$
4. B. Perform the operation in the radical first. $25 - 9 = 16$, which is a perfect square. $\sqrt{16} = 4$.
5. B. $100 - 36 = 64$, which is a perfect square. $\sqrt{64} = 8$.
6. A. $144 + 25 = 169$, which is a perfect square. $\sqrt{169} = 13$.
7. A. Simplify the expression. $\sqrt{25} - (\sqrt{16} + \sqrt{9}) = 5 - (4 + 3) = 5 - 7 = -2$.
8. A. A rational number is one that can be expressed as a whole number or fraction. Irrational numbers can be expressed as a decimal that neither terminates nor repeats. Do the operation in each radical to find out if any of these are perfect squares, and keep in mind that $\sqrt{-1}$ is irrational. Only $\sqrt{64 + 36}$ is irrational because $64 + 36 = 100$, which is a perfect square.
9. C. Since both the numerator and denominator are perfect squares, this expression can be rewritten as $\sqrt{\frac{1}{4}} = \frac{\sqrt{1}}{\sqrt{4}} = \frac{1}{2}$.
10. D. Since both the numerator and denominator are perfect squares, this expression can be rewritten as $\frac{\sqrt{9}}{\sqrt{25}} = \frac{3}{5}$.

11. C. Since both the numerator and denominator are perfect squares, this expression can be rewritten as $\frac{\sqrt{144}}{\sqrt{81}} = \frac{12}{9} = \frac{4}{3}$.
12. B. 27 is not a perfect square. In order to find the value, we can factor out a number that is a perfect square. Since $27 = 9 \times 3$, and 9 is a perfect square, we can find the square root of 9 and put it outside the radical. So $\sqrt{27} = \sqrt{9 \times 3} = 3\sqrt{3}$.
13. B. 48 is not a perfect square. However, 16 is a perfect square and a factor of 48. So, $\sqrt{48} = \sqrt{16 \times 3} = 4\sqrt{3}$.
14. B. 75 is not a perfect square. Factor out a perfect square. $\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$.
15. B. Each term of the expression has a perfect square that can be factored out. So, $\sqrt{50} + \sqrt{18} = \sqrt{25 \times 2} + \sqrt{9 \times 2}$. This simplifies to $5\sqrt{2} + 3\sqrt{2}$. In addition, since both terms have a common factor (which is $\sqrt{2}$), the numbers outside the radicals can be added together. Since $5 + 3 = 8$, so the answer is $8\sqrt{2}$.
16. A. It may be helpful to look for factors that result in both terms of the expression having a common radical. $\sqrt{108} = \sqrt{36 \times 3} = 6\sqrt{3}$. $\sqrt{75} = \sqrt{25 \times 3} = 5\sqrt{3}$. $6\sqrt{3} - 5\sqrt{3} = \sqrt{3}$.
17. C. Factor out perfect squares. $\sqrt{32} = \sqrt{16 \times 2} = 4\sqrt{2}$; $\sqrt{98} = \sqrt{49 \times 2} = 7\sqrt{2}$. Thus, $4\sqrt{2} + 7\sqrt{2} = 11\sqrt{2}$.
18. B. Factor out the perfect squares first, then combine like terms. This expression can be rewritten as $4\sqrt{5} + 2\sqrt{5} + 3\sqrt{5} = 9\sqrt{5}$.
19. D. The terms in this expression do not have perfect square factors in common. This expression can be rewritten as $3\sqrt{11} + 2\sqrt{11} + 6\sqrt{2} = 5\sqrt{11} + 6\sqrt{2}$.
20. C. This expression can be rewritten as $5\sqrt{10} - 4\sqrt{5} - 3\sqrt{10} = 2\sqrt{10} - 4\sqrt{5}$.
21. A. This expression can be rewritten as $4\sqrt{3} - 2\sqrt{2} + 9\sqrt{2} = 4\sqrt{3} + 7\sqrt{2}$.
22. A. Since $\sqrt{8} = 2\sqrt{2}$, multiplying this by $\sqrt{2}$ effectively squares the $\sqrt{2}$. So, this expression is equal to $2 \times (\sqrt{2} \times \sqrt{2})$ or $2 \times 2 = 4$.
23. B. Multiplying a number by itself is equal to squaring the number. If you square a square root, the result is the number inside the radical. So, this expression is equal to 7.
24. A. This expression can be rewritten as $2\sqrt{5} \times \sqrt{5} = 2(5) = 10$.
25. B. This expression can be rewritten as $\sqrt{15 \times 5} = \sqrt{75} = 5\sqrt{3}$.
26. D. This expression can be rewritten as $\sqrt{20 \times 80} = \sqrt{1600} = 40$.
27. A. This expression can be rewritten as $\sqrt{80 \div 2} = \sqrt{40} = \sqrt{4 \times 10} = 2\sqrt{10}$.
28. A. This expression can be rewritten as $\frac{1}{3} \times (\sqrt{27 \div 3}) = \frac{1}{3} \times (\sqrt{9}) = \frac{1}{3} \times 3 = 1$.

Roots of Variables

1. A. If x is a positive integer and \sqrt{x} is also an integer, x must be a perfect square. The only choice for which this is true is 1.
2. B. If x is a positive integer, in order for $\sqrt{\frac{x}{2}}$ to also be an integer, $x \div 2$ must be a perfect square. Of the choices, this is only true for 18. ($18 \div 2 = 9$). $\sqrt{9} = 3$.
3. C. If x is a positive integer, then the value of $\frac{3x}{2}$ must be a perfect square. Of the choices, the only value for which this is true is 24: $\frac{3(24)}{2} = \frac{72}{2} = 36$ and $\sqrt{36} = 6$.
4. A. The square root and the square cancel each other out (since you'd square the number and then find the square root). So, $\sqrt{x^2} = x$.

5. C. Since we know a square and a square root cancel each other out, in order to find the value of this expression, we must divide the exponent by 2. (This expression is equal to $(x^{16})^{\frac{1}{2}}$. So, the correct answer is x^8 .)
6. B. This expression is equal to $(x^{100})^{\frac{1}{2}}$ or x^{50} .
7. A. We can take apart the expression to find the value. The square root of 9 is 3 and the square root of x^2 is x so the value of this expression is $3x$.
8. B. We can take apart the expression to find the value. The square root of 36 is 6 and the square root of x^{18} is x^9 , so the value is $6x^9$.
9. D. We can take apart the expression to find the value. The square root of 25 is 5 and the square root of x^{50} is x^{25} . So, the value of the expression is $5x^{25}$.
10. B. The square root of 64 is 8 and the square root of x^{128} is x^{64} , so the value of the expression is $8x^{64}$.
11. B. To solve, find the square root of the coefficient and divide the variable by 2 (since the square root of x is equal to $x^{\frac{1}{2}}$). So, the value of this expression is $4x^8$.
12. B. Since the exponent cannot be divided by 2, factor out the part of the expression that is a perfect square. $\sqrt{x^3} = \sqrt{x^2 \times x} = x\sqrt{x}$.
13. B. Since the exponent cannot be divided by 2, factor out the part of the expression that is a perfect square. This expression can be rewritten as $x^3\sqrt{x}$.
14. C. Since the exponent cannot be divided by 2, factor out the part of the expression that is a perfect square. This expression can be rewritten as $x^7\sqrt{x}$.
15. C. To solve, find the square root of the coefficient and divide the variable by 2 (since the square root of x is equal to $x^{\frac{1}{2}}$). Since, this variable cannot be divided by 2, factor out the perfect square. So, this expression can be rewritten as $7x^4\sqrt{x}$.
16. B. Neither the coefficient or the variable are perfect squares, so factor out the squares. We know that $12 = 4 \times 3$, and that 4 is a perfect square. If we factor this out, we are left with $\sqrt{4} \times \sqrt{3}$ or $2\sqrt{3}$.
Similarly, we know that x^{13} is simply $(x^{12})(x)$. If we square both terms, we arrive at $\sqrt{x^{12}} \times \sqrt{x}$, or $x^6\sqrt{x}$. This expression can be rewritten as $2x^6\sqrt{3x}$.
17. A. Neither the coefficient or the variable are perfect squares. Following a similar process as above, we factor out the squares. This expression can be rewritten as $3x\sqrt{5x}$.
18. A. y^4 is a perfect square and so can be factored out of the radical. x^3 is not, but x^2 can be factored out. So, this expression is equal to $xy^2\sqrt{x}$.
19. D. Factor out the parts of the expression that are perfect squares. This expression can be rewritten as $a^3b^2\sqrt{b}$.

Imaginary Numbers

1. D. The square root of -1 cannot be calculated (because no two negatives multiplied together can give us a negative number), so it is referred to as an imaginary number, or i . Since there is a negative sign under the square root with the number, the answer is the positive value of the square root multiplied by i . So, the value of $\sqrt{-25} = \sqrt{25}\sqrt{-1}$ or $5i$.
2. D. The square root of 196 is 14, so the value of this expression is $14i$.
3. B. The square and the square root cancel each other out, so since $i = \sqrt{-1}$, $i^2 = -1$.
4. D. $i^2 = -1$, so $i^3 = -1(\sqrt{-1})$.
5. D. We can find the solution set, or set of possible values for x that make the equation true, for $x^2 = -100$ by solving for x . Remember that the square root of -1 is equal to i . $x = \sqrt{-100} = \pm 10i$.
6. D. Solve for x . If $x^2 + 144 = 0$, $x^2 = -144$, and $x = \pm 12i$.

7. B. Evaluate each square root individually, then multiply. The square root of -9 gives $3i$, and the square root of -4 gives $2i$. Take the product of $3i$ and $2i$ to get $6i^2$. Since $i^2 = -1$, the correct answer is -6 .
8. B. This expression can be simplified to $\sqrt{50 \times -2} = \sqrt{-100}$ or $10i$.

Percents

1. B. 10% of 20 is equal to $20 \times 0.1 = 2$. So if the length increases by 10%, it is 2 meters greater, or 22 meters, and if the width decreases, it is 2 meters less, or 18 meters. The new area is $22 \times 18 = 396$ square meters. Alternatively, a 10% increase can be calculated by multiplying the base number by 1.1 (since $1.1 \times 100\% = 110\%$). A 10% decrease can be calculated by multiplying the base number by 0.9 (since $0.9 \times 100\% = 90\%$).
2. C. The area of this rectangle is $50 \times 40 = 2,000$ square feet. If the length decreases by 20%, it decreases by 50×0.2 , or 10. If the width increases by 20%, it increases by 40×0.2 , or 8. So, the new length is $50 - 10 = 40$ and the new width is $40 + 8 = 48$. The new area is $40 \times 48 = 1,920$ square feet. The second rectangle has an area 80 square feet less than the first rectangle. To find the percent decrease, divide the difference by the original size: $80 \div 2,000 = 0.04$, or 4%.
3. B. To find 20% off \$30, we can either decrease 20% of the total price, or multiply the price by the remaining percent. In this case, 80% of \$30 is equal to $0.8 \times 30 = \$24$. If this price is then marked back up 30%, then the new price is equal to $24 \times 130\%$ or $24 \times 1.3 = \$31.20$. This is \$1.20 greater than the original price, so the percent increase is equal to $1.2 \div 30 = 0.4 = 4\%$.
4. A. Let x represent the unknown number. If it is decreased by 20%, we can express this as $x \times 0.8$, or $x(1 - 0.2)$. If this is then decreased by another 20%, we are not taking 20% off of x , but off of $x \times 0.8$. So, we could express this as $x \times 0.8 \times 0.8 = x \times 0.64$. $0.64x$ is $0.36x$ less, or 36% less.
5. B. Let x represent the unknown number. A 30% increase can be expressed as $1.3x$. A further 30% increase could be expressed as $1.3 \times 1.3 \times x$, or $1.69x$. This is a 69% increase from the original, x .
6. C. If Juan slept 6 hours on Thursday night, he slept 1.5 times as many hours on Friday (50% more = $1 + 0.5$ times more), or $6 \times 1.5 = 9$ hours.
7. B. If John ran 150% of 8 miles, he ran $1.5 \times 8 = 12$ miles.
8. A. If Kevin watched 2 hours of TV on Saturday and 100% of that number on Sunday, then he also watched 2 hours of TV on Sunday. (100% is the same as the original number.) The question does not say "100% more".
9. A. If the price of the shirt is x , and it goes up 20% one month ($1.2x$) and then decreases 20% ($1.2x \times 0.8 = 0.96x$), then the decrease from the original price is 4% ($x - 0.96x = 0.04x$). Thus, the value in Column A is greater.
10. B. The length is equal to 130% of the original and the width is equal to 80% of the original, so the area is $130\% \times 80\% = 104\%$ of the original, or increases by 4%. This is less than the value in Column B.
11. B. The price of the book after the decrease is 80% of \$25, or $0.8 \times 25 = 20$. If the price then increase by 20%, it is equal to $20 \times 1.2 = \$24$. Thus, the value in Column B is greater.
12. A. The first year she owns the novel, it will increase to $(100)(1.2) = \$120$. The second year, it will increase another 20% to $(120)(1.2) = \$144$. She will earn a profit of $144 - 100 = \$44$ if she sells it after 2 years. The value in Column A is greater.

Percent Change

1. C. The surefire way of calculating percent change is to use the formula: $\frac{\text{new} - \text{old}}{\text{old}}$. In this case the "new" value is 180, and the "old" value is 90. So, we have $\frac{180 - 90}{90} = \frac{90}{90} = 1$, or 100%. Whenever something doubles, it increases by 100%.

2. B. The number of pizzas is halved, so it decreases by 50%, or half of the original. Using our formula:
$$\frac{90 - 180}{180} = -\frac{90}{180} = -\frac{1}{2}, \text{ or } 50\%.$$
3. D. Flynn did $6 - 4 = 2$ more hours of community service this week. The percent increase is equal to the difference (new – old) divided by the original (old). $2 \div 4 = 0.5$, or 50%.
4. C. The percent change is equal to the difference divided by the original. $750 - 600 = 150$. So, $150 \div 600 = 0.25$, or a 25% increase.
5. C. The percent change is equal to the difference divided by the original. $750 - 600 = 150$. So, $150 \div 650 = 0.25$, or a 25% decrease.
6. C. The percent change is equal to the difference divided by the original. $600,000 - 420,000 = 180,000$, which is the difference in price. $180,000 \div 600,000 = 0.3$, or a 30% decrease.
7. D. The percent change is equal to the difference divided by the original. The value in 2010 was \$420,000. In 2015, it has increased by $630,000 - 420,000 = 210,000$. $210,000 \div 420,000 = 0.5$, or a 50% increase.
8. A. The percent change is equal to the difference divided by the original. The price in 2006 is 600,000 and in 2015 is 630,000, which is an increase of 30,000. The percent increase is equal to $30,000 \div 600,000 = 0.05$, or a 5% increase.
9. B. The percent change is equal to the difference divided by the original. The difference in price is $6.00 - 2.40 = 3.60$. Find the percent change: $3.60 \div 2.40 = 1.5$, or a 150% increase. Therefore, the value in Column B is greater.
10. C. The percent change is equal to the difference divided by the original. The difference in price is $6.00 - 2.40 = 3.60$. Find the percent change: $3.60 \div 6.00 = 0.6$, or a 60% decrease. Therefore, the values in both columns are equal.
11. B. The percent change is equal to the difference divided by the original. The difference in price is $40 - 30 = 10$. Find the percent change: $10 \div 40 = 0.25$, or a 25% decrease. Therefore, the value in Column B is greater.
12. A. The percent change is equal to the difference divided by the original. The difference in price is $9,200 - 7,000 = 2,200$. Find the percent change: $2,200 \div 7,000 = 0.314$, or more than a 30% increase. Therefore, the value in Column A is greater.

Scientific Notation

1. D. When a number is written in scientific notation, an integer or a number expressed in decimal between 0 and 10 (exclusive) is multiplied by 10 raised to some exponent. The value of the exponent is the number of times the decimal in the integer or number should be moved “to the right” (if a positive exponent) or “to the left” (if a negative exponent). If there are no digits to the right of the decimal point (as with an integer), zeros are added to the number. So, in this case, 5 is the integer, and it is multiplied by 10^4 . Remember that 5 can be written as 5.0. The 4 in the exponent means that we should move the decimal point over “to the right” by 4 places, adding a zero each time. So, we end up with 50,000.
2. A. The decimal point moves when you multiply by powers of 10. The exponent in this expression is 6, which means we must move the decimal point over to the right by 6 places. If we move the decimal point over to the right by 2 places from 8.52, we get 852. Another 4 places gives us 8,520,000.
3. D. The negative exponent indicates that the decimal point moves to the left when converting to standard form. So, 5×10^{-3} is equal to 0.005 because the decimal moves 3 places to the left from 5.0.
4. B. The decimal point moves 7 places to the left in this equation. So, $3.695 \times 10^{-7} = 0.0000003695$.
5. D. We express a number written in standard form in scientific notation by finding an integer or number expressed in decimal form and multiplying it by 10 raised to some power. In this case, we can see that an integer would be 9.0. Then, we count to see how many decimal places to the right or left the number in standard format is. In the case of 90,000.0, we see that the decimal is 4 places to the right of 9.0. This means the value of the expression in scientific notation is 9×10^4 .

6. A. As shown previously, find a number expressed in decimal form. In this case, we must take every non-zero number. We could use 5.28 and see that 5,280,000.0 is 6 decimal places to the right. This means 5.28×10^6 is the way to express this value in scientific notation. We could use 52.8×10^5 , but scientific notation seeks to use numbers expressed in decimal form between 0 and 10, exclusive.
7. A. Be careful to preserve numbers in their proper place value – the 0 in 2088 would not move or be eliminated. So, the number expressed in decimal form would be 2.088. Compared with 0.0002088, we see that we would need to move the decimal point over to the left by 4 places, meaning our exponent will be negative. Thus, 2.088×10^{-4} .
8. C. It may help to find the value of each term of the expression in standard form first. This expression is equal to $50,000 + 6,000 = 56,000$. In scientific notation, this is 5.6×10^4 .
9. A. It may help to find the value of each term of the expression in standard form first. This expression is equal to $74,100,000 + 14,700 = 74,114,700$, or 7.41147×10^7 .
10. A. As shown previously, when multiplying multiples of ten, we can add the exponents. When multiplying numbers written in scientific notation, then, we can multiply the coefficients and add the exponents. $3 \times 2 = 6$ and $3 + 2 = 5$, so the product can be written as 6.0×10^5 .
11. A. As shown previously, when multiplying multiples of ten, we can add the exponents. When multiplying numbers written in scientific notation, then, we can multiply the coefficients and add the exponents. $5 \times 6 = 30$ and $4 + 4 = 8$, so the result would be 30×10^8 . However, in scientific notation, the coefficient must be represented by an integer between 0 and 10, so we can move the decimal point one place to the left and add 1 to the exponent, resulting in 3.0×10^9 .
12. C. As shown previously, we can solve this question by multiplying the coefficients and adding the exponents. $5 \times 5 = 25$ and $-5 + -5 = -10$, so the result would be 25×10^{-10} . However, since, in scientific notation, the coefficient must be represented by an integer between 0 and 10, we can move the decimal point one place to the left and add 1 to the exponent, resulting in 2.5×10^{-9} .
13. D. To solve, we can follow the procedure of multiplying the coefficients and adding the exponents, then, if needed, shifting the decimal to express the product in scientific notation. $2.5 \times 2.4 = 6$ and $2 + 3 = 5$, so the product can be written as 6.0×10^5 .
14. C. Recall that a negative exponent means that the number should be flipped over the fraction line. In this case, we could express this number as $5 \times \frac{1}{10^6} = \frac{5}{10^6}$.
15. A. To multiply numbers written in scientific notation, we multiply the coefficients and add the exponents. Likewise, to divide, divide the coefficients and subtract the exponents. (In other words, we can factor out common multiples when dividing.) Since $9 \div 3 = 3$ and $6 - 4 = 2$, this value can be written as 3.0×10^2 .
16. B. When dividing numbers written in scientific notation, divide the coefficients and subtract the exponents. $4.8 \div 1.2 = 4$ and $6 - 4 = 2$, so this value can be written as 4.0×10^2 .
17. C. As with multiplication where we shift the decimal to show an integer between 0 and 10, we should shift the decimal for quotients less than 1. $3.6 \div 6.0 = 0.6$ and $8 - 4 = 4$, which would be 0.6×10^4 , but we need to shift the decimal and subtract 1 from the exponent, so this value can be written as 6.0×10^3 .

Vocabulary

1. D. 9 is a perfect square ($\sqrt{9} = 3$). Thus, $\sqrt{9}$ is a rational number. A rational number is any number that can be expressed as the ratio or fraction $\frac{p}{q}$ of two integers, where $q \neq 0$. Since q may be equal to 1, every integer is a rational number. Since the square root of 9 is an integer, it is also a rational number.
2. C. 19 is prime and is not a perfect square, so it cannot be represented as a ratio $\frac{p}{q}$. An irrational number is a real number that cannot be expressed as a ratio.

3. B. An imaginary number is defined as the square root of -1 and is usually written as i . Thus, $\sqrt{-1}$ is by definition an imaginary number.
4. A. A complex number is defined as $a + bi$ where a and b are real numbers and i is the imaginary number, $\sqrt{-1}$. So, $5 + 3i$ is a complex number.
5. C. Subtracting two positive integers will result in a difference that is also an integer, so it is a rational number.
6. D. An integer is a whole number, so this is not an integer. $\frac{2}{3}$ is a ratio, or $\frac{p}{q}$, of two integers where $q \neq 0$. In other words, it is a rational number.
7. C. The zero exponent rule states that any base number with an exponent of 0 is equal to 1. A counting number is any positive integer. 1 is a positive integer, so both Mary and Peter are correct.
8. C. Remember that i is the imaginary number, equal to $\sqrt{-1}$. Thus, $i^2 = (\sqrt{-1})^2 = -1$. Since -1 is an integer, it is also a rational number. Whole numbers are never negative.
9. B. The conjugate is an expression in which the sign is changed. So, the conjugate of $x + 4$ is $x - 4$. We can apply FOIL: $(x + 4)(x - 4) = x^2 - 4x + 4x - 16$, or $x^2 - 16$.
10. C. Although $20 - 10 = 10$, since Thomas is also reading page 10, he actually reads 11 pages. (He reads pages 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, and 20, which is 11 numbers.)
11. C. $952 - 751 = 201$, but since Daphne also reads page 751, we add 1 to the difference, so she reads 202 pages.
12. B. The sum of all integers from 1 to 5, inclusive, is z , so the sum of all integers from 3 to 5, inclusive, is equal to $z - 1 - 2$, or $z - 3$.
13. C. If the sum of all integers from 1 to 20, inclusive, is equal to x , then the sum of all integers from 1 to 18, inclusive, is equal to $x - 20 - 19$, or $x - 39$.
14. B. If the sum of all integers from 1 to 500, inclusive, is equal to a , then the sum of all integers from 1 to 499, inclusive, is equal to $a - 500$.
15. C. As we have seen, when calculating the difference between the sum of a series of consecutive integers and a smaller series of consecutive numbers, subtract the numbers that are not included in the smaller set. So, if the sum of all the integers from 1,000 to 10,000, inclusive, is y , then the sum of all the integers from 1,003 to 10,000 is equal to $y - 1,000 - 1,001 - 1,002 = y - 3,003$.
16. D. If the sum of all integers from 1 to 100, inclusive, is b , then the sum of all integers from 1 to 104, inclusive is equal to $b + 101 + 102 + 103 + 104 = b + 410$.
17. D. The product of $\sqrt{8}$ and $\sqrt{18}$ is $\sqrt{144}$ which reduces to 12. This is a rational number, a whole number, and an integer, but not an irrational number.
18. B. The value of i is $\sqrt{-1}$. Therefore, $i^2 = -1$. So, $i^2 \times i^2 = -1 \times -1 = 1$. This is a whole number.

Consecutive Integers

1. B. Consecutive numbers are integers in a sequence: 1, 2, 3, 4, for example. The consecutive number that follows a number x is $x + 1$. So, three consecutive numbers can be expressed as $x, x + 1, x + 2$.
2. D. Consecutive even numbers are multiples of 2: 2, 4, 6, 8, etc. The even number that comes in a consecutive sequence after a number y is $y + 2$. So, three consecutive even numbers can be expressed as $y, y + 2, y + 4$.
3. C. Consecutive odd numbers, like consecutive even numbers, increase by 2, but show odd numbers in a sequence: 1, 3, 5, 7, etc. The first number in a sequence of odd numbers is odd, so the consecutive numbers can be expressed the same way even numbers would be: $z, z + 2, z + 4$.
4. A. As we have seen, three consecutive even integers can be expressed as $x, x + 2$, and $x + 4$. The sum of these numbers is $x + x + 2 + x + 4$, or $3x + 6$. Thus, if the sum of three consecutive numbers is 54, we can express that as $3x + 6 = 54$.

5. A. As we have seen, the sum of four consecutive odd numbers is $x + x + 2 + x + 4 + x + 6$ or $4x + 12$. To find the smallest integer, or the first integer in the sequence, solve for x . Since we know the sum is 64, $4x + 12 = 64$. Thus: $4x = 52$ and $x = 13$.
6. D. As we have seen, two consecutive numbers can be represented by x and $x + 1$. So, the product would be $x(x + 1)$. Since we know the product is 30, this can be expressed as $x(x + 1) = 30$.
7. B. We know the product is 72, and the product of two consecutive numbers can be represented as $x(x + 1) = 72$. You can solve for x , although you can also recall that 8 and 9 are consecutive numbers, and $8 \times 9 = 72$. The larger number is 9.
8. A. The product of the 3 consecutive odd integers could be represented as $x(x + 2)(x + 4) = 105$. We could solve for x , but since the product is a multiple of 5, one of the integers must be a 5. We can test this: $3 \times 5 \times 7 = 105$.
9. B. The product of the 3 consecutive even integers could be represented as $x(x + 2)(x + 4) = 480$. We could solve for x , but since 480 is a multiple of 10, one of the integers must be a multiple of 5 or 10. $6 \times 8 \times 10 = 480$. The sum of these integers is $6 + 8 + 10 = 24$.
10. D. The sum can be represented as $x + x + 2 + x + 4 + x + 6 = 4x + 12 = 16$. Solving for x , or the first integer in the sequence, we get $x = 1$. So, the four integers are 1, 3, 5, and 7. The product is $1 \times 3 \times 5 \times 7 = 105$.
11. B. Remember that the average of a set of integers is equal to the sum of integers divided by the number of integers. In the case of 3 consecutive integers, the average will be the middle integer. If the average is 8, the three consecutive integers are 7, 8, and 9. (We can test this: $7 + 8 + 9 = 24$; $24 \div 3 = 8$.)
12. A. The sum can be expressed as $x + x + 1 + x + 2 + x + 3 = 4x + 6 = 54$. Therefore, x , or the smallest integer in the sequence, is equal to 12. The largest number would be 15, so the value in Column A is greater.
13. B. If the product is equal to 120, because the number ends in 0, one of the numbers in the sequence must be a multiple of 5 or 10. $4 \times 5 \times 6 = 120$. The largest integer is 6, so the value in Column B is greater.
14. B. As we have seen, the sum of 4 consecutive odd integers can be represented as $4x + 12$. In this case, $4x + 12 = 72$. The smallest integer can be found by solving for x ; $4x = 60$, and $x = 15$. Therefore, the value in Column B is greater.
15. C. The average of 7 consecutive numbers will be the middle term, so this sequence is 20, 21, 22, 23, 24, 25, 26. The smallest integer is 20, so the values in the columns are equal.

Matrices

1. B. To add two matrices, add the corresponding terms. In this case, $13 + 2 = 15$, $2 + -19 = -17$, $-3 + 41 = 38$, $17 + 67 = 84$. Expressed as a matrix, the sum is $\begin{bmatrix} 15 & -17 \\ 38 & 84 \end{bmatrix}$.
2. D. To subtract two matrices, subtract the corresponding terms. In this case, $1 - 9 = -8$, $5 - -5 = 10$, $9 - 1 = 8$, $-4 - -2 = -2$, $3 - 3 = 0$, $-2 - -4 = 2$. Expressed as a matrix, the difference is $\begin{bmatrix} -8 & 10 & 8 \\ -2 & 0 & 2 \end{bmatrix}$.
3. C. To add two matrices, add the corresponding terms: $4 + 4 = 8$, $2 + 2 = 4$, $-8 + -8 = -16$, $9 + 9 = 18$, $3 + 3 = 6$, $-5 + -5 = -10$. As a matrix: $\begin{bmatrix} 8 & 4 \\ -16 & 18 \\ 6 & -10 \end{bmatrix}$.
4. A. The 3 functions as a coefficient, so multiply every term in the matrix by 3. $3 \times -6 = -18$, $3 \times 0 = 0$, $3 \times 2 = 6$, $3 \times 1 = 3$, $3 \times -9 = -27$, $3 \times 8 = 24$, $3 \times 7 = 21$, $3 \times 0 = 0$, $3 \times -6 = -18$. Expressed as a matrix, this is: $\begin{bmatrix} -18 & 0 & 6 \\ 3 & -27 & 24 \\ 21 & 0 & -18 \end{bmatrix}$.

5. A. Multiply the coefficients (multiply every term in the matrix by the number that appears before it), then add the matrices (add corresponding terms). In this case, after multiplying, we can rewrite the matrices as $\begin{bmatrix} 20 & 4 \\ 12 & 28 \end{bmatrix} + \begin{bmatrix} 4 & 18 \\ 8 & 12 \end{bmatrix} = \begin{bmatrix} 24 & 22 \\ 20 & 40 \end{bmatrix}$.
6. C. To multiply matrices, the number of columns in the first must match the number of rows in the second. The product will have the same number of rows as the first and the same number of rows as the second. Therefore, the resulting matrix will be 2 rows and one column. To find the term in the first row, multiply the top row of the first matrix with the numbers in the second matrix as such: $(2 \times 6) + (-3 \times 7) = -9$. To find the term in the second row, multiply the bottom row of the first matrix with the numbers in the second matrix as such: $(4 \times 6) + (1 \times 7) = 31$. Expressed as a matrix, this is: $\begin{bmatrix} -9 \\ 31 \end{bmatrix}$
7. C. Subtracting matrices with variables follows the same procedure as subtracting matrices with integers. Subtract corresponding terms: $10a - 6a = 4a$, $a^2 - 5a^2 = -4a^2$, $c^3 - c^3 = 0$, $-b - 7b = -8b$. Expressed as a matrix, this is: $\begin{bmatrix} 4a & -4a^2 \\ 0 & -8b \end{bmatrix}$.
8. A. Multiplying matrices with variables follows the same procedure as multiplying matrices with integers. In this case, multiply every term in the matrix by the coefficient: $-3 \times 6 = -18$, $-3 \times 9x = -27x$, $-3 \times 2y = -6y$, $-3 \times -5 = 15$, $-3 \times -7x^2 = 21x^2$, $-3 \times 0 = 0$. Expressed as a matrix, this is $\begin{bmatrix} -18 & -27x \\ -6y & 15 \\ 21x^2 & 0 \end{bmatrix}$.
9. A. Multiply each term by the coefficient: $5x \times 2 = 10x$, $5x \times 2x = 10x^2$, $5x \times -7 = -35x$, $5x \times 1 = 5x$, $5x \times -9y = -45xy$, $5x \times 8x^2 = 40x^3$. Expressed as a matrix, this is $\begin{bmatrix} 10x & 10x^2 & -35x \\ 5x & -45xy & 40x^3 \end{bmatrix}$.
10. B. Multiply each term by the coefficient, then add the corresponding terms in the matrices. So, after multiplying, this equation can be rewritten as $\begin{bmatrix} 30x & -6x \\ 18x & 42x \end{bmatrix} + \begin{bmatrix} -2x & 9x \\ -3x & 6x \end{bmatrix} = \begin{bmatrix} 28x & 3x \\ 15x & 48x \end{bmatrix}$.

Algebraic Concepts

Algebraic Relationships

1. C. Remember that x represents a real number, and thus will be the same on both sides of the equation. If $3x = ax$, then a must equal 3.
2. C. In this equation, b represents a real number. There's a clear pattern on both sides of the equation. If $10c + 10 = bc + b$, then b must equal 10.
3. A. This equation is asking which number can be multiplied by another number and still be equal to the original number? Only 0 makes this equation true.
4. B. If $m + n = 8$, we can subtract m from both sides of the equation to isolate n . So, $n = 8 - m$.
5. B. $15x - 9 = 3(5x - 3)$. Since $5x - 3 = 4$, $15x - 9$ must be equal to 3×4 , or 12.
6. D. We can simplify the second part of the equation to $5y + 25$ by applying the distributive property. Then subtract one from each side: $x - 1 = 5y + 25 - 1 = 5y + 24$.
7. D. Apply the distributive property to simplify: $m(a + b) = 80 = ma + mb$. If $am = 20$ (and the associative property says $am = ma$), then we can substitute that in the equation: $20 + mb = 80$. If we subtract 20 from both sides, $mb = 60$.
8. D. We can determine without doing the arithmetic that $5^2 + 10^2$ is the greatest value, since we know $5 < 5^2$ and $10 < 10^2$.
9. D. If x and y are positive integers, we can conclude that $x < x^2$ and $y < y^2$, so $x^2 + y^2$ is the expression with the greatest value.

10. B. If x and y are positive integers, and x is greater than y , then $y - x$ will result in a negative integer. So, this is the expression with the smallest value. Substitute values (for instance, $x = 2$ and $y = 1$) to check.
11. C. Based in the information given in the question, we can conclude that $x < xy < x^2$. Since adding will yield a greater value than subtracting, the expression with the greatest value is $x^2 + xy - y^2$. Substitute values (for instance, $x = 2$ and $y = 1$) to check.
12. B. It may help to apply FOIL to $(x + y)^2$: $(x + y)(x + y) = x^2 + 2xy + y^2$. If $x^2 + y^2 = 50$, we can substitute this in the equation: $50 + 2xy = 100$. So, $2xy = 50$ and $xy = 25$.
13. C. If $x - y = 3$, then $x = 3 + y$. If we replace this in the first equation, $3 + y + y = 10$, and $2y = 7$, so $y = 3.5$. $x + 3.5 = 10$, so $x = 6.5$. $6.5^2 - 3.5^2 = 42.25 - 12.25 = 30$.
14. A. Look at the numerators. If $x \times x = 10$, then $x^2 = 10$. Thus $x^2 - 1 = 10 - 1 = 9$.
15. C. Apply the Pythagorean theorem. If $a^2 + b^2 = c^2$, in the case of this triangle, $x^2 + x^2 = 8^2$. So, $2x^2 = 64$, and $x^2 = 32$.
16. A. Apply the Pythagorean theorem. If $(2x)^2 + (2x)^2 = 8^2$, then $8x^2 = 64$ and $x^2 = 8$.
17. B. If $x < 4$, then the only numbers that can make the equation true are $y \geq 4$. So, $y > x$ and $y > |x|$ and the value of Column B is greater. Substitute any permitted values to test. For example, if $x = -7$, then $|-7| = 7$, and $y = 14$. We can see that $y > x$ and $y > |x|$.
18. A. Substitute and solve. In Column A, $y = 3(5) + 4$, or $y = 19$. In Column B, $19 = 3x + 4$. We can see immediately that the equations are the same. However, Column A asks the value of y , which is greater than the value of x .
19. D. Without knowing the value of x and y , and without enough information to solve, we can't evaluate whether the value in Column A or Column B is greater. Both variables might be negative, or only x might be negative while y is positive, for example.
20. D. In this situation, x and/or y could be negative, so we cannot determine whether x^2 or y^2 will be greater.
21. A. If we apply the distributive property, $4,200 = 40x + 200$. Simplifying gives us $4,000 = 40x$, and $x = 100$. Since $100 > 10$, the value in Column A is greater.
22. A. The variable x is positive, and any positive number squared gives a positive answer. The variable y is negative, and any negative number cubed gives a negative answer. Therefore, Column A is greater.
23. A. The value of y when $x = -0.5$ is 6. The value of x when $y = 5$ is 0. Therefore, the value of Column A is greater.
24. B. If we apply the distributive property, $360 + 20x = 720$. Simplifying gives us $20x = 360$, and $x = 18$. Since $18 < 36$, the value in Column B is greater.

Simplifying Expressions

1. C. A number added to itself is the same as that number multiplied by 2, so $x + x = 2x$.
2. B. When terms have like variables, we can add and subtract the coefficients as if they were integers. $5 + 4 - 3 = 6$, so $5y + 4y - 3y = 6y$.
3. D. We can add or subtract terms with like variables, but not terms with unlike variables. $x^2 + 2x^2 = 3x^2$ and $4x + 3x = 7x$, so this expression is equivalent to $3x^2 + 7x$.
4. B. Combine terms with like variables. $9a^3$ cannot be combined with any other terms, but the other terms with like variables can be added together to yield $9a^3 - 5a^2 + 12a$.
5. C. Combine terms with like variables. $7a + a = 8a$ and $4b - 2b = 2b$, so this expression is equal to $8a + 2b$.
6. B. Combine terms with like variables. $3m - 5m = -2m$ and $6n - 3n = 3n$, so this expression is equal to $-2m + 3n$.
7. A. mn is the common variable in all three terms. Since $3 + 5 + 8 = 16$, this expression is equal to $16mn$.
8. B. Remember that only terms with like variables can be combined. xz occurs twice, xy occurs twice, and yz occurs once, so this expression is equal to $2xy + 2xz + yz$.
9. A. Combine terms with like variables. $3ab + 9ab + 4ab = 16ab$; $5bc - 6bc = -1bc$; $-7ac - 2ac = -9ac$, so this expression is equivalent to $16ab - 9ac - bc$.

10. A. Remember that only terms with like variables can be combined. $6x^2y + 3x^2y = 9x^2y$ and $-4xy^2 - 3xy^2 = -7xy^2$, so this expression is equivalent to $9x^2y - 7xy^2$.
11. C. Remember that only terms with like variables can be combined. Adding together these terms yields $5x^2y^2 + 7x^2y + 4xy^2 + 6xy$.
12. A. Be careful about applying the negative sign. The second term can be treated like $-1(x + 2)$, so this expression is equal to $3x - x - 2 = 2x - 2$.
13. A. Be careful about applying the negative sign. The second term can be treated like $-1(5x^2 + 5x)$, so this expression is equivalent to $9x^2 - 5x^2 - 5x = 4x^2 - 5x$.
14. A. Remember that only like terms can be combined and that a negative sign before parentheses must be applied to the terms within the parentheses. So, this expression can be rewritten as $3x + 4y - 5x - 6y = -2x - 2y$.
15. B. Apply the distributive property to multiply the coefficient before the parentheses. This expression can be rewritten as $5x^3 - 2x^2 - 4x^3 + 3x^2 = x^3 + x^2$.
16. A. Only $9pq^3$ and $7pq^3$ have like variables and can be combined, so this expression is equivalent to $-8p^3q + 16pq^3 - 6pq$.
17. B. Apply the distributive property to the expressions in parentheses and combine like terms. This expression can be rewritten as $3x^2y^3 + 4x^3y^2 + 3x^2y^3 + 4x^3y^2 = 6x^2y^3 + 8x^3y^2$.
18. A. Apply the distributive property to the expressions in parentheses and combine like terms. This expression can be rewritten as $3x^2y^3 - 4x^3y^2 - 3x^2y^3 + 4x^3y^2$. In this case, the terms with like variables cancel each other out, so this expression is equivalent to 0.

Distributing

1. B. Applying the distributive property to an equation in the form $a(b + c)$ means that, in order to find the product, we must multiply the number outside the parentheses to each number within the parentheses and then adding. So, $a(b + c) = ab + ac$. So, to find the equivalent expression to $6(x + 4)$, multiply each term in parentheses by 6: $6x + (6 \times 4) = 6x + 24$.
2. A. Remember, multiply each term in parentheses by the number outside the parentheses: $12(y - 12) = 12y - 144$.
3. D. Remember, multiply each term in parentheses by the number outside the parentheses: $9(n - 2) = 9n - 18$.
4. C. Following the order of operations (PEMDAS), do the multiplication first. Remember to multiply each term in parentheses by the term outside the parentheses. $9(x + 4) = 9x + 36$. So, this expression can be rewritten as $x + 9x + 36 = 10x + 36$.
5. A. The subtraction in this equation can be treated as a negative 1. $-1(c + 7) = -c - 7$. So, the whole expression can be rewritten as $c + 6 - c - 7 = -1$ (since the $c - c = 0$).
6. A. Following the order of operations, we'd multiply first; the second term is $-2(h + 2) = -2h - 4$. So, the whole expression can be rewritten as $2h + 2 - 2h - 4 = -2$.
7. D. When multiplying two binomials, like in this equation, we must multiply each term in the first parentheses by each term in the second parentheses. We can apply FOIL: First, Outer, Inner, Last. For the equation $(a + 9)(a + 4)$: first: $a \times a = a^2$; outer: $a \times 4 = 4a$, inner: $9 \times a = 9a$, last: $9 \times 4 = 36$. Now combine like terms: $a^2 + 4a + 9a + 36$ for $a^2 + 13a + 36$.
8. D. Apply FOIL to find the equivalent expression: $(m - 2)(m - 18) = m^2 - 18m - 2m + 36$. Combine like terms for $m^2 - 20m + 36$.
9. C. Apply FOIL to find the equivalent expression: $(p + 1)(p - 36) = p^2 - 36p + 1p - 36$. Combine like terms for $p^2 - 35p - 36$.
10. C. Since we know $a \times a = a^2$, then we know that $(c - 6)^2 = (c - 6)(c - 6)$. Now we can apply FOIL: $c^2 - 6c - 6c + 36$. Combine like terms for $c^2 - 12c + 36$.
11. C. Apply FOIL to find the equivalent expression: $(x + 3)(x - 12) = x^2 - 12x + 3x - 36$. Combine like terms for $x^2 - 9x - 36$.
12. D. We can apply FOIL even if all of the terms are variables. $(x + c)(x + c) = x^2 + cx + cx + c^2$. Combine like terms for $x^2 + 2cx + c^2$.

13. C. Apply FOIL To find the equivalent expression, and pay attention to signs: $(x + z)(x - z) = x^2 - zx + zx - z^2$. Combine like terms for $x^2 - z^2$. Notice that when the two binomials are the same, but have opposite signs, that the outer and inner terms will cancel when simplified. This leaves us with only the product of the first and the product of the last terms.
14. B. Apply FOIL to find the equivalent expression. $(x + 5)(x - 5) = x^2 - 5x + 5x - 25$. Combine like terms for $x^2 - 25$.
15. D. Apply FOIL to find the equivalent expression. $(x - 9)(x + 9) = x^2 + 9x - 9x - 81$. Combine like terms for $x^2 - 81$.
16. B. Apply FOIL to the left part of the equation. We can rewrite this expression as $(x + 2)(x + 2) = x^2 + 2x + 2x + 4$. Combine like terms for $x^2 + 4x + 4$. If this is equal to $x^2 + ax + 4$, then a must be equal to 4, since the rest of the expressions are the same.
17. D. Apply FOIL to the left part of the equation. We can rewrite this expression as $(x - 5)(x - 5) = x^2 - 5x - 5x + 25$. Combine like terms for $x^2 - 10x + 25$. If this is equal to $x^2 + ax + 25$, then a must be equal to -10 , since the rest of the expressions are the same.
18. B. Apply FOIL to the left part of the equation. We can rewrite this expression as $(x + 8)(x + 8) = x^2 + 8x + 8x + 64$. Combine like terms for $x^2 + 16x + 64$. So, c must be equal to $16x$, since the rest of the expressions are the same.
19. D. Apply FOIL to the left part of the equation: $(x + 4)(x - 4) = x^2 - 4x + 4x - 16$. Combine like terms for $x^2 - 16$. Substitute into the original equation, $x^2 - 16 = 10$, so we can add 16 to both sides of the equation to isolate x^2 . $x^2 = 26$.
20. D. Apply FOIL to the left part of the equation: $(2x + 3)(2x - 3) = 4x^2 - 6x + 6x - 9$. Combine like terms for $4x^2 - 9$. Substitute this into the original equation for $4x^2 - 9 = 7$. If we add 9 to both sides, we get $4x^2 = 16$.
21. D. Apply FOIL to the left part of the equation: $(3x + 4)(3x - 4)$. As we've seen when we multiply two binomials that are the same except for the sign, the outer and inner terms will cancel each other out. So, we can quickly find that this expression is equivalent to $9x^2 - 16$. Substitute this into the original equation for $9x^2 - 16 = 16$, so $9x^2 = 32$.
22. C. As with equations where we use FOIL, each term in the first expression in parentheses must be multiplied by each expression in the second: $(x + y)(x^2 - xy + y^2) = x^3 - x^2y + xy^2 + x^2y - xy^2 + y^3$. Combine like terms: $x^3 + y^3$. ($x^2y - x^2y$ and $xy^2 - xy^2$ both cancel each other out.)
23. D. In the case of 3 expressions in parentheses, it may help to FOIL the first two and then multiply the third. As we've seen, in cases like $(x + 2)(x - 2)$, we know the inner and outer terms will cancel each other out, so this is equal to $x^2 - 4$. Now, multiply the rest of the expression: $(x^2 - 4)(x^2 + 4)$. Again, the inner and outer expressions will cancel each other out, so this expression is equivalent to $x^4 - 16$.
24. A. To find an equivalent expression for Column A, do the multiplication first (notice no parenthesis around $x + 7$). This expression can be rewritten as $x + 7x + 42$. Combine like terms for $8x + 42$. This value will be greater than $8x + 6$ no matter what the value of x (you can substitute any numbers to check).
25. D. We can start by applying FOIL to $(x + 9)^2$ for $(x + 9)(x + 9) = x^2 + 18x + 81$. At first glance this expression may seem larger than the expression in Column B. However, we don't know whether or not x is a positive or negative number, so we don't have enough information to determine which is greater. To check this, simply set $x = 1$ or $x = -1$ to see what happens.
26. D. We can start by applying FOIL to $(x - 10)^2 = (x - 10)(x - 10) = x^2 - 20x + 100$. So, the expressions in Columns A and B are not equivalent. However, without knowing the value of x , we don't have enough information to determine which is greater.
27. B. We know that $(x - 1)(x + 1) = x^2 - 1$. Multiply the first term as well to find an equivalent expression for Column A: $(x^2 + 1)(x^2 - 1) = x^4 - 1$. Since $x^4 = x^4$, then the value in Column B is greater, because we'd be adding 1 to x^4 instead of subtracting 1. Since x^4 will always be positive (multiplying a negative number by itself an even number of times will always give us a positive number), we know that the value will always be positive.

28. C. To find an equivalent expression for Column A, do the multiplication first (notice no parenthesis around $x - 1$). This expression can be rewritten as $x - x - 5$. Combine like terms for -5 . This is the same value as Column B, so the columns are equal.

Factoring

1. B. Finding the factors of a polynomial is similar to finding the factors of a number. What could be multiplied to result in the expression? Since 10 and 5 have a common factor of 5, we can factor 5 out of this expression to get $5(2x + 1)$.
2. B. 8 and 6 have a common factor of 2, and we see the variable x in each term, so we can factor out both of them. Factoring out the 2 gives us $2(4x^2 + 3x)$ and factoring out x from that expression gives us $2x(4x + 3)$. Only the former is given as a choice.
3. D. $6x$ is a common factor for each coefficient. Divide each expression in the expression by $6x$ to find the equivalent expression $6x(2x^2 + x + 3)$. Remember that dividing exponential terms by exponential terms with the same base allows us to simply subtract the two exponents.
4. A. When finding factors for an expression in the form $x^2 + ax + b$, it can help to find numbers that add to the middle term and multiply to the last term. That is, in $(x + y)(x + z) = x^2 + ax + b$, then $y + z = a$ and $yz = b$. In this case, $4 + 6 = 10$ and $4 \times 6 = 24$. So, the factors of this expression are $(x + 6)(x + 4)$. You can apply FOIL to check.
5. B. If the last term in the expression has a negative sign, then one of the factors will show a negative number (or subtraction). In this case $12 + -2 = 10$ and $12 \times -2 = 24$, so the factors are $(x + 12)(x - 2)$.
6. C. As we have seen, we can find the factors of this expression by finding two numbers that add to 2 and multiply to -24 . $6 + -4 = 2$ and $6 \times -4 = -24$, so the factors are $(x + 6)(x - 4)$.
7. D. As we have seen, we can find the factors of this expression by finding two numbers that add to 14 and multiply to 24. $12 + 2 = 14$ and $12 \times 2 = 24$, so the factors are $(x + 12)(x + 2)$.
8. B. Remember to find numbers that add to the middle coefficient and multiply to the last coefficient. In this case $2 - 9 = -7$ and $-2 \times 9 = -18$, so the factors of this expression are $(x + 2)(x - 9)$.
9. D. Remember to find numbers that add to the middle coefficient and multiply to the last coefficient. $8 + -3 = 5$ and $8 \times -3 = -24$, so the equivalent expression is $(x + 8)(x - 3)$.
10. B. Remember to find numbers that add to the middle coefficient and multiply to the last coefficient. $-48 + -1 = -49$ and $-48 \times -1 = 48$. So, the equivalent expression is $(x - 48)(x - 1)$.
11. A. Remember to find numbers that add to the middle coefficient and multiply to the last coefficient. $3 + -2 = 1$ and $3 \times -2 = -6$, so the equivalent expression is $(x + 3)(x - 2)$.
12. A. Remember to find numbers that add to the middle coefficient and multiply to the last coefficient. $6 + 6 = 12$ and $6 \times 6 = 36$, so the equivalent expression is $(x + 6)(x + 6)$ or $(x + 6)^2$.
13. A. Remember to find numbers that add to the middle coefficient and multiply to the last coefficient. $-4 + -4 = -8$ and $-4 \times -4 = 16$, so the equivalent expression is $(x - 4)(x - 4)$ or $(x - 4)^2$.
14. C. As we have seen, when an expression takes the form $(x + a)(x - a)$, the inner and outer terms will cancel each other out. Since $49 = 7^2$, the equivalent expression is $(x + 7)(x - 7)$.
15. C. As we have seen, when an expression takes the form $(x + a)(x - a)$, the inner and outer terms will cancel each other out. Since $144 = 12^2$, the equivalent expression is $(x + 12)(x - 12)$.
16. C. We've seen that when $(x + y)(x + z) = x^2 + ax + b$, then $y + z = a$ and $yz = b$. So, we can find the value of a in the given equation by adding $2 + 4$. So, the value of $a = 6$.
17. D. We've seen that when $(x + y)(x + z) = x^2 + ax + b$, then $y + z = a$ and $yz = b$. In this expression, $y = 3$ and $z = 5$. So, $a = 3 + 5 = 8$ and $b = 3 \times 5 = 15$. Therefore, $a + b = 8 + 15 = 23$.
18. D. We've seen that when $(x + y)(x + z) = x^2 + ax + b$, then $y + z = a$ and $yz = b$. In this expression, $y = 4$ and $z = -3$. So, $a = 4 + -3 = 1$ and $b = 4 \times -3 = -12$. $a - b = 1 - -12 = 13$.
19. C. Remember that when an expression takes the form $(x + a)(x - a)$, the inner and outer terms will cancel each other out. This expression is equivalent to $(x^2 + 4)(x^2 - 4)$, but since that's not an option, we need to further factor this out. Since $x^2 - 4 = (x + 2)(x - 2)$, the equivalent expression is $(x^2 + 4)(x + 2)(x - 2)$, or $(x - 2)(x + 2)(x^2 + 4)$.

20. A. Remember that when an expression takes the form $(x + a)(x - a)$, the inner and outer terms will cancel each other out. This expression is equivalent to $(x^2 + 1)(x^2 - 1)$, but since that's not an option, we need to further factor this out. Since $x^2 - 1 = (x + 1)(x - 1)$, the equivalent expression is $(x^2 + 1)(x + 1)(x - 1)$, or $(x + 1)(x - 1)(x^2 + 1)$.
21. A. FOIL to find that $(x - 3)(x - 3) = x^2 - 6x + 9$. Set this equal to the given polynomial $x^2 - 6x - b$ to find that $-b = 9$. So, $b = -9$.

Creating Expressions & Equations

1. B. If Julie has twice as many books as Simon, we can multiply the number of books Simon has by 2 to find the number of books Julie has. So, we can write $2x$ to show the multiplication of $2 \times x$.
2. A. Remember PEMDAS. The phrase "twice as many as" means we need to multiply. The phrase "more than" means we need to add. We know from PEMDAS that we should multiply/divide first before adding/subtracting. To find the number of books Percy has, multiply by 2 and add 5 to the number of books Jack has, or $2y + 5$.
3. A. Let x represent Lucy's cookies. If Bob baked 3 times as many, we can represent Bob's cookies with $3x$. If they have 36 cookies in all, we can show this by writing $3x + x = 36$, or $4x = 36$. (To find the number of cookies Bob baked, we'd need to solve for x then multiply by 3.)
4. B. If Brent mowed m lawns, Jane mowed $m + 6$ lawns. If they mowed 30 lawns altogether, we can find the value of m by solving $m + m + 6 = 30$ or $2m + 6 = 30$.
5. C. Let b represent the number of bumpers welded by humans. If a machine welds 10 times as many, the machine welds $10b$. So, the total number of bumpers can be represented by the equation $10b + b = 550$, or $11b = 550$. Divide both sides by 11 to solve for b : $b = 50$. So, humans welded 50 bumpers. Multiply by 10 to find the number of bumpers machines weld: 500.
6. D. Let m represent the number of miles Bob biked. Al biked 3 times as many, or $3m$. The total number of miles they biked can be represented by the equation $3m + m = 72$, or $4m = 72$. So, $m = 18$. Al biked $3m$ miles, or $3 \times 18 = 54$.
7. B. Let p represent the number of pages Tom read last week. Sue read 12 more than twice the number of pages, or $2p + 12$. If Sue read 90 pages, then $2p + 12 = 90$. Solve for p to find Tom's pages: $2p = 78$, $p = 39$.
8. C. The sum of the three angles in a triangle is 180 degrees. Since angle C is 90 degrees, the remaining angles must add up to 90 degrees. Let x represent the number of degrees in angle A . Angle B is 18 less than twice angle A , or $2x - 18$. So, $2x - 18 + x = 90$, or $3x - 18 = 90$. Combine like terms for $3x = 108$, and isolate the variable for $x = 36$. Remember, though, that x is the number of degrees in angle A . Angle $B = 2x - 18$, or $2(36) - 18 = 54$.
9. D. Remember, the sum of the angles in a triangle is 180 degrees. If the ratio of angles is 1:2:3, then the angles must have the measures 30, 60, and 90 degrees. ($1x + 2x + 3x = 180$, or $6x = 180$. So, $x = 30$.) So, the largest angle measures 90 degrees.
10. B. Remember, the sum of the angles in a triangle is 180 degrees. If the ratio of the angles is 3:4:5, then the angles must measure 45, 60, and 90 degrees. ($3x + 4x + 5x = 180$, or $12x = 180$, so $x = 15$.) So, the smallest angle measures 45.
11. C. Let x represent the number. If it is doubled and increased by 12 to get 34, $2x + 12 = 34$. $2x = 22$, and $x = 11$.
12. C. Let x represent the number. If five more than twice the number is 23, $2x + 5 = 23$. So, $x = 9$. The question is asking what 4 times this number is, or $4x$: $4(9) = 36$.
13. D. In terms of 1 dollar, a quarter is equal to 0.25 and a nickel is equal to 0.05. 10 quarters (10×0.25) and 5 nickels (5×0.05) can be added as: $10 \times 0.25 + 5 \times 0.05$.
14. C. In terms of 1 dollar, a penny is equal to 0.01 and a nickel is equal to 0.05. If Wendy has 3 more pennies than nickels, then let x be the number of nickels: she has $0.05x + 0.01(x + 3)$.
15. C. In terms of 1 dollar, a dime is equal to 0.10 and a penny is equal to 0.01. If Carl has 9 fewer pennies than dimes, then let x be the number of dimes. His money can be shown by the equation $0.10x + 0.01(x - 9)$.

16. A. In terms of 1 dollar, a nickel is equal to 0.05 and a dime is equal to 0.10. Let x represent the number of dimes. This question can be represented as $0.10x + 0.05(x + 6) = 1.05$.
17. A. Let x equal the number of quarters. Since the total amount of money is given in dollars, we should represent nickels and quarters in the question in terms of dollars. We can write this question as $0.25x + 0.05(x + 5) = 1.75$. Applying the distributive property, this can be rewritten as $0.25x + 0.05x + 0.25$, or $0.30x + 0.25 = 1.75$. Solve for x : $0.30x = 1.50$, so $x = 5$. So, James has 5 quarters. If James has 5 more nickels than quarters, he has 10 nickels.
18. B. Since the total amount of money is given in dollars, we should represent dimes and nickels in the question in terms of dollars. Let x represent the number of nickels. Julie's coins can be represented as $0.05x + 0.10(2x) = 3.00$, or $0.05x + 0.2x = 3.00$. Solving for x yields $x = 12$. So, Julie has 12 nickels.
19. D. In terms of dollars, 30 cents is equal to 0.3. The call is equal to 2 dollars for the first minute and 0.3 for each minute after that. Let x represent the total number of minutes of a call: $2 + 0.3(x - 1)$. (Remember to subtract the first minute from x .)
20. B. Column A can be represented by the equation $0.25x + 0.05(3x) = 0.25x + 0.15x$. This simplifies to $0.40x = 2.40$, where x is equal to the number of quarters. Solving for x yields 6. If Harry had 3 times as many nickels as quarters, and he has 6 quarters, then he has 18 nickels. Since a nickel is equal to 0.05 dollars, 18 nickels are equal to \$0.90. So, the value in Column B is greater.
21. A. The scenario can be represented by the equation $5x + x = 300$, or $6x = 300$ where x is equal to the number of square meters mowed with the man-powered motor. $x = 50$. If the motorized mower mows 5 times that, then it mows 250 square meters. So, the value in Column A is greater.

Function Notation

1. C. Substitute 5 into the function. This gives us $-2(5) + 3(5)$, or $-10 + 15 = 5$.
2. C. Substitute -3 into the equation for $-|-3| + (-3)^2$. This simplifies to $-3 + 9 = 6$.
3. B. $f(g(4))$ is called a composite function, since the result of one function becomes the input for the other. First, substitute 4 into the equation for $g(x)$. This gives us $0.5(4) = 2$. This becomes the input for $f(x)$. So, we must find the value of $f(2)$. Substitute 2 into the equation for $2(2) = 4$.
4. C. $g(f(2))$ is called a composite function, since the result of one function becomes the input for the other. First, substitute 2 into the equation for $f(x)$. This gives us $3(2) + 5 = 11$. This becomes the input for $g(x)$. So, we must find the value of $g(11)$. Substitute 11 into the equation for $(11 - 1)^2$, which simplifies to $10^2 = 100$.
5. C. $n(m(q)) = 1$ can be rewritten as $q + 1 - 1$, since the output of $m(q)$ will be substituted into $n(p)$. Since the entire function is equal to 1, we need simply to solve for q : $q + 1 - 1 = 1$. This leaves us with $q = 1$.
6. A. In this case, we do not know the value of u or t . However, we know that when we substitute a value of t into $g(t)$, that becomes the input for $h(u)$; in other words, t^2 becomes u . We can substitute t^2 into the equation of $h(u)$, giving us $\sqrt{t^2}$. We're told that this is equal to 2, so we simply need to solve for t in the equation $\sqrt{t^2} = 2$. We can square both sides then take the square root, but these cancel out on the left side of the equation, giving us 2 or -2 . The equation states the $t > 0$, so the answer is 2.
7. A. In the function $f(x)$, for some value of x (which could be negative, positive, a fraction, etc.), we must square it. This is because $f(x) = x^2$. If we square any positive or negative number, we will always get a positive number (try this with -1 and 1). We're told that x does not equal 0, which is neither positive nor negative. We also won't necessarily always get an even number. We can square an odd number (for example, 3) and end up with an odd answer.
8. A. When a positive whole number is cubed, we end up with a larger positive whole number. However, when a positive fraction (some number greater than 0 but less than 1) is cubed, we get a smaller fraction. In this case, $2^3 = 8$, while $(\frac{1}{2})^3 = \frac{1}{8}$.
9. C. $\square b$ is not a function that exists in math. Imaginary operations as you to simply substitute a different number into a given expression or equation. If $\square b = 6b - 12$, then $\square 4 = 6(4) - 12$, or $24 - 12 = 12$.

10. C. Plug in the given values to find the value of the expression. $\square 7 = 6(7) - 12 = 30$ and $\square 3 = 6(3) - 12 = 6$. So, $\square 7 - \square 3 = 30 - 6 = 24$.
11. B. $\square b = 6b - 12$. If $\square b = 42$, then $6b - 12 = 42$. Solve for b by adding 12 to both sides and dividing by 6. This gives us $6b = 54$ and then $b = 9$.
12. D. If $x^2 = x^2 + 6x + 9$, then $2^2 = 2^2 + 6(2) + 9$. Simplify for $4 + 12 + 9 = 25$.
13. B. If $x^2 = x^2 + 6x + 9$, then $(-3)^2 = (-3)^2 + 6(-3) + 9$. Simplify for $9 - 18 + 9 = 0$.
14. A. To find the square root, it may help to find the factors first. $x^2 + 6x + 9 = (x + 3)(x + 3)$ or $(x + 3)^2$. Therefore, the square root of the expression is $x + 3$.
15. B. If $x@y = 2x - 3y$, then $5@3 = 2(5) - 3(3)$, or $10 - 9 = 1$.
16. B. If $x@y = 2x - 3y$, then $7@y = 2(7) - 3y$, or $14 - 3y$.
17. C. If $x@y = 2x - 3y$, then $x@7 = 2x - 3(7)$, or $2x - 21$.
18. C. If $c\#d = c^2 + cd + 16$, then $2\#3 = (2)^2 + 2(3) + 16$, or simply $4 + 6 + 16 = 26$.
19. C. Find the value of each expression first. If $c\#d = c^2 + cd + 16$, then $10\#4 = (10)^2 + (10)(4) + 16$. This simplifies to $100 + 40 + 16 = 156$. Next would be $4\#10 = (4)^2 + (4)(10) + 16$, or $16 + 40 + 16 = 72$. So, $10\#4 - 4\#10 = 156 - 72$, or 84.
20. B. If $c\#d = c^2 + cd + 16$, then $4\#d = 4^2 + 4d + 16$, or $16 + 4d + 16$. This simplifies to $4d + 32$.
21. C. If $p \sim \frac{1}{p} + p^2$, then $(-5) \sim \frac{1}{-5} + (-5)^2$. This simplifies to $\frac{1}{-5} + 25 = 24\frac{4}{5}$.
22. B. If $p = 0$, then the value in the denominator would be 0, which is not defined. Therefore, $p = 0$ would not result in an integer.
23. A. If $0 = \frac{1}{p} + p^2$, then we can solve to find the value of p . If we subtract $\frac{1}{p}$ from both sides, we get $-\frac{1}{p} = p^2$. The only possible value that makes this equation true is -1 , since $-\frac{1}{-1} = -1^2$.
24. C. As we've seen, we can substitute 1 into the expression. This gives us $\frac{1}{4} + 1^2 = 1\frac{1}{4}$ or $\frac{5}{4}$.
25. C. We can substitute and solve for x . $\frac{x}{4} + x^2 = 17$. Multiplying both sides by 4 gives us $x + 4x^2 = 68$. If we subtract 68 from both sides, we see a familiar pattern: $4x^2 + x - 68 = 0$. Find factors for $4x^2 + x - 68$, which are $(4x + 17)(x - 4) = 0$. From this equation, we can see that $x = 4$ will make the equation true.
26. C. Substitute -2 in the expression for $-\frac{2}{4} + (-2)^2$. This simplifies to $-0.5 + 4 = 3.5$.
27. A. The value of Column A is equal to $\frac{1}{1/2} + 1$, where the first term equals 2 (when we divide 1 by a fraction, we are finding the fraction's reciprocal). $2 + 1 = 3$, and $3 > 2$, so the value of Column A is greater.
28. A. The value of Column A is $(3)^2 + 3$, or $9 + 3 = 12$. $12 > 9$, so the value in Column A is greater.
29. B. The value of Column A is $5(10) - 4$, or $50 - 4 = 46$. $46 < 54$, so the value in Column B is greater.
30. B. The value of Column A is equal to $3a + 4 = 10$. Solving for a yields $a = 2$. The value of Column B is $3(10) + 4$, or $30 + 4 = 34$. $10 < 34$, so the value in Column B is greater.

Solving for Zero

- A. If $x + 5 = 0$, subtract 5 from both sides to solve for x . This leaves us with $x = -5$.
- B. If $5y = 0$, then y must be equal to 0; any number multiplied by 0 results in 0.
- C. Only III must be true; in order for the product to be 0, either a or b must be 0.
- D. None of these statements **must** be true. If $a + b = 0$, then either a and b are both equal to zero, or else any number of possible values could be true, as long as the sum were 0.

5. A. $5 - 5 = 0$, so if $x = 5$, this expression must be equal to 0, because the second term in parentheses will be equal to 0. Or: $(5 + 5)(5 - 5) = (10)(0) = 0$.
6. D. In order for this expression to be equal to 0, one of the expressions in parentheses must be equal to 0. -3 or -5 would make this expression true.
7. B. A number with 0 in the denominator is undefined and therefore not a real number (one cannot divide by nothing). But a fraction can have 0 in the numerator (which makes it equal to 0). Therefore, only II is true, because it shows an undefined number.
8. A. All real numbers make the expression $5x$ true. A real number includes all numbers except for imaginary numbers. (An imaginary number is $\sqrt{-1}$ or i .) 0 is also a real number, so x could be any real number to make $5x$ a real number.
9. B. In order for this expression to be equal to 0, one of the expressions in parentheses must be equal to 0. -9 would make the first expression in parentheses equal to 0 ($-9 + 9 = 0$), so this is a possible value of x to make this equation equal to 0. (9 is the only other possible value.) Any other real number would not make the equation true.
10. D. One way to solve this is to find the factors of the equation. Remember, we can find the factors that add to the middle term and multiply to the last term. So, this expression is equal to $(x + 5)(x + 2)$. Thus, either -2 (not given as a choice) or -5 would make the equation equal to 0. One could also substitute answer choices into the equation.
11. A. Find the factors of the equation. Remember, we can find the factors that add to the middle term and multiply to the last term. So, this expression is equal to $(x + 10)(x - 2)$. Thus, either 2 (not given as a choice) or -10 would make the expression equal to 0. One could also substitute answer choices into the equation.
12. D. Find the factors of the equation. Remember, we can find the factors that add to the middle term and multiply to the last term. So, this expression is equal to $(x - 9)(x + 4)$. Thus, either -4 (not given as a choice) or 9 would make this equation true. One could also substitute answer choices into the equation. One could also substitute answer choices into the equation.
13. D. Find the factors of the equation. Remember, we can find the factors that add to the middle term and multiply to the last term. So, this expression is equal to $(x - 11)(x - 1)$. Thus, either 1 (not given as a choice) or 11 would make the equation true. One could also substitute answer choices into the equation.
14. C. Find the factors of the equation. Remember, we can find the factors that add to the middle term and multiply to the last term. We can rewrite this equation as $(x + 10)(x - 10)$. Thus, 10 or -10 would make the equation true.
15. D. The commutative property tells us that $x + 8 = 8 + x$, so this expression is actually equal to 1, unless $x = -8$, in which case the denominator would be equal to 0 and the value is undefined. So, there are no values for x that make this equation true.
16. C. If we find the factors, we see that the numerator is equal to $(x + 4)(x - 4)$. So, 4 or -4 would make the numerator equal to 0, and hence the whole fraction equal to 0. -6 or 8 would make the denominator equal to 0, which would make the value of the fraction undefined, so we could immediately rule out all choices that contain any one of those values. Thus, only 4 or -4 would make the equation true.
17. C. The correct answer will give a zero value in the numerator and a non-zero value in the denominator. -2 is the only answer that will give this result. The numerator becomes $(-2)^3 + 8 = 0$. The denominator would be $2(-2) = -4$.
18. C. Set each factor equal to zero and solve to find the answers. This gives $(2x - 6) = 0$, and $4x = 0$. Therefore, $x = 3$ or 0.

Inequalities

1. D. Remember that an open dot on the graph means that number is not included in the set of possible solutions. A solid dot includes that it is. If $x \geq 4$, then the graph would show the set of numbers greater than or equal to 4, so the graph with the solid dot pointing to the right is correct.
2. A. If x is less than -6 , then the graph should have an open dot and will point to the left, since -6 is not included in the set of possible solutions.
3. B. The \leq indicates less than or equal to, so the graph will show a solid dot at -3 , and the number line shaded in until the solid dot at 9.
4. B. Treat the greater-than-or-equal-to sign as an equal sign. Isolate x . Subtract 3 from both sides of the equation and then divide by 3, giving us $x \geq 1$. The correct graph will show a solid dot at 1 and an arrow pointing to the right.
5. C. Isolate x . Subtract 10 from both sides and then divide by -5 , and remember to switch the direction of the inequality sign when multiplying or dividing by a negative number. So, $x < -1$. There will be an open dot at -1 and an arrow pointing to the left.
6. B. Isolate y , and remember to switch the sign if dividing by a negative number. $y \geq -\frac{2}{4}$ or $y \geq -\frac{1}{2}$.
So, the correct graph will show a solid dot halfway between 0 and -1 , and the arrow will point to the right.
7. C. Isolate a . Subtract 5 from both sides, then divide by -3 , and remember to switch the sign because you're dividing by a negative number. This gives us $a \leq 4$, so the correct graph will show a solid dot at 4 and an arrow pointing to the left.
8. A. To isolate h , first subtract 3 from all 3 terms, then divide by 3. This yields $-4 \leq h \leq 1$. So, the graph will show a solid dot at -4 and a line drawn to the solid dot at 1.
9. A. Isolate f . Subtract 1 from all 3 terms, then divide all terms by -2 . Remember to switch the signs, since you're dividing by a negative number. This yields $1 \geq f \geq -4$. The correct graph will have a solid dot at -4 and a line drawn to the solid dot at 1.
10. A. Isolate p . Subtract 3, then divide by -1 , and remember to switch the signs. This yields $-3 \leq p \leq 4$. The correct graph will have a solid dot at -3 and a line to the solid dot at 4.
11. A. Isolate w . Subtract 3 from all 3 terms, then divide by -3 , and remember to switch the signs. This yields $\frac{-8}{-3} > w > \frac{-4}{-3}$. The negatives in the fraction cancel out. Since the small tickmarks on the number line show that the space between integers is divided into thirds, the graph will show an open dot between 1 and 2, since $\frac{4}{3} = 1\frac{1}{3}$, then a line to the open dot between 2 and 3, since $\frac{8}{3} = 2\frac{2}{3}$.

Absolute Value Inequalities

1. C. Since the absolute value of a number is the value of that number regardless of the sign, If $|x| > 5$, then $x > 5$, or $x < -5$. (Since x could be positive or negative, in order for the inequality to be true, the value of x must be greater than 5 or less than -5 .)
2. D. Create a compound inequality to solve. This inequality can be rewritten as $-5 < x + 3 < 5$. Isolate x by subtracting 3 from all 3 terms: $-8 < x < 2$. Of the answer choices, only -7 makes this inequality true. Otherwise, one can simply plug answer choices into the inequality to see which is true.
3. D. This inequality can be rewritten as $-24 > -3x > 24$. Divide all 3 terms by -3 , and remember to switch the sign because we're dividing by a negative number: $-8 < x < 8$. Therefore $x > -8$ or $x < 8$.
4. D. This inequality can be rewritten as $-5 < 2y + 3 < 5$. Isolate the variable by subtracting 3 from all 3 terms, then dividing by 2. $-4 < y < 1$. So, $y > -4$ or $y < 1$.
5. B. This inequality can be rewritten as $-7 < 4a - 5 < 7$. Isolate the variable by adding 5 to all three terms, then divide by 4 (and simplify the fraction). This gives us $-\frac{1}{2} < a < 3$. So, $a > -\frac{1}{2}$ or $a < 3$.

6. A. This inequality can be rewritten as $-10 < -3m + 8 < 10$. Isolate the variable by subtracting 8 from all 3 terms and then dividing by -3 . $6 > m > -\frac{2}{3}$. So, $m < 6$ and $m > -\frac{2}{3}$.
7. A. This inequality can be rewritten as $-3 < x < 3$. The correct graph will show an open dot at -3 and a line to an open dot at 3 .
8. C. This expression can be rewritten as $-30 > 5x > 30$. Divide all terms by 5 . $-6 > x > 6$. The correct graph will show an open dot at -6 and an arrow to the left, and an open dot at 6 and an arrow to the right.
9. C. This expression can be rewritten as $-5 > y + 3 > 5$ or $-8 > y > 2$. The correct graph will show an open dot at -8 and an arrow to the left and an open dot at 2 and an arrow to the right.
10. A. Add the 5 to both sides before dealing with the absolute value. This expression can be rewritten as $|x + 5| < 7$. This can be rewritten as $-7 < x + 5 < 7$ which we simplify to $-12 < x < 2$. The correct graph will show an open dot and a line connecting it to an open dot at 2 .
11. D. Subtract 2 before dealing with the absolute value $|x - 8| < 4$, so $-4 < x - 8 < 4$ or $4 < x < 12$. The correct graph will show an open dot at 4 and a line to an open dot at 12 .
12. D. Divide both sides by 3 before applying the inequality. $|x - 1| < \frac{7}{3}$. So, $-\frac{7}{3} < x - 1 < \frac{7}{3}$ which we simplify to $-\frac{4}{3} < x < \frac{10}{3}$. Simplifying the fractions gives us $-1\frac{1}{3} < x < 3\frac{1}{3}$. So, the correct graph will show an open dot between -1 and -2 and a line to an open dot between 3 and 4 .
13. C. Multiply both sides by 4 before applying the absolute value. $|x - 1| < 20$. This can be rewritten as $-20 < x - 1 < 20$. So, $-19 < x < 21$. The correct graph will show an open dot at -19 and a line to an open dot at 21 .
14. B. Add 3 to both sides, then multiply by 2 before dealing with the absolute value. This yields $|x + 3| < 8$. This inequality can be rewritten as $-8 < x + 3 < 8$ or $-11 < x < 5$. The correct graph will have an open dot at -11 and a line to an open dot at 5 .
15. A. This graph shows that the value of x is $-2 < x < 2$, or $|x| < 2$.
16. A. Find the midpoint of the line segment. In this case, the midpoint is -2 . Then, subtract that number from x , giving us $x + 2$. The distance from this point to either endpoint is 5 , so the inequality must be set to be no greater than 5 .
17. D. Find the midpoint of the line segment. In this case, the midpoint is -1 . Then, subtract that number from x , giving us $x + 1$. The distance from this point to either endpoint is 2 , so the inequality must be set to be no greater than 2 .
18. B. Find the midpoint of the line segment. In this case, the midpoint is 0 . Then, subtract that number from x , giving us x . The distance from this point to either endpoint is 5 , so the inequality must be set to be no greater than 5 . Notice that the choices here do not give this answer in that format. Instead, we are given a choice where we must divide both sides by -2 (and flip the direction of the inequality).
19. A. Find the midpoint of the line segment. In this case, the midpoint is 0 . Then, subtract that number from x , giving us x . The distance from this point to either endpoint is 6 , so the inequality must be set to be no greater than 6 . Notice that the choices here do not give this answer in that format. Instead, we are given a choice where we must divide both sides by -0.5 (and flip the direction of the inequality).
20. A. This inequality can be rewritten as $-7 < 2m - 7 < 7$. Isolate the variable by adding 7 to all three terms to give us $0 < 2m < 14$. Then, divide all terms by 2 to get: $0 < m < 7$. Thus, $m > 0$, or $m < 7$.

Slope

1. C. The slope of a line is found as $\frac{y_2 - y_1}{x_2 - x_1}$, where each variable represents an x or y coordinate in a coordinate pair. Simply put, slope is the difference between the y coordinates divided by the difference between the x coordinates of two points on a line. Pick two easy-to-work with coordinates

(choosing x and y intercepts is generally a good idea). We can use the points at $(0, -3)$ and $(3, 3)$.

Substitute for $\frac{3 - (-3)}{3 - 0} = \frac{6}{3} = 2$.

2. C. This line has points at $(-4, 0)$ and $(0, 2)$. Use $\frac{y_2 - y_1}{x_2 - x_1}$ to find the slope: $\frac{2 - 0}{0 - (-4)} = \frac{2}{4} = \frac{1}{2}$.
3. B. This line has points at $(-3, -1)$ and $(0, 1)$. Use $\frac{y_2 - y_1}{x_2 - x_1}$ to find the slope: $\frac{1 - (-1)}{0 - (-3)} = \frac{2}{3} = \frac{2}{3}$.
4. B. Linear equations have the form $y = mx + b$ where m is the slope of the line. In this equation, $m = 5$, so the slope is 5.
5. A. This line has points at $(1.5, -0.5)$ and $(1, 1)$. Use $\frac{y_2 - y_1}{x_2 - x_1}$ to find the slope: $\frac{1 - (-0.5)}{1 - 1.5} = -\frac{1.5}{0.5} = -3$. Remember that dividing by a number between 0 and 1 is the same as multiplying the numerator by the reciprocal of the denominator. In this case, $0.5 = \frac{1}{2}$, and its reciprocal is 2.
6. B. This line has points at $(0, -2)$ and $(-2, -0.5)$. Use $\frac{y_2 - y_1}{x_2 - x_1}$ to find the slope: $\frac{-0.5 - (-2)}{-2 - 0} = \frac{1.5}{-2}$.

This simplifies to $-\frac{3}{4}$.

7. B. Parallel lines have the same slope. Since this equation is in the form $y = mx + b$, the slope of the line for the given equation is $-\frac{2}{3}$. Parallel lines have the same slope, so the slope of a parallel line is

also $-\frac{2}{3}$.

8. B. When graphed, lines with a positive slope rise from left to right, lines with a negative slope decline from left to right, and lines with a slope of 0 are flat. On this graph, the only section with a negative slope is between $x = -2$ and $x = 1$, or $-2 < x < 1$.
9. A. Use $\frac{y_2 - y_1}{x_2 - x_1}$ to find the slope: $\frac{-5 - 3}{2 - 0} = \frac{-8}{2} = -4$.
10. D. Use $\frac{y_2 - y_1}{x_2 - x_1}$ to find the slope: $\frac{11 - 6}{3 - 2} = \frac{5}{1} = 5$.
11. B. Use $\frac{y_2 - y_1}{x_2 - x_1}$ to find the slope: $\frac{5 - 1}{3 - (-3)} = \frac{4}{6} = \frac{2}{3}$.
12. C. Use $\frac{y_2 - y_1}{x_2 - x_1}$ to find the slope: $\frac{-9 - (-7)}{-10 - (-2)} = \frac{-2}{-8} = \frac{1}{4}$.

13. B. The equation for slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$. If we substitute the given values, we get $2 = \frac{5 - 1}{3 - x} = \frac{4}{3 - x}$.

In order for this equation to be true, the denominator in the fraction must equal 2, or $3 - x = 2$, so $x = 1$.

14. D. The equation for slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$. If we substitute the given values, we get

$\frac{1}{2} = \frac{4 - 7}{-5 - x} = \frac{-3}{-5 - x}$. In order for the equation to be true, the denominator must be twice the numerator, so $-6 = -5 - x$. Therefore, $x = 1$.

15. D. The equation for slope is $m = \frac{y_2 - y_1}{x_2 - x_1}$. If we substitute the given values, we get
- $$\frac{3}{4} = \frac{y - (-4)}{8 - (-8)} = \frac{y + 4}{16}$$
- In order for this equation to be true, $y + 4 = 12$. ($16 = 4 \times 4$, so the numerator is equal to 4×3 .) So, $y = 8$.
16. B. This equation is in the format $y = mx + b$, so the slope of the given line is 2. Find the reciprocal of a number by flipped that number over the fraction line. The reciprocal of 2 is $\frac{1}{2}$ so the negative reciprocal of 2 is $-\frac{1}{2}$.
17. A. Since the slope of a line perpendicular to another line is the latter's negative reciprocal, then product of the two slopes will always be -1 . Where m is the slope of the latter, and $-\frac{1}{m}$ is the slope of the line perpendicular to it, then $-\frac{1}{m} \times \frac{m}{1} = -1$. In this case, the slope of line a is 4, and the slope of the line perpendicular to it is $-\frac{1}{4}$. Therefore, the product of any slope and the slope of a line perpendicular to it is -1 .
18. A. As we've seen, since the slope of perpendicular lines are negative reciprocals, the product of the slope of a line and the slope of its perpendicular line will be -1 .
19. C. The pictured line passes through $(0, 0)$ and $(5, 5)$, so the slope of this line is 1. If the other line passes through M and N as well as the origin $(0, 0)$, then it has a smaller rise than the pictured line and will thus have a smaller slope. Of the choices, 0.5 is the only one that fits this criteria.
20. B. Since the slope of a perpendicular line is the negative reciprocal, the slope of the line perpendicular to a line with a slope of m has a slope of $-\frac{1}{m}$.
21. B. The pictured line passes through $(0, 0)$ and $(3, -6)$, so the slope is equal to $-\frac{6}{3} = -2$. Because the other line passes through M and N it will also have a negative slope. However, it has a lower rise than the pictured line, so the slope will be less than the slope of the pictured line. The only remaining choice for which this is true is -0.5 .
22. C. The pictured line has a slope of 1. (It passes through $(0, 0)$ and $(1, 1)$.) Thus $2m = 2$.
23. A. The equation in Column A is in the form $y = mx + b$, so the slope of the line is 2. Convert the equation in Column B to the same format by solving for y . $2y = -4x + 8$. Divide all terms by 2 to arrive at $y = -2x + 4$. So this line has a slope of -2 . Since $2 > -2$, the value of Column A is greater.
24. C. Parallel lines have the same slope, so the values of both columns are the same.
25. A. The slope of a perpendicular line is the negative reciprocal, so the slope of the perpendicular line is -4 . Thus the value of Column A is greater.
26. A. Without calculating the actual slope, you can tell that the slope of line q is a positive value. The perpendicular line would be the negative reciprocal. Thus, the value of Column A is greater.

Equation of a Line

1. B. First find the slope. $m = \frac{y_2 - y_1}{x_2 - x_1}$, so this line has a slope of $\frac{5 - 3}{9 - 3} = \frac{2}{6} = \frac{1}{3}$. There is only one equation where the slope is $\frac{1}{3}$, and there is no need to perform additional calculations.

2. B. The slope of the line is $\frac{0 - (-6)}{3 - 0} = \frac{6}{3} = 2$. There are two choices with a slope of 2, so we can eliminate the other two choices. Simply substitute into one of the two choices either x -coordinate given (choose 0, since it is easier to work with than 3). Substituting 0 into the first choice gives us a y -coordinate of -3 , which is not correct. Substituting 0 into the second choice gives us a y -coordinate of -6 , which is correct.
3. A. The slope of the line is $\frac{-5 - 4}{8 - (-4)} = \frac{-9}{12} = -\frac{3}{4}$. There are two equations with this slope, so we must test which value of x results in the corresponding value of y . Work with -4 , since the slope is $-\frac{3}{4}$. Substitute -4 into the first equation and solve for y . We find that it equals 4, which is the corresponding y -coordinate when the x -coordinate is -4 .
4. D. Since the slope is the same, we can plug in the x -coordinate given into both of the choices with a slope of 3. If we do so to the first of these choices, we end up with $y = 15 - 5$, or $y = 10$. This is not 5, as we know from the given y -coordinate. If we do the same to the second of these choices, we end up with $y = 15 - 10$, or $y = 5$. This is true based on the information given.
5. A. Since the slope is the same, we know we are looking for an equation with a slope of -2 . This rules out the last two choices. Plug in the value of x into the two remaining equations to see which equation gives us a y value of 6. This is only true in the first equation.
6. A. Since the slope is the same, we know we are looking for an equation with a slope of $\frac{1}{2}$. This rules out the last two choices. Plug in the value of x into the two remaining equations to see which equation gives us a y value of -1 . This is only true in the first equation.
7. C. The slope of a line that is perpendicular to another is the negative reciprocal of the latter. Since the slope of the line is 3, we know we are looking for an equation where the slope is $-\frac{1}{3}$. This rules out the first two choices. Plug in the value of x into the two remaining equations to see which equation gives us a y value of -7 . This is only true in the third equation.
8. C. The slope of a line that is perpendicular to another is the negative reciprocal of the latter. Since the slope of the line is -2 , we know we are looking for an equation where the slope is $\frac{1}{2}$. This rules out the first two choices. Plug in the value of x into the two remaining equations to see which equation gives us a y value of 0. This is only true in the third equation.
9. B. The slope of a line that is perpendicular to another is the negative reciprocal of the latter. Since the slope of line m is $\frac{1}{2}$, we know we are looking for an equation where the slope is -2 . This rules out the last two choices. Plug in the value of x into the two remaining equations to see which equation gives us a y value of 0. This is only true in the second equation.
10. B. Choose two easy coordinates in order to calculate the slope. Generally, the intercepts on either axis will be a good choice. In this case, we can work with $(0, -3)$ and $(1.5, 0)$, so the slope of this line is equal to $\frac{0 - (-3)}{1.5 - 0} = 2$. This rules out the last two choices. Simply plug in an x -coordinate to see which equation gives the related y -coordinate (choose $x = 0$, since it is easy to work with). Only in the second choice is this true.
11. C. There is no x -intercept here, so we must look for another easy-to-use coordinate. We see the y -intercept at $(0, -3)$. We can find another point at $(-4, -2)$. The slope is $\frac{-2 - (-3)}{-4 - 0} = -\frac{1}{4}$. This rules

out the first two choices. Simply plug in an x -coordinate to see which of the remaining equations gives the related y -coordinate (choose $x = 0$, since it is easy to work with). Only in the third choice is this true.

12. B. This line passes through $(0, 2)$ and $(-4, 0)$. The slope is equal to $\frac{0-2}{-4-0} = \frac{1}{2}$. This rules out the last two choices. Simply plug in an x -coordinate to see which of the remaining equations gives the related y -coordinate (choose $x = 0$, since it is easy to work with). Only in the second choice is this true.
13. A. The line passes through $(0, -3)$ and $(2, 1)$. The slope is $\frac{1-(-3)}{2-0} = \frac{4}{2} = 2$. So the perpendicular equation has a slope of $-\frac{1}{2}$. This rules out the last two choices. Simply plug in the x -coordinate 4 to see which of the remaining equations gives the related y -coordinate of 5. Only in the first choice is this true.
14. B. The slope of line l is the opposite reciprocal of line k since it is stated that they are perpendicular. This means that the slope is 3. Immediately eliminate choices A and D. Next, we know that the two lines intersect at $(3, 6)$, which means we can plug 3 in for the value of x into choices B and C to see which gives us a y -value of 6. Doing so for choice B gives us $3(3) - 3 = 6$.

Sequences

- D. Although each term in the sequence is two more than the previous, if you plug numbers into $n + 2$, you get a series of consecutive numbers. ($1 + 2 = 3$, $2 + 2 = 4$, $3 + 2 = 5$, etc.) However, if you plug the same numbers into $2n - 1$, you get $(2(1) - 1)$, $(2(2) - 1)$, $(2(3) - 1)$, etc., which yields 1, 3, 5, 7, 9.
- C. Although each term in the sequence is four more than the previous, as we have seen, the expression $n + x$ will yield consecutive numbers, not numbers in the sequence. In this case, $4n - 5$ will yield the correct results: $4(1) - 5 = -1$; $4(2) - 5 = 3$; $4(3) - 5 = 7$, etc.
- C. The easiest way to solve may be to plug numbers between 1 and 5 into each answer choice. In this case, $-5n + 20$ yields the sequence: $-5(1) + 20 = 15$; $-5(2) + 20 = 10$; $-5(3) + 20 = 5$; etc.
- A. The easiest way to solve may be to plug numbers between 1 and 5 into each answer choice. In this case, $n - \frac{1}{2}$ yields the sequence: $1 - \frac{1}{2} = \frac{1}{2}$; $2 - \frac{1}{2} = \frac{3}{2}$; $3 - \frac{1}{2} = \frac{5}{2}$, etc.
- C. Each term in the sequence is 2 greater than the one before, and they are all odd numbers, so the 10th term will be the 10th odd number: 19.
- A. The hint is helpful here; since we know the sequence is skip counting by 5, we can write an expression to represent the sequence. One that would yield this sequence is $5n - 3$. Plug in values to test this: $5(1) - 3 = 2$; $5(2) - 3 = 7$; $5(3) - 3 = 12$; $5(4) - 3 = 17$. We can find the 50th term by plugging 50 into the expression: $5(50) - 3 = 247$.
- B. As we have seen, we can find an expression to represent the sequence. In this case, $3n - 7$ will yield the sequence: $3(1) - 7 = -4$; $3(2) - 7 = -1$; $3(3) - 7 = 2$, etc. We can plug 100 into this expression to find the 100th term: $3(100) - 7 = 293$.
- C. Unlike an arithmetic sequence, a geometric sequence increases exponentially. We can follow the same process of plugging numbers 1–5 into each expression to find the one that works. In this case $2(3)^{n-1}$ is correct. $2(3)^{1-1} = 2$ since $(3^0 = 1)$; $2(3)^{2-1} = 6$ since $3^1 = 3$; etc. Remember to apply the exponent before performing the multiplication!
- C. We can treat this geometric sequence like an arithmetic sequence, and plug 1–5 into the answer choices to find the one that fits. $-5(3)^{n-1}$ will yield the sequence: $-5(3)^{1-1} = -5$ since $3^0 = 1$; $-5(3)^{2-1} = -15$ since $3^1 = 3$; etc.

10. A. If you look at the sequence, you can see that each term is equal to $\frac{1}{2}$ the term that comes before it.

So the expression that represents this sequence is $12\left(\frac{1}{2}\right)^{n-1}$, which will yield a term that is one half of the previous term. $12\left(\frac{1}{2}\right)^{1-1} = 12$; $12\left(\frac{1}{2}\right)^{2-1} = 6$, etc.

11. A. In this sequence, each term is $\frac{1}{3}$ of the term before it, so the correct answer will show $\left(\frac{1}{3}\right)^{n-1}$.

We only see this in the first expression, and should test the truth of this by plugging in some values (for instance, from 1 through 5).

12. C. Each term in this sequence is twice the previous term, so we can find an expression to represent this sequence. In this case, since we're only looking for the 6th term, we need only multiply by 2 two more times: $32 \times 2 = 64$, and $64 \times 2 = 128$. This saves time rather than finding an algebraic expression that represents each term in the sequence.

13. A. Each term is $\frac{1}{2}$ the previous term. Since we are being asked for the 8th term, and are already given 5, we need only divide by 2 three more times to find the resulting term. Remember that when dividing a fraction by an integer, we need only multiply the denominator of the fraction by the integer. This gives us: $\frac{5}{4} \div 2 = \frac{5}{8}$, then $\frac{5}{8} \div 2 = \frac{5}{16}$, and finally $\frac{5}{16} \div 2 = \frac{5}{32}$.

Permutations & Combinations

1. D. Billy has 10 shirts and 4 pairs of pants and it's possible for shirts and pants to repeat. For each of the 4 pairs of pants, there are 10 shirt possibilities, so there is a total of $10 \times 4 = 40$ possible outfits.
2. D. Brad can repeat digits, so there are 10 possibilities for the first digit, 10 possibilities for the second digit, and 10 possibilities for the third digit. This is equal to $10 \times 10 \times 10$ or $10^3 = 1,000$ possible codes.
3. B. There are 26 letters in the alphabet, so there are 26 possibilities for each of the 4 slots in his password (since he can repeat letters), so we can multiply 26 by itself four times ($26 \times 26 \times 26 \times 26$) or find 26^4 .
4. C. There are 4 answer choices for each of the 3 questions, so multiply 4 by itself three times: $4 \times 4 \times 4 = 4^3 = 64$.
5. D. There are 3 kinds of bread, 4 kinds of meat, and 3 types of cheese, so there are $3 \times 4 \times 3$ possibilities, or 36 different possibilities.
6. B. Each of the first 3 letters have 26 possibilities, or $26 \times 26 \times 26$. Each of the last 3 numbers have 10 possibilities, or $10 \times 10 \times 10$. So this permutation is equal to $26^3 \times 10^3$.
7. C. Brad cannot repeat digits, so the first digit in his code has 10 possibilities, the second has 9 (since the one used in the first place cannot be repeated), and the third has 8. So, the total number of possible codes is equal to $10 \times 9 \times 8 = 720$.
8. D. The password has 26 possible letters for the first place, 25 for the second, 24 for the third, and 23 for the fourth, so the total number of possibilities is equal to $26 \times 25 \times 24 \times 23$.
9. C. There are 4 letters in MATH. The total number of ways they can be arranged is equal to $4 \times 3 \times 2 \times 1 = 24$.
10. C. Polly wants to arrange 3 books and she has 7 total books. Since none can repeat, the total number of possibilities is equal to $7 \times 6 \times 5 = 210$.

11. C. No numbers or letters can repeat, so the first 3 letters can be represented by $26 \times 25 \times 24$ and the 3 letters can be represented by $10 \times 9 \times 8$. So the total number of possibilities can be represented by $26 \times 25 \times 24 \times 10 \times 9 \times 8$.
12. C. There are 4 kids, and none can come in two different places in the same race. So there are $4 \times 3 \times 2 \times 1 = 24$ possibilities.
13. A. There are four pieces of fruit and Dorothy wants to make combinations of two. Let the fruit be represented as O, B, A, P. You can list the possible combinations of two: OB, OA, OP, BA, BP, AP. That's 6 combinations.
14. B. There are 5 teammates, and the team wants to send a combination of 2. You can use the formula $\frac{n!}{r!(n-r)!}$ where n is the number of things to choose from and r is the number we choose. The explanation point shows the factorial ($5 \times 4 \times 3 \times 2 \times 1 = 5!$), So this question can be represented by $\frac{5!}{2!(5-2)!} = \frac{5 \times 4 \times 3 \times 2 \times 1}{(2 \times 1)(3 \times 2 \times 1)}$. This simplifies to $\frac{120}{2(6)} = \frac{120}{12} = 10$.
15. B. We can apply the combination formula to this question. Mrs. Thompson wants 2 of her 7 children to clean, so $n = 7$ and $r = 2$. This gives us $\frac{7!}{2!(7-2)!} = \frac{7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1}{(2 \times 1)(5 \times 4 \times 3 \times 2 \times 1)}$. We see that we can actually cancel out the $5 \times 4 \times 3 \times 2 \times 1$ in both the numerator and the denominator, leaving us with $\frac{7 \times 6}{2 \times 1} = \frac{42}{2} = 21$.
16. A. Tina's sister can take 4 of her 5 dolls on vacation, so in the combination formula, $n = 5$ and $r = 4$. This gives us $\frac{5!}{4!(5-4)!} = \frac{120}{24(1)} = 5$.
17. B. Six chess players can take two spots, so in the combination formula, $n = 6$ and $r = 2$. This gives us $\frac{6!}{2!(6-2)!} = \frac{720}{2(24)} = 15$.
18. D. We can apply the combination formula to this question. She has a total of 6 games and can take 3, so $n = 6$ and $r = 3$. This gives us $\frac{6!}{3!(6-3)!} = \frac{6 \times 5 \times 4 \times 3 \times 2 \times 1}{(3 \times 2 \times 1)(3 \times 2 \times 1)}$. We can see that we can cancel the $3 \times 2 \times 1$ on top and bottom to get $\frac{6 \times 5 \times 4}{3 \times 2 \times 1}$ which can reduce further to $2 \times 2 \times 5 = 20$.

Proportions

1. D. Use cross multiplication to solve, giving us $12x = 15 \times 16$. Isolate x to find its value: $12x = 240$, so $x = 20$.
2. D. Use cross multiplication to solve, giving us $3x = 4 \times 5$. Solve for x : $3x = 20$, so $x = \frac{20}{3} = 6\frac{2}{3}$.
3. B. Similar triangles will have proportional corresponding sides. The information in the question tells us that $\frac{LM}{PQ} = \frac{MN}{QR}$. We can substitute the known values given, and use x to represent the unknown value: $\frac{8}{4} = \frac{6}{x}$. Cross multiply to solve: $8x = 24$, so $x = 3$ cm.
4. A. We can represent this proportional relationship as $\frac{6}{20} = \frac{x}{120}$. The corresponding measures should align with each other in the equation. Use cross multiplication to solve. $20x = 720$, so $x = 36$ feet.

5. D. We can represent this proportional relationship as $\frac{16}{3} = \frac{x}{48}$ (be sure that the numbers representing tickets and points are in the right place – either both in the numerator, or both in the denominator). Use cross multiplication to solve: $3x = 768$, so $x = 256$.
6. C. First convert everything to the same unit. 1 hour is 60 minutes, so 2 hours is $60 \times 2 = 120$ minutes. If Ingrid usually runs 12 miles in 2 hours, or 120 minutes, we can represent this proportional relationship as $\frac{12}{120} = \frac{x}{40}$. Use cross multiplication to solve. $120x = 480$, so $x = 4$.
7. D. Similar triangles have proportional corresponding sides. If CB is x and FE is $\frac{1}{2}x$, then AB is twice the length of DE . So the length of AB is $2 \times 8 = 16$.
8. D. Similar triangles have proportional corresponding sides. If GI has a length of $\frac{3}{4}x$, then GH is $\frac{3}{4}$ the length of JK , or $\frac{3}{4}y = 12$, where y is the length of JK . Solving for y gives a length of JK of 16.
9. D. An inch represents 6 yards, which is the same as 18 feet. 2.5 feet is equal to 30 inches. So, $30 \times 18 = 540$ feet.
10. A. In this question, you can use process of elimination to find the answer. If 4 cows can eat all the grass in 10 days, 5 cows will eat the same amount of grass in less time. So, of the choices, 8 is the only possible answer.
11. A. In this question, you can use process of elimination to find the answer. If 2 hoses can fill a bucket in 5 hours, 5 hoses will fill it in less time. So, 2 hours is the only possible answer.
12. A. Use cross-multiplication to solve for x . $6x = 225$, so $x = 37.5$. The value in Column A is greater.
13. C. All sides of DEF $\frac{4}{3}$ length of ABC , so the length of AB can be shown as $12 = \frac{4}{3}y$, where y is equal to the length of AB . Solving for y shows that the length of AB is 9. Thus, the two amounts are equal.
14. B. If a 6-foot man casts a 4-foot shadow, then the proportion of person-height to shadow-length can be shown as $\frac{6}{4} = \frac{40}{x}$, where x is the length of the shadow of the flagpole. Cross multiply to solve: $6x = 160$, so $x \approx 26.67$. Thus, the value in Column B is greater.
15. B. Each textbook costs $51 \div 3 = \$17$. If Jennifer were to be able to buy 8 textbooks, it would cost her $8 \times 17 = \$136$. Column A says she only has \$130, so she can only buy fewer than 8 books (the exact amount doesn't matter). Thus, the value in Column B is greater.
16. B. Note that the units in Columns A and B are given in minutes, but the question gives Cho's reading rate in hours. If it takes Cho 1 hour = 60 minutes to read 6 pages, and he has to read 78 pages, then it will take Cho $78 \div 6 \times 60 = 780$ minutes to finish reading these pages.

Geometry

Geometry with Variables

- A. The area of a rectangle is equal to the length times the height. Because a square has equal length and width, the area of this square is $x \times x$ or x^2 .
- A. The perimeter of a shape is the sum of the length of each side. Since a square has 4 equal sides, the perimeter is equal to $y + y + y + y$ or $4y$.
- D. The volume of a rectangular prism is equal to the length times the width times the height. Since all sides of a cube have the same measurement, the volume of this cube is $a \times a \times a$ or a^3 .

4. C. The formula for the area of a rectangle is $A = lw$, where l represents the length and w represents the width. In this case, we are told that the length and width are a and b , so the area of this rectangle is ab .
5. C. The perimeter of this rectangle is $c + d + c + d$ or $2c + 2d$.
6. B. As we have seen, the perimeter of a rectangle is $2l + 2w$, so the perimeter of this rectangle is $2(1.5m) + 2(0.5n) = 3m + n$.
7. C. $A = lw$, so the area of this rectangle is $A = 0.5f \times 1.5g = 0.75fg$.
8. A. The volume of this cube would be equal to abc , but since this is a cube, $a = b = c$. So the volume of the cube could be expressed as $b \times b \times b$ or b^3 .
9. D. The volume of the rectangular prism on top is equal to y^3 ($V = lwh$). The length and width of the rectangular prism on the bottom are both equal to $y + 2$, so the area is $y(y + 2)(y + 2)$. The height, however, is y . This means the volume of the rectangular prism on the bottom is $y(y + 2)(y + 2)$. The combined areas of the rectangular prisms is therefore $y^3 + y(y + 2)(y + 2)$.
10. B. If the radius were doubled, the formula would be $A = \pi(2r)^2$. Since $2^2 = 4$, the area would be multiplied by 4. We can test this using simple values for r . If $r = 1$, then $A = \pi$. If $r = 2$ (doubling r), then $A = 2^2\pi$, or 4. This is 4 times more than 1.
11. C. Let the area of the blue circle equal πr^2 . The area of the red circle would have an area of $9\pi r^2$, since it is 9 times bigger than that of the blue circle. Thus, if the radius of the blue circle is r , the radius of the red circle is $3r$, since $\sqrt{9r^2} = 3r$. If the radius of the red circle is 3 times the radius of the blue circle, the circumference of the blue circle will be $\frac{1}{3}$ the circumference of the red circle (since the circumference of the blue circle would be $2\pi r$, and the circumference of the red circle would be $2\pi(3)r$, or $6\pi r$).
12. D. Simply let the radius of Sphere B equal 1. This means the volume of Sphere B = $\frac{4}{3}\pi$. If the radius of Sphere A is 3 times that of Sphere B, then it is 3. The volume of Sphere A would be $\frac{4}{3}\pi 3^3$.
Comparing these volumes, we can see that the only difference is the fact Sphere A has a volume that is $3^3 = 27$ times greater than Sphere B.
13. C. Each side of Cube R is twice the length of Cube Q, so if the length of each side of Cube Q is q , then the volume of Cube Q is q^3 and the volume of Cube R is $(2q)^3$. So the volume of Cube R will be 2^3 or 8 times the volume of Cube Q. We can prove this using the values given: $2^3 = 8$, and $4^3 = 64$. $64 \div 8 = 8$.
14. C. We can use simple substitution to help us solve this. If $x = 1$, then the volume of Cube W = $1^3 = 1$. Each side of Cube V would be $3(1) = 3$, so its volume would be $3^3 = 27$. $27 \div 1 = 27$.
15. C. The area of a circle is equal to πr^2 . So, the area of Circle A = $\pi(12)^2 = 144\pi$. This means that the area of Circle B is $144\pi \div 4 = 36\pi$. Next, simply solve for the radius using the formula for the area of a circle: $36\pi = \pi r^2$. The π cancels out, leaving us with $r^2 = 36$, and $r = 6$.
16. D. Without knowing the values of x or y , there's no way to determine which square is greater.
17. D. Without knowing the values of a or b , there's no way to determine which perimeter is greater.
18. D. The perimeter of Rectangle M is $2(5x + 6) + 2(2x + 4) = 14x + 20$. The perimeter of Rectangle N is $2(7x + 5) + 2(x + 5) = 16x + 20$. While the side lengths of both rectangles must be positive, the value of x does not. If $x > 0$, then Rectangle N has the greater perimeter. However, x could equal 0, in which case the side lengths are still positive, and the perimeters are equal. The only limitation is that x must be greater than $-\frac{5}{7}$. Therefore, there is no way to determine which perimeter is greater.
19. D. The volume of the rectangular prism is $x(x + 2)(x + 3) = x(x^2 + 5x + 6)$. We can distribute further for $x^3 + 5x^2 + 6x$. At first glance, Column A looks greater, but if $x \leq 1$, Column B could be greater. Since we don't know the value of x , we don't have enough information to determine which is greater.

20. D. If the square has a side length of 6, it has a perimeter of 24. (The perimeter of a square is equal to $4l$.) The triangle also has a perimeter of 24, but since we don't know what kind of triangle it is, we don't have enough information to determine the length of the base.

Angle Sums

1. B. The sum of the 3 interior angles of all triangles is 180° . Applying the given formula, where n represents the number of sides of the polygon: $180(3 - 2) = 180(1)$.
2. B. The sum of the 5 interior angles of a pentagon is 540° . Applying the given formula, where n represents the number of sides of the polygon: $180(5 - 2) = 180(3) = 540$.
3. C. The sum of the 8 interior angles of an octagon is $1,080^\circ$. Applying the given formula, where n represents the number of sides of the polygon: $180(8 - 2) = 180(6) = 1,080$.
4. B. A dodecagon has 12 sides and 12 angles. Applying the given formula, where n represents the number of sides of the polygon: $180(12 - 2) = 180(10) = 1,800$. So, the sum of the interior angles of a dodecagon is $1,800^\circ$. A single angle is equal to $1,800 \div 12 = 150^\circ$, since we are told that the dodecagon is regular.
5. B. Find the sum of the 6 interior angles of a hexagon using. We know that the sum is $180(6 - 2) = 720^\circ$. Since the hexagon is described as regular, we know that each of the angles (and sides) have an equal measure. This means we can divide the total sum of angles by the number of sides to find the measure of each individual angle. This is $720 \div 6 = 120^\circ$. Since AB is a straight line with a degree measure of 180° , and since x is a supplementary angle to the angle inside the hexagon, we know that $180^\circ = 120^\circ + x^\circ$. This means $x = 60^\circ$.
6. A. The sum of the interior angles of a triangle is 180° . So, $180 = 45 + 54 + x$, where x is the measure of the unknown angle. $180 = 99 + x$, so $x = 81^\circ$.
7. C. A right angle has a measure of 90° . So the unknown interior angle of the triangle is equal to y in the equation $90 + 55 + y = 180$. So, $y = 35$. A straight line is equal to 180° , so $35 + x = 180$, and $x = 145$. This is because x is a supplementary angle to the unknown interior triangle angle.
8. D. As we have seen, the sum of the interior angles of a quadrilateral is 360° . An isosceles trapezoid has two sets of equal angles. So, if the two base angles are 65° , the measure of one of the remaining two angles have a sum that can be represented by x in the equation $2(65) + 2x = 360$, or $2x + 130 = 360$. Solving for x yields $x = 115^\circ$.
9. D. Adjacent angles in a parallelogram are supplementary, meaning they have a sum of 180° . If angle A has a measure of 32° , angle B has a measure of $180 - 32$, or 148° .
10. C. The sum of the interior angles of a pentagon is 540° (since $180(5 - 2) = 540$). Let x represent the unknown angle measure. $141 + 82 + 131 + 103 + x = 540$. So, $x = 83^\circ$.
11. D. The sum of the interior angles of an octagon is $1,080^\circ$ (since $180(8 - 2) = 1,080$). Let x represent the unknown angle measure. $125 + 132 + 134 + 125 + 115 + 165 + 106 + x = 1,080$. So, $x = 178^\circ$.

Polygons on a Coordinate Grid

1. D. In order for the shape to be a rectangle, the fourth point would need to be at $(-5, -2)$. Remember that a rectangle has two sets of parallel sides. Quickly sketch a coordinate plane and plot the given points to visualize where the last coordinate would need to be.
2. A. The fourth point is at $(6, -2)$. Remember that a square has four equal sides meeting at right angles. Quickly sketch a coordinate plane and plot the given points to visualize where the last coordinate would need to be.
3. B. A rhombus has two sets of parallel sides. Since the distance between $(3, 1)$ and $(3, -4)$ is 5 spaces on the grid, the unknown point must be 5 spaces below $(-1, 4)$, which puts the point at $(-1, -1)$.
4. B. Quickly sketch the vertices given and eliminate options where the resulting triangle is obviously not a right triangle, which has an angle of 90° . Then, confirm that the two legs of the triangle have perpendicular slopes. In this case, it appears that $(2, -5)$ could be the correct vertex. The slope of the

line from $(2, -5)$ to $(-2, 1)$ is $\frac{1-(-5)}{-2-2} = -\frac{3}{2}$. The slope of the line from $(1, 3)$ to $(-2, 1)$ is $\frac{1-3}{-2-1} = \frac{2}{3}$.

The two slopes are negative reciprocals, showing that we have a right angle.

5. A. The remaining point must be the same distance from $(2, 3)$ as $(-3, 3)$ is from $(-4, -2)$, so the remaining point is located at $(3, -2)$.
6. D. If the third point is at $(-4, 3)$, we know this must be an obtuse triangle (a triangle with an angle measuring more than 90°) because a right angle would form if the bottom-left-most vertex was anywhere that $x = -4$. Since the third vertex is located at $x = -5$, the resulting angle is obtuse. We can see that a right angle forms if the vertex is at $(0, 0)$, because the slope of the intersecting lines is -1 and 1 . We can see visually that the remaining choices result in acute triangles.

Distance on a Coordinate Grid

1. C. This is a straight line 3 units above the x axis, since y is the same for both points. Therefore, to find the length, find the difference between the x coordinates: $6 - 2 = 4$.
2. D. This is a vertical line 4 units from the y axis, since x is the same for both points. Therefore, to find the length, find the difference between the y coordinates: $5 - (-2) = 7$ coordinate units.
3. C. This line will be diagonal, since neither coordinate shares the same x or y value. We can find the length of a diagonal line by thinking of it like the hypotenuse of a right triangle. If the points are located at $(3, 3)$ and $(7, 6)$, we can draw a right angle at $(7, 3)$. One side of this triangle is 4 units long, and one side is 3 units long. If we apply the Pythagorean theorem, this is a 3-4-5 triangle: $(3^2 + 4^2 = c^2)$, or $9 + 16 = 25$, so $c^2 = 25$ and $c = 5$.) Therefore, the distance between the points is 5 units.
4. C. As we have seen, we can treat a diagonal line like the hypotenuse of a right triangle. The right angle would be located at $(4, 3)$. One side is 8 units long and one side is 6 units long, so this makes a 3-4-5 triangle in which each side is multiplied by 2. The distance between the two points is 10 units.
5. A. As we have seen, we can treat a diagonal line like the hypotenuse of a right triangle. The right angle would be located at $(4, 3)$. One side is 12 units long, and one side is 9 units long, so this makes a 3-4-5 triangle in which each side is multiplied by 3. The distance between the two points is 15 units.
6. C. XZ has a length of 8 and XY has a length of 6. This makes a 3-4-5 triangle in which each side is multiplied by 2. So the hypotenuse is equal to 10.
7. B. The diameter of a circle will be a line between the two points on the circle's circumference. We can draw a right angle at $(2, 13)$. So the length of one side is 12 units and the length of the other is 16. This makes a 3-4-5 triangle in which each side is multiplied by 4. So the distance between the two points is 20 units.
8. A. Sketch the given information. The triangle described is another 3-4-5 triangle in which each side is multiplied by 4. The missing side is 16 units long. The y coordinate will be 18, since this is a horizontal line (the x value for both G and H is 5), and the x coordinate can be found by adding or subtracting 16 from 5 (which equals -11). So the point is located either at $(-11, 18)$ or $(21, 18)$; only the latter is given as an answer choice.
9. B. The midpoint of the line will have an equal number of units on either side of the point. Since the line is 8 units long, the midpoint will be 4 units from each point and will be located at $(1, 2)$. Since m is the x coordinate, $m = 1$.
10. D. We can test the values of each coordinate to find that the equation is only true for the coordinates of Point D, which is located at $(-5, -1)$. In this instance $|-5| - |-1| = 5 - 1 = 4$.
11. C. Think of the diagonal distance between the two points as a hypotenuse of a right triangle. The length of the leg on the x -axis is $7 - (-5) = 12$ and the length of the leg on the y -axis is $3 - (-2) = 5$. This makes the shape a right triangle with legs of 5 and 12. Recalling our special right triangles, we therefore know that the hypotenuse is 13.

Nets

1. D. To match a net with its cube, make sure the positions of the images on each face match. Start with known information, like the fact that the bottoms of 4 and 5 are pointing to a common edge. This is only the case in the last two choices. From there, we can see the relationship between 6 and 5. This time, the top of the 6 is pointing in the same direction as the bottom of the 5, which is only the case in the last choice.
2. B. We know that A, B, and C cannot line up in a row, as in the first choice, because they are on the 3 visible sides of the cube shown. A and C are oriented in the same direction, which means when flattened, C will not be oriented in the same direction. This rules out the third choice. The last choice shows us that the bottoms of A and C would be facing the same common edge, which is not the case in the cube. Therefore, all that remains is the second choice.
3. D. In this case, all that matters is the side with the 1, 3, and 4 dots. When folded up, the net must show that those three shapes will be converging at a single vertex of the cube. This is not the case in the first choice (1 and 4 end up on different sides of the cube). This is not the case in the third choice (1, 3, and 4 end up in a row, not touching at a vertex). 1, 3, and 4 touch at a vertex in the second choice, but the orientation is switched (4 and 3 are swapped). It may not look like it, but only in the last choice do 1, 3, and 4 touch in a vertex. If rotated, the resulting cube would show 1 “on top” and 4 “in front” with 3 “on the side.”
4. A. We must have a cross, triangle, and circle converging at one vertex. We can see that this is true in the first choice. The last choice puts the cross in the wrong place. The third choice is incorrect, as the orientation of the triangle is would be wrong, though the three shapes converge on a vertex. This is the same reason the second choice is incorrect.
5. C. There is only one star, so we can use this as a clue. In the first choice, though the hexagon, heart, and star converge on a vertex, the orientation of the heart is incorrect. In the second choice, it is not possible for the two hearts to be folded such that they are side by side. We can, however, have two hearts in a row, with a hexagon on the other side. This is correct in the third choice. The last choice shows the hearts with bottoms facing the same edge, which is impossible.
6. A. The only possible result shows two circle adjacent (the rightmost and bottommost squares would touch).
7. A. Use the points of the arrows as a way to help in this case. In no case will three sides of the cube have arrows facing in the same direction, eliminating all but the first choice.
8. A. Since two parts of the net are pentagons, the net will make a pentagonal prism.

Congruent Triangles

1. A. The diagram shows that the triangles have a congruent angle, a second congruent angle, and a congruent side, in that order. This proves the AAS postulate.
2. D. Using the Pythagorean Theorem, you can find that the missing side of the first triangle is 5 and the missing side of the second triangle is 3. Then, the diagram would prove three side congruent using the SSS postulate.
3. C. The diagram shows two pairs of congruent sides. The angles between those two sides are vertical angles formed by two intersecting lines, and vertical angles are congruent. Therefore, the diagram shows that the triangles have a congruent side, a congruent angle, and a congruent side, in that order. This proves congruence using the SAS postulate.
4. C. The diagram shows that the triangles have a congruent side, a congruent angle, and a congruent side, in that order. This proves congruence using the SAS postulate.
5. A. Dilation is the only transformation that can change the size of an image. When an image’s size changes, it is no longer congruent to the original image.
6. D. Using the Pythagorean Theorem, you can find that the missing side of the first triangle is 13 and the missing side of the second triangle is 12. With the missing measure, all three postulates can be used to prove that the two triangles are congruent.

Trigonometry

1. D. The abbreviation SOH CAH TOA can help you remember that the sine is the opposite side over the hypotenuse, the cosine is the adjacent angle over the hypotenuse, and the tangent is the opposite over the adjacent side. Since $\frac{4}{3}$ shows the opposite over the adjacent sides, of x , it must be $\tan x$.
2. A. In this triangle, $\frac{5}{13}$ the adjacent side over the hypotenuse (the CAH in SOH CAH TOA), so it must be $\cos x$.
3. A. The given value is the adjacent side over the hypotenuse, so it must be $\cos x$.
4. C. The given value is the opposite side over the hypotenuse, so it must be $\sin x$.
5. D. We've been given an angle and an adjacent side and need to find the hypotenuse. So, $\cos 64^\circ = \frac{4}{x}$, where x is the length of DF . Therefore, $x = \frac{4}{\cos 64^\circ}$.
6. B. HI is the adjacent side to the 34° angle, and we've been given the opposite side. So, $\tan 34^\circ = \frac{2}{x}$, where x is the length of HI . So, $x = \frac{4}{\tan 34^\circ}$.
7. A. We've been given the hypotenuse and need to find the opposite side. So, $\sin 22^\circ = \frac{x}{4}$, where x is the length of KL . So, $x = 4(\sin 22^\circ)$.
8. C. We've been given the adjacent side and need to find the opposite side. So, $\tan 42^\circ = \frac{x}{11}$ and $x = 11(\tan 42^\circ)$.
9. B. We've been given the opposite side and need to find the hypotenuse. So, $\sin 46^\circ = \frac{9}{x}$ and $x = \frac{9}{\sin 46^\circ}$.
10. B. We've been given the adjacent side and need to find the hypotenuse. So, $\cos 10^\circ = \frac{8}{x}$ and $x = \frac{8}{\cos 10^\circ}$.

MeasurementsAppropriate Units

1. D. Water in a bathtub is best measured in units of liquid capacity. Of the units given, only liters measure liquid capacity.
2. D. Ants are very small, so a smaller unit of length is the most reasonable unit. Feet are too large. Milliliters and milligrams measure liquid capacity and mass, respectively, so millimeters is the correct answer.
3. D. Cars are very heavy, so the most reasonable unit should be a large unit of mass. Cubic feet and square feet measure volume and area, respectively, and grams are too small. Tons is the correct answer.
4. B. A skyscraper is very tall and should be measured in units of length. Kiloliters measure capacity, tons measure mass, and square feet measure area, so meters is the correct answer.

5. C. The distance between cities would be significant and should be measured in a unit of length. Tons is a measure of mass. Centimeters and feet are both small units of length. The most reasonable unit is kilometers.
6. C. An eyedropper holds only a small amount of liquid. It requires a unit of capacity. Gallons is too large, so milliliters is the correct answer.
7. D. Grams and mass are a unit of mass and acres are a unit of area. Since we're measuring the length of a city block, yards is the correct unit.

Area & Perimeter

1. B. A square has 4 equal sides. The area of a square is equal to the square of a length of a side, so each side of this square measures 6 centimeters. This is because $6 \times 6 = 36$, which is also $6^2 = 36$. The inverse operation of $6^2 = 36$ is $\sqrt{36} = 6$. This gives us the length of one side. The perimeter is equal to the sum of each side, so the perimeter of this square is equal to $6 + 6 + 6 + 6 = 4(6) = 24$ centimeters.
2. D. A square has 4 equal sides. The perimeter is the sum of each side, so to find the length of each side, divide the perimeter by 4: $36 \div 4 = 9$ centimeters. The area of a square is equal to one side squared, so the area is equal to $9^2 = 81$ square centimeters.
3. D. Subtract the area of the cutout square from the larger square. We can represent the area of the larger square as 10^2 and the smaller square as 2^2 , so the area of the shaded part is $10^2 - 2^2$.
4. C. Subtract the area of the cutout square from the larger square. The area of the larger square is a^2 and the area of the cut out is b^2 , so the area of the shaded part is $a^2 - b^2$.
5. D. The area of a circle is equal to πr^2 . In this circle, $r = 2$, so the area is equal to 4π . The larger square has an area of $5^2 = 25$, so the area of the shaded region is $25 - 4\pi$.
6. D. The rounded corners each represent a fourth of a circle with a radius of 2, so the area of this circle is equal to 4π . ($\pi(2)^2 = 4\pi$). If you draw lines to mark the four circle quarters, you will see that what remains are two 2×2 squares and a rectangle that measure 4×6 . So, the area of this shaded region is equal to $(2 \times 2) + (2 \times 2) + (4 \times 6) + 4\pi$. This simplifies to $4 + 4 + 24 + 4\pi = 32 + 4\pi$.
7. D. If the length is 10 inches, so is the diameter of the circle, so the radius of the circle is 5, and the area of the circle is equal to $\pi(5)^2$ or 25π . The area of the square is equal to $10^2 = 100$. The shaded region is equal to the area of the square minus the area of the circle, or $100 - 25\pi$.
8. C. The trapezoid is equal to the area of the rectangle minus the right triangle cut out of it. In this case, the rectangle has an area of $12 \times 4 = 48$. The triangle has one side equal to $12 - 9 = 3$ and one side equal to 4. Therefore, the triangle has an area of $\frac{1}{2}(3 \times 4) = 6$. The trapezoid has an area of $48 - 6 = 42$ units.
9. C. The gray square has a side length of 6 inches, or is 4 triangles that each have two sides that measure 3 inches. The area of one of these triangles, then, is equal to $\frac{1}{2}(3 \times 3) = 4.5$. The four triangles combined have an area of $4.5 \times 4 = 18$ square inches.
10. C. If each square has a perimeter of 16 in^2 , then each square measures 4 inches on each side. There are 16 4-inch sides, so the perimeter is equal to $4 \times 16 = 64$ inches.
11. A. There are 11 shaded squares. If the figure has a total area of 99 ft^2 , then the area of each square is equal to $99 \div 11 = 9 \text{ ft}^2$. Each square has a side equal to the square root of the area of each small square, or 3 ft.
12. A. The area of each square is equal to $\frac{1}{2}^2 = \frac{1}{4}$. There are 16 shaded squares, so the area of the shaded portion is equal to $16 \times \frac{1}{4} = 4 \text{ in}^2$.

13. A. The area of the shaded region is equal to the area of the smaller rectangle subtracted from the area of the larger rectangle. So, the area is equal to $(7 \times 14) - (5 \times 2) = 98 - 10 = 88 \text{ in}^2$. So, the value in Column A is greater.
14. B. The shaded area is a 3 foot-wide path around the pool, so the area of the shaded portion is equal to the area of the larger rectangle minus a rectangle with the measurements $28 - 6$ and $15 - 6$ (remember 3 feet on each side!). The area of the larger rectangle is $28 \times 15 = 420$. The area of the smaller rectangle is $22 \times 9 = 198$. So, the area of the shaded region is $420 - 198 = 222$ square feet. Therefore, the value of Column B is greater.
15. A. The area of the shaded region will be equal to the area of the larger square minus the area of the smaller square, or $x^2 - y^2$. $(x - y)^2$, the value in Column B, represents the area of only the top-right portion of the shaded area, or $x - y$ times $x - y$. This will always be less than the entire shaded area.
16. A. The area of the path is equal to the area of the larger rectangle minus the area of the garden. The garden has an area of 16 by 12 or $16 \times 12 = 192$. The path is 4 feet greater on each side, so the larger rectangle has an area of 20 feet by 24 feet, or $20 \times 24 = 480$. So, the area of the path is equal to $480 - 192 = 288$ feet. This is greater than the value of Column B.

Speed

1. C. If Train B traveled twice the distance in half the time, it must have traveled 80 miles in 1 hour, so the miles per hour is also 80. (Twice the distance is $40 \times 2 = 80$, and half the time is $2 \text{ hours} \div 2$.)
2. B. If Alice is jogging 6 miles per hour, she will jog 15 miles in 2.5 hours. ($15 \div 6 = 2.5$.) Bob jogs the same distance in 3 hours ($15 \div 5 = 3$) So, Bob takes 0.5 hours longer, which is equal to 30 minutes.
3. C. Nancy skates 36 kilometers in $36 \div 12 = 3$ hours. Mike skates 36 kilometers in $36 \div 8 = 4.5$ hours. So it takes Mike 1.5 hours longer to skate the same distance. 1.5 hours = 90 minutes.
4. A. Owen swims 200 meters in $200 \div 50 = 4$ minutes. Penny swims $200 \div 40 = 5$ minutes. So it takes Penny 1 minute more.
5. B. Caitlin drives 50 kilometers per hour for 6 hours, or $50 \times 6 = 300$ kilometers. David drives 70 kilometers per hour for 6 hours, or $70 \times 6 = 420$ kilometers. So, David drives 120 kilometers ($420 - 300$) farther.
6. C. Frances bikes 12 miles per hour for 2.5 hours, she travels $12 \times 2.5 = 30$ miles. Elon rides at 10 miles per hour, so it will take him 3 hours to bike 30 miles. ($30 \div 10 = 3$) So Frances will wait for 0.5 hours, or 30 minutes, for Elon to catch up.
7. D. Katherine's speed is 2 miles per hour, so if she travels for x hours, her distance will be $2x$. Leon travels 6 mph, and he starts 1 hour after Katherine, so his distance is $6(x - 1)$. We want to know when he catches up to her (in other words, when they've traveled the same distance), so $2x = 6(x - 1)$, which simplifies to $2x = 6x - 6$. Solving for x results in $4x = 6$, or $x = 1.5$ hours. However, that is the time Katherine traveled. Leon traveled $x - 1$, or 0.5 hours (which is 30 minutes).
8. C. It took Helen 40 minutes to canoe 3 miles, so she canoes 1 mile in $13\frac{1}{3}$ minutes ($40 \div 3$). $60 \div 13\frac{1}{3} = 4.5$, so Helen canoes at 4.5 miles per hour. Alternatively, set up a proportion:

$$\frac{1 \text{ mile}}{13.333 \text{ minutes}} = \frac{60 \text{ minutes}}{1 \text{ hour}}$$
9. B. In order to know who won, we need to know how long they each took to skate across the lake. We are told Randy's time was 10 minutes. We only know that Quentin's speed was 10 mph. In order to know how much time it took for him to finish, we need to know the distance. Only choice B gives us the distance.
10. D. We know only that Ursula raced at a faster speed and that Victor started a mile ahead. If we know how much faster Ursula is traveling than Victor, we can calculate the time it will take her to catch up. Choice D tells us that Ursula was travelling 2 mph faster than Victor, so each hour she will gain 2 miles on him.

11. C. As we have seen, if Sue is traveling at a faster speed, but Thomas has a 1 mile head start, we need to know how much faster Sue is traveling than Thomas in order to calculate the time it will take her to catch up.

Converting Units

1. B. To convert the units start by writing the given rate as a ratio. The man walks $\frac{90 \text{ yards}}{1 \text{ minute}}$, so to convert to feet per second, multiply 90 by 3 (3 feet in a yard) and 1 by 60 (60 seconds in a minute). The resulting expression is $\frac{90 \times 3}{60}$.
2. A. First write as a ratio: $\frac{2 \text{ meters}}{1 \text{ second}}$. We want to convert this to feet per minute. Multiply 2 by 60 to find how many meters the woman jogs in a minute, then divide by 0.305 to find how many feet she traveled in 1 minute. Written as an expression, this is $\frac{2 \times 60}{0.305}$.
3. A. The cheetah's speed is $\frac{70 \text{ miles}}{1 \text{ hour}}$. To convert this to yards per minute, multiply 70 by 1,760 and 1 by 60: $\frac{70 \times 1,760}{60}$.
4. D. Brenda runs $\frac{6 \text{ miles}}{1 \text{ hour}}$. To convert to feet per second, multiply 6 by the number of feet in a mile and 1 hour by the number of seconds in an hour (there are 60 minutes in an hour and 60 seconds in a minute). So, we can write this as the expression: $\frac{6 \times 5,280}{1 \times 60 \times 60}$, or simply $\frac{6 \times 5,280}{60 \times 60}$.
5. D. The snail travels at 2 inches per minute. Multiply by 60 to find how many inches the snail travels in 1 hour: $2 \times 60 = 120$. This is equal to 10 feet ($120 \div 12 = 10$). So the snail travels at 10 feet per hour.
6. A. The cyclist bikes at $\frac{5 \text{ yards}}{1 \text{ second}}$. Multiply the yards by 3 to find how many feet per second, then multiply by 12 to find inches per second. Then, multiply by 60 to find the number of inches per minute, then by 60 again to find the inches per hour. This can be represented by the expression $5 \times 3 \times 12 \times 60 \times 60$.
7. D. The plane travels at a rate of $\frac{614 \text{ miles}}{1 \text{ hour}}$. Multiply the distance by 5,280 and then by 12 to find the number of inches. Then, multiply the hours by 60 to convert to minutes, and by 60 again to convert minutes to seconds. This can be represented by the expression $\frac{614 \times 5,280 \times 12}{1 \times 60 \times 60}$, or simply $\frac{614 \times 5,280 \times 12}{60 \times 60}$.

Formulas

1. C. We can plug numbers into the formula and then solve for the remaining variable. With the given information, this equation is $100 = \frac{1}{2}(2)b$, which simplifies to $b = 100$.
2. C. The radius is half of the diameter, so the area of this circle is equal to $A = \pi(18 \div 2)^2$, which simplifies to $A = 81\pi$.
3. B. Plug the numbers into the formula: $A = \frac{1}{2}(10 + 12)4$, which simplifies to $A = 44$.
4. B. Plug the numbers into the formula. Since the area of the base is given, we know that $lw = 24$. Therefore, $V = \frac{(24)(6)}{3} = 48 \text{ feet}^3$.

5. C. Plug the numbers into the formula: $SA = 2(6)(8) + 2(6)(10) + 2(10)(8)$, which simplifies to $96 + 120 + 160 = 376$ square meters.
6. B. Plug the numbers into the formula. $24\pi = 2(4\pi) + 4\pi h$. So, $16\pi = 4\pi h$, and $h = 4$. So, the vertical height of the cylinder is 4 inches. Remember that you cannot simply subtract $4\pi h$ from both sides.
7. D. Plug the numbers into the formula: $F = \frac{9}{5}(25) + 32$, which simplifies to $45 + 32 = 77^\circ\text{F}$.
8. B. Plug the numbers into the formula: $70\pi = 10\pi + \frac{1}{2}(12\pi)l$. Or, $60\pi = 6\pi l$, and $l = 10$ inches.
9. D. Plug the numbers into the formula: $100 = \frac{5}{9}(F - 32)$. Multiplying both sides by 9 and distributing the remaining 5 gives us $900 = 5F - 160$, so $1,060 = 5F$, and $F = 212^\circ\text{F}$.
10. C. Plug the numbers into the formula: $16\pi = \frac{1}{3}\pi r^2(3)$. So, $16\pi = \pi r^2$. Therefore, $r^2 = 16$ and $r = 4$.
11. C. Plug the numbers into the formula: $36\pi = \pi r^2$. The π on both sides cancel, leaving $36 = r^2$, so $r = 6$. Since the diameter is $2r$, we know that the diameter is $2(6) = 12$ cm.

Data & Probability

Probability

1. C. The probability of an event happening is equal to the number of ways it can happen over the total number of outcomes. There are 12 marbles in all, and 4 of them are white, so there are 4 ways this outcome can happen and 12 possible outcomes. Therefore, the probability that a white marble will be chosen is equal to $\frac{4}{12} = \frac{1}{3}$.
2. B. The probability of an event happening is equal to the number of ways it can happen over the total number of outcomes. In this case, the total number of outcomes is equal to the total number of candies in the bowl. Based on the given ratio, the total number of candies must be a multiple of 6, so 15 is not a possible total number of candies.
3. B. The probability of a coin landing on heads is $\frac{1}{2}$, since heads is one of two sides of the coin. The probability of that happening twice is equal to $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.
4. B. If the probability of drawing a boy's name is $\frac{2}{5}$, then $\frac{2}{5}$ of the students are boys. If there are 35 students in the class, multiply by $\frac{2}{5}$ to find the total number of boys: $35 \times \frac{2}{5} = 14$.
5. A. The probability of a coin landing on heads is $\frac{1}{2}$. The probability of that happening 4 times in a row is equal to $\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$.
6. C. In order to roll a 12, Dustin will need to roll a 6 on each die. The probability of rolling any number on a six-sided die is $\frac{1}{6}$ so the probability of rolling 2 sixes is $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.
7. B. The probability of pulling a queen out of the deck is $\frac{4}{52} = \frac{1}{13}$. Since the first card is chosen then replaced back into the deck, the probability of pulling two queens is $\frac{1}{13} \times \frac{1}{13}$.

8. C. The probability of pulling an ace from the complete deck is $\frac{4}{52} = \frac{1}{13}$. If the card is not put back in the deck, then the probability of pulling a second ace is $\frac{3}{52} = \frac{1}{17}$. So, the probability of pulling an ace, keeping it (not putting it back) and then pulling an ace again is equal to $\frac{1}{13} \times \frac{1}{17}$.
9. A. The probability of picking a green grape after eating a red grape (not replacing it into the bowl) can be represented by x in the equation $\frac{1}{4} \times x = \frac{1}{8}$. Solving for x yields $\frac{1}{2}$.
10. C. If there are 10 apples in the bucket, then the probability of pulling a red apple is equal to the total number of red apples over 10. The probability of pulling the second red apple is equal to the total number of apples remaining over 9. So, $\frac{a}{10} \times \frac{a-1}{9} = \frac{12}{90}$, where a is equal to the total number of red apples. So, $a(a-1) = 12$. Solve for a : $a^2 - a - 12 = 0$, so $(a+3)(a-4) = 0$, and a could be equal to -3 or 4 . Since there can't be a negative number of red apples, the number of red apples must be 4 .
11. B. Find the square root of the ratio to find the probability of hitting the green once. If the probability of hitting it twice is $\frac{9}{100}$, this is equal to $x \times x = \frac{9}{100}$, where x is the probability of hitting it once. So, the probability of hitting the green once is equal to $\frac{3}{10}$.
12. A. The probability of rolling a particular number is $\frac{1}{6}$, so out of 30 rolls, the number of 3s expected is $30 \div 6 = 5$.
13. C. The probability of flipping tails once is $\frac{1}{2}$. So, the expected number of tails in 10 flips is $10 \div 2 = 5$.
14. A. The probability of rolling a particular number is $\frac{1}{6}$, so out of 15 rolls, the number of 5s expected is $15 \div 6 = 2.5$.
15. B. The probability of flipping heads once is $\frac{1}{2}$. So, the expected number of heads in 7 flips is $7 \div 2 = 3.5$.
16. B. The probability of flipping a coin to heads or tails is equal, so in five flips, the expected number of heads or tails is equal to 2.5 .
17. B. Sandra has 21 shirts in all, so the probability of her choosing an orange shirt $\frac{6}{21}$. This is less than $\frac{2}{5}$, so the value in Column B is greater.
18. C. If the probability of picking out a name that starts with M is equal to x , then $\frac{2}{15} \times x = \frac{1}{45}$. Solving for x , we find that $x = \frac{15}{90} = \frac{1}{6}$. The values in both columns are equal.
19. A. In 25 flips, the number of expected tails is 12.5 , since the probability of tails is half. This is greater than the value in Column B, so Column A is greater.

20. A. Given the probability, $\frac{1}{4}$ of the apples are red. If twice as many are yellow, then $\frac{1}{2}$ are yellow, and the remaining $\frac{1}{4}$ are green. So, the probability of picking a green apple is $\frac{1}{4}$. This is greater than $\frac{1}{8}$ so the value in Column A is greater.
21. A. The color of the first ball doesn't matter. The only thing that matters is that the second ball is the same as the first. The probability of picking any particular color in his second draw is $\frac{1}{10}$.
22. B. The probability of any side on a 6-sided die being rolled is $\frac{1}{6}$. Therefore, if a die is rolled 85 times, the expected number of 5's is approximately $\frac{1}{6} \times 85 = 14.2$. Thus, Column B is greater.

Probability – Conditional

1. D. The probability of rolling a 6 doesn't change based on previous events. Since there are 6 sides to the number cube, there is a 1 in 6 chance of rolling a six each time it is rolled. Therefore, the probability of rolling a 6 is $\frac{1}{6}$.
2. C. If Tom has removed three 10s from the deck, there is only 1 ten remaining and 49 cards. So the probability that Tom will pull the last 10 is $\frac{1}{49}$.
3. C. The probability of picking a red candy or a white candy is $\frac{1}{2}$, since there are a total of 10 candies and 5 each of red and white candies. Picking one and then picking the other has a probability of $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$.
4. C. The question simply asks what the odds are of picking out a white candy if there are 5 white candies in a bowl of 10 candies. It doesn't matter that the first candy picked was red, since she puts it back into the bowl. This is $\frac{1}{2}$.
5. D. There are 18 marbles in all. The probability of Paul picking the first blue marble is $\frac{7}{18}$. Since he removed one blue marble, the probability of Paul picking a blue marble now is $\frac{6}{17}$, since the numerator represents the number of blue marbles, and the denominator represents the total number of marbles. If he does it again, the probability of picking a blue marble is now $\frac{5}{16}$. Therefore, the probability of picking 3 blue marbles in a row is the product of all 3, or $\frac{7 \times 6 \times 5}{18 \times 17 \times 16}$.
6. A. In this question, we only need to find the probability that Paul will pick a blue marble after 2 blue marbles have been removed from the bag. As we saw in the previous question, the probability is $\frac{5}{16}$.
7. C. There are 12 lollipops in the bag. If Jackie takes out 1 green lollipop, there are now 11 lollipops. So the probability of Jackie picking a purple lollipop from the bag is equal to $\frac{6}{11}$.

8. B. There are 12 lollipops in the bag. The probability that Jackie will pick a green lollipop is $\frac{2}{12}$. If she picks lollipops without replacement, that means we are permanently reducing the total number of lollipops in the bag. If she picks out a green lollipop, then there are only 11 lollipops in the bag. If she then picks again, she has a $\frac{6}{11}$ chance of picking out a purple lollipop. The question asks the probability of both things happening, which is $\frac{2}{12} \times \frac{6}{11} = \frac{12}{132} = \frac{1}{11}$.
9. C. The probability that Kevin will roll a 2 is not dependent on his rolling a 1 first, so the probability that Kevin will roll a 2 is the same as Kevin rolling any other number: $\frac{1}{6}$.
10. B. There are 30 pencils. The probability of Damon picking a yellow pencil first is simply the number of yellow pencils divided by the total number of pencils, or $\frac{10}{30} = \frac{1}{3}$. If he does not put the pencil back, the probability of picking a black pencil is $\frac{10}{29}$, since the total number of pencils was reduced from 30 to 29. Thus, the probability of Damon picking a yellow pencil and then picking a black pencil is $\frac{1}{3} \times \frac{10}{29} = \frac{10}{97}$.
11. C. In both scenarios, the previous roll does not affect the current roll. Since a 6-sided die has 3 even numbers, the probability of rolling an even number is equal to $\frac{1}{2}$. The six-sided die also has 3 sides that are prime numbers (2, 3, and 5), so the probability that a prime number will be rolled is also $\frac{1}{2}$. So, the value of both columns is equal.
12. A. The probability that the coin will land on heads is $\frac{1}{2}$ and the probability that an even number will be rolled is also $\frac{1}{2}$, since 3 out of the 6 sides are even. The probability of both things happening is equal to $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. The probability of the coin landing on tails is $\frac{1}{2}$ and the probability of the die landing on 4 is $\frac{1}{6}$, so the probability of both things happening is equal to $\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}$. So, the value in Column A is greater.

Averages

1. D. The average is equal to the sum of all the terms in a set divided by the number of terms. This question is asking you to find the inverse of that: the sum of the terms. If James has an average of 84 on 4 quizzes, then the sum of all 4 quizzes is equal to $84 \times 4 = 336$.
2. A. The average is equal to the sum of all the terms in a set divided by the number of terms. Over 3 games, Bonnie scored 36 points, which is the sum of the terms. Divide 36 by 3 to find the average: $36 \div 3 = 12$.
3. D. This question gives us the average, 12, and the number of games, 4. So the sum of the points Daphne scored is equal to $12 \times 4 = 48$.
4. D. The average height is x inches, and there are 6 students. So, the sum of the heights (or the total height of all six students) is equal to $6 \times x$, or $6x$.

5. C. To earn a 95 average, the sum of Ellen's test scores must be 95 multiplied by the number of tests.
 $95 \times 5 = 475$.
6. A. "Mean" and "average" mean the same thing. The sum of Susie's first four tests was 374 and she earned a 96 on the last test. The sum of all her tests scores, then, is $374 + 96 = 470$. So, the average of all 5 tests is $470 \div 5 = 94$.
7. C. Let x stand for the last test. The average is equal to the sum of the tests divided by the number of tests, so $\frac{92 + 85 + 87 + x}{4} = 90$. Solve for x by first multiply both sides by 4, then add the terms on the left side of the equation: $360 = 264 + x$. So, $x = 96$. Therefore, Paul needs a 96 on the last test.
8. B. A set of six numbers has a mean of 12. If the mean is 3 more, or 15, and x represents the unknown new number, then $15 = \frac{(12 \times 6) + x}{7}$. Solve for x : $105 = 72 + x$, so $x = 33$. In other words, adding 33 to the original set will yield a mean that is 3 more than the original mean.
9. B. If the mean is 4 less than 20, then it is equal to 16. Find the sum of the original set of numbers, which is 9×20 . If we add a new number, x , to this set, we then have 10 numbers. Therefore,
 $16 = \frac{(9 \times 20) + x}{10}$. Solve for x : $160 = 180 + x$, and $x = -20$.
10. A. Let x be Brandon's score on his last test. The question can be represented by the equation
 $85 = \frac{(3 \times 90) + x}{4}$, where x represents the fourth test score, and $3 \times 90 = 270$ represents the sum of the first three test scores. Simplifying gives us $340 = 270 + x$, and $x = 70$. So Brandon's last scores must have been 70.
11. C. Daniel scored an average of 10 points over 5 games for a total number of 50 points. He also scored at least 1 point per game. If he scored only one point in each of 4 games, and then scored 46 points in the fifth game, he'd have scored 50 points total over the 5 games, and his average would still be 10 points per game.
12. D. We've been given the average and the number of terms, so we can represent this question as
 $30 = \frac{x + 2x + 3x}{3}$, which simplifies to $90 = 6x$, so $x = 15$.
13. D. The median is a number in the midpoint of a data set. There are many different combinations of numbers that could give us an average of 90 (infinitely many, actually). Without knowing the individual terms in a set, we can't determine the median. Therefore, we do not have enough information to determine which is greater.
14. A. If each student in the class read an average of 12 books each and there are 24 students, then the total number of books read by the class is equal to $12 \times 24 = 288$. This is greater than 252, so the value of Column A is greater.
15. B. If Julie scored 12 points in her first 3 games, she scored 36 total points. If she scored 15 points in her fourth game, then her point total is now $36 + 15 = 51$. So her new point average is equal to $51 \div 4 = 12.75$. This is less than 15, so the value of Column B is greater.
16. B. If Donny's fourth test raised his average by 5 points, then the average of the first three tests must have been 88. The sum of his first 3 tests, then, is equal to $88 \times 3 = 264$. This is less than 274, so the value of Column B is greater.
17. A. We can solve for x by representing this question as $18 = \frac{x + 5x}{2}$. So, $36 = 6x$ and $x = 6$. This is greater than 3, so the value in Column A is greater.
18. A. Subtract Dot's cookies from the total to find that Bill and Rosie baked 64 together. The average is $64 \div 2 = 32$, so the average number of cookies each baked was 32. This is greater than 22, so the value in Column A is greater.

Standard Deviation

1. D. Standard Deviation measures the spread of a data distribution. For standard deviation to equal zero, all the values in the data set must be the same. Both Options II and III have a standard deviation equal to zero.
2. D. Standard Deviation measures the spread of a data distribution. The formula is: standard deviation = $\sqrt{\frac{\sum|x-u|^2}{n}}$. Here, x = mean, u = value, and n = the number of values in the data set. First, we can find the mean by taking the average of the five numbers: $\frac{(5+7+9+11)}{4} = 8$. Next, we subtract each data value by 8, and square the results. We divide this value by the number of data in the set which is 4. The square root is the standard deviation, as shown below: $\sqrt{\frac{(5-8)^2 + (7-8)^2 + (9-8)^2 + (11-8)^2}{4}}$
3. B. Standard Deviation measures the spread of a data distribution. The closer to the mean the values lie, the smaller the standard deviation. The mean of all three histograms is 6. Graph I has the smallest standard deviation, as there are more data points closer to the mean, followed by Graph III, then Graph II.
4. A. Standard Deviation measures the spread of a data distribution. The mean of all three histograms is 11, and the data in all three graphs are symmetrical around this value. The more bunched up the data, the smaller the standard deviation. Graph I has the smallest standard deviation, as there are fewer data points furthest from the mean, followed by graph II, and then graph III.
5. C. Standard Deviation measures the spread of a data distribution. The formula is: standard deviation = $\sqrt{\frac{\sum|x-u|^2}{n}}$. Here, x = mean, u = value, and n = the number of values in the data set. First, we can find the mean by taking the average of the five numbers: $\frac{(1+5+9+13+17)}{5} = 9$. Next, we subtract each data value by 9, and square the results. We divide this value by the number of data in the set which is 5. The square root is the standard deviation, as shown below: $\sqrt{\frac{(1-9)^2 + (5-9)^2 + (9-9)^2 + (13-9)^2 + (17-9)^2}{5}}$
6. B. Standard Deviation measures the spread of a data distribution. The formula is: standard deviation = $\sqrt{\frac{\sum|x-u|^2}{n}}$. Here, x = mean, u = value, and n = the number of values in the data set. First, we can find the mean by taking the average of the five numbers: $\frac{(11+12+13+14+15)}{5} = 13$. Next, we subtract each data value by 13, and square the results. We divide this value by the number of data in the set which is 5. The square root is the standard deviation, as shown below: $\sqrt{\frac{(11-13)^2 + (12-13)^2 + (13-13)^2 + (14-13)^2 + (15-13)^2}{5}}$
7. B. The means of sets A-D are all 7. The spread of the data around the mean is different in each case, which means that the standard deviations are different. Therefore, the means are the same, and the standard deviations are different.
8. C. The question asks for the sets in decreasing order of standard deviations. First, notice that the sets are all symmetrical around 60, which tells us that the mean of each set is 60. Since the means are equal, we can use the spread of the data as a way of evaluating the standard deviation. Set II has the greatest, then Set I, and finally Set III.
9. A. Standard Deviation measures the spread of a data distribution. We do not need to calculate standard deviation for this question. The mean for each student is the same: 90. The more the data is clustered around the mean, the smaller the standard deviation. Eli's scores are more clustered near the mean than Ian's. This means that Ian's scores has a greater standard deviation.
10. B. Standard Deviation measures the spread of a data distribution. We do not need to calculate standard deviation for this question. The mean for each plot is given. The more the data is clustered around the mean, the smaller the standard deviation. The plot in Column B has the greater standard deviation over Column A, as there are more data points further from the mean.

11. A. Standard Deviation measures the spread of a data distribution. The mean of both data sets is 50. However, the values for the first data set vary more widely than the values for the second data set. The standard deviation will therefore be greater for Column A.
12. A. Standard Deviation measures the spread of a data distribution. We do not need to calculate the standard deviation for this question. The mean for each plot is given. The closer to the mean the values lie, the lower the standard deviation. The plot in Column B shows values closer to the mean, so Column A has the greater standard deviation.
13. A. Standard Deviation measures the spread of a data distribution. While both sets have eight values that are evenly distributed, the first data set has numbers 2 units apart and a range of 14, while the second data set has numbers only 1 unit apart and a range of 7. The numbers in set A are spread out more, so its standard deviation is greater.

Interpreting Graphs

1. C. Be careful to understand the relationship between the axes. The x -axis represents time, while the y -axis represents speed. As the value of the x -axis increases, that means time is moving onward. As the value of the y -axis increases, that means speed is increasing. We see between time 3 through time 6 that the line decreases vertically before flattening vertically. At 3 minutes, Michelle is driving at 30 mph, at 4 minutes, she is driving 10 mph, at 5 minutes, she is driving 10 mph, and at 6 minutes she is still driving at 10 mph. Michelle has not stopped, but she has slowed down.
2. D. The graph shows time (x -axis) and elevation (y -axis). At 3 minutes, she is at 30 m; at 4 minutes, she is at 10 m; at 5 minutes, she is at 10 m; and at 6 minutes she is still at 10m. This means that her elevation decreased from 3 to 4 minutes, and then stayed the same from 4 to 6 minutes. As time went on, from minutes 4 to 6, her elevation did not change.

Frequency Charts & Graphs

1. A. The median of data is the midpoint data. This chart shows the number of books and the number of students who read that number of books. So, 10 students read 1 book, 9 students read 2 books, and so on. So, the data set would be something like 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, etc. But we also know that 31 one students responded (the sum of all students). So, the 16th student will have the median response. 15 students responded that they read 2 or fewer books, and 15 students responded that they read 2 or more books, so the median of this set is 2.
2. A. Multiply the number of birds by the number of days that number was spotted for each column and add them together: $6 + 12 + 3 + 24 + 30 = 75$. This is the total number of birds. Then add the total number of days: $6 + 6 + 1 + 6 + 6 = 25$. Divide the total number of birds by the total number of days to yield the mean: $75 \div 25 = 3$.
3. A. The mode of a data set is the number that occurs most frequently. The number of slices that got the most responses was 1 slice (9 people ate 1 slice of pizza, according to the chart), so the mode is 1.
4. A. The mode of a data set is the number that occurs most frequently. The largest number of kids (7) responded that they spent 3 hours trick or treating, so 3 is the mode of the data.
5. C. The range of data is equal to the largest value in a data set subtracted by the smallest number. This data set has 500 as its greatest value and 20 as its lowest, so the range is equal to $500 - 20 = 480$.
6. A. To find the mean, add up the total number of peas discarded (by multiplying the number of peas discarded by the number of days): $(0 \times 3) + (1 \times 1) + (2 \times 2) + (3 \times 5) + (4 \times 1) = 0 + 1 + 4 + 15 + 4 = 24$. This is the total number of peas. Then count the number of days: $3 + 1 + 2 + 5 + 1 = 12$. So the mean is equal to $24 \div 12 = 2$.
7. C. The median is the midpoint of a data set. There are 18 data points in this set, or 18 total days that Jason biked ($5 + 1 + 3 + 7 + 2 = 18$). On 9 days, Jason biked 15 or fewer miles, and on 9 days Jason biked 20 or greater miles, so the median is halfway between 15 and 20, or 17.5.
8. B. First, find the total number of apples shown in the graph: $(1 \times 1) + (2 \times 5) + (3 \times 2) + (4 \times 1) + (5 \times 4) + (7 \times 3) + (8 \times 4)$. This simplifies to $1 + 10 + 6 + 4 + 20 + 21 + 32 = 94$. If he picked a total of 100 apples, that means that the table does not account for 6 apples. So the number of days he picked 6 apples must have been 1.

Circle Graphs

1. B. Brent spent the largest percentage of his budget on clothes (35%) so the correct graph will show clothes as the largest piece of the circle. This is only true of the graph in choice B.
2. B. If the section labeled apples has a 90° angle, it accounts for one fourth of the total number of shoppers. This is because there are 360° in a circle, and $360 \div 90 = 4$. So, divide the total by 4 to find how many people this represents: $240 \div 4 = 60$ people.
3. A. A circle has a total of 360° . So a 60° portion of the circle represents one sixth of the circle ($360 \div 60 = 6$). If there are 90 students, one sixth ($90 \div 6$) is equal to 15. So, the blue portion of the graph would be 60° .
4. D. The portion labeled Cow and the portion labeled Horse both look like they could each measure about 90° , or one fourth, of the total. The portion labeled Pig is much greater than one fourth, and the portion labeled Goat is much less than one fourth. Graph D shows that this is true: there are 140 total animals. One fourth is equal to 35 animals; there are 35 cows and 35 horses. There is a greater number of pigs than any other animal and a smaller number of goats. This fits with the circle graph.
5. C. If Hunter ran the same number of miles every month, the circle graph will have 12 sections that are equal inside. Since a circle has 360° , if we divide 360 by 12, each section will be 30° .
6. D. 40° represents $\frac{1}{9}$ of the graph $\left(\frac{40}{360}\right)$. If this also represents 50 people, then the total number of people interviewed will be $50 \times 9 = 450$.
7. C. The Saturday section of the circle graph is one half of the graph. Friday is one quarter, and Sunday is one eighth. If 48 people chose Saturday, that means 96 people were interviewed in total. One eighth of that total is $96 \div 8 = 12$. So, 12 people chose Sunday.
8. B. The D section of the graph is greater than an eighth but less than a quarter of the graph, so we can estimate is about one sixth of the graph. If 3,000 students earned a D, then $3,000 \times 6 = 18,000$ students took the test in all. It definitely could not be 12,000, since 3,000 would represent one quarter. Neither could it be 24,000, since 3,000 would represent one-eighth.
9. B. 40 people chose peach. If this represent 120° of the circle graph, then this is one third of the total. This means there are $40 \times 3 = 120$ students in the class. The graph shows $30 + 20 + 40 + 10 = 100$ students. So, the differences is 20 students, who chose orange.
10. B. Half of the graph is equal to 180° ($360 \div 2$). One third of the graph is equal to 120° ($360 \div 3$). So, in this graph, rent takes up 60° more than food.
11. A. One sixth of the graph (transportation) is equal to $360^\circ \div 6 = 60^\circ$. One eighth of the circle (utilities) is $360^\circ \div 8 = 45^\circ$. So, the angle for transportation is 15° more than the angle for utilities.
12. D. If the sections of the circle graph have a 2:3:4 ratio, then we could represent the sum of the angles as $2x + 3x + 4x = 9x$. If we set this equal to 360° , we get $9x = 360^\circ$ and $x = 40^\circ$. Therefore, the sections are equal to 80° , 120° , and 160° .
13. C. Nigel ate $\frac{3}{8}$ of the pizza. $360 \times \frac{3}{8} = 135$, so the central angle of the slices Nigel ate is 135° .

Box-and-Whisker Plots

1. C. A box-and-whisker plot graphs the lowest number and the highest number at either end of the plot. So the highest number of this plot is the rightmost point, which is 95.
2. B. The leftmost point graphed is the least number. In this box-and-whisker plot, the least number is less than 20, but more than 10, or 18.
3. B. The range of the data is the difference between the greatest and least numbers in a data set. So, the range for this data set is $90 - 20 = 70$.
4. C. The least value is 3 and the greatest is 11, so the leftmost point of the plot should be at 3 and the rightmost should be at 11. We can thus eliminate the second choice. The median of this data, or the midpoint, is 7. This will be the middle line in the box, so we can eliminate the first choice. The

median, or midpoint, of the numbers less than 7 is 5. The median of the numbers greater than 7 is 9. So, the leftmost line of the box will be at 5 and the rightmost line of the box will be at 9.

5. C. It may help to put the data in order: 1, 3, 4, 5, 6, 7, 8, 8, 9, 9, 12, 12. The least point is at 1 and the greatest point is at 12. This rules out the last choice. The median is 7.5, which rules out the first and second choices. We can tell right away that the third choice is correct.
6. A. Find the least and greatest numbers, which are 35 and 85, respectively. This rules out the second and last choices. Next, find the median (cross out the next greatest and least values, until we arrive at the median). The median is 58, which rules out the third choice.

Stem-and-Leaf Plots

1. B. The mode of a data set is the number that occurs most frequently. There are 4 ones in the first row. Since the number at the beginning of the row (the stem) is 0, this means the number 1 occurs 4 times and is the most frequent number, or mode.
2. B. The median is the midpoint of the data. There are 31 points in this data set, so the 16th point will be the median. This is the leaf 3 in the row in which 5 is the stem, so the median is 53.
3. D. The mean is the sum of the data points divided by the number of points. Applying stems to leave shows that this data set consists of 3, 5, 5, 6, 10, 10, 11, 12, 20, 21. The sum is 103. There are 10 data points, and $103 \div 10 = 10.3$, so the mean is 10.3.
4. C. The mode is the number that occurs most frequently. In this plot, that number is 13. (The three 3s in the second row each represent 13.)
5. B. This stem and leaf plot shows 1, 1, 2, 2, 2, 3, 5, 10, 12, 20. The sum is 58. There are 10 numbers in this set. So the mean is $58 \div 10 = 5.8$.
6. C. Let x represent the unknown value. The greatest value in this data set is 69. If the range is 67, then $69 - x = 67$. So, x must be equal to 2. The question is asking for the mode of the data. The number 2 and the number 5 each occur 3 times, which is more than the other values in the set, so the mode is 2 and 5.

Histograms

1. C. The number of people who weighed less than 160 pounds is represented by the first two bars in the histogram. This is $3 + 1 = 4$ people.
2. B. The range is equal to the difference between the greatest and least values. The least value will fall between 51 and 60, and the greatest value will be between 91 and 100. So, 40 is a possible range for the data. The range cannot be 30 because we know that that several students scored between a 91-100, and even if only one student scored a 60, $91 - 60 \neq 30$.
3. B. A histogram shows ranges of values in each column of the graphic. The value of column A is the sum of days represented by the first two columns: $3 + 1 = 4$ days. The exact value of column B cannot be determined because the third column of the graphic gives the range 30-39, inclusive. However, since 40-49 is a range greater than 30, and the number of days is 5, we know that column B must be greater than column A.
4. D. Because the graph shows ranges, we can't determine which number occurs the most in the data set. Therefore, we cannot determine the mode and do not have enough information to know which value is greater.
5. D. Because the graph shows ranges, we can't determine the exact value of the median. Therefore, we do not have enough information to determine which value is greater.

Survey Samples

1. C. April surveyed 30 students as they walked out of the library, meaning she interviewed 30 students who are more likely to read more and not a random sample of students. Ben surveyed people outside the school, who might not all have been students, so his sample may not be of all students. Daniel surveyed people in the teachers' lounge, meaning he likely interviewed teachers not students. Only Cindy, who surveyed students on their way out of the cafeteria, asked a random sample of students, and thus will have the most reliable results.

2. B. The best sample will be a random sample of people who live in the city. Middle school students can't vote, so that choice can be ruled out. The 2,000 people who attended the opponent's event are likely biased in favor of the opponent. Sampling people from across the country will not yield accurate results for the city. So the best sample is of people chosen from the city's phone book.
3. A. If the survey is intended to determine favorite animals among students in the eighth grade, the best sample will include only eighth grade students, so the sample should include only their class.
4. D. If the scientist wants to test the drinking water, the best sample is the water coming out of the faucets, which will take into account the mountain, the river, the reservoir, and the plumbing system.

Final Practice Test (Form B)

Verbal Reasoning

1. D. "Mock" generally means "make fun of." "The bully mocked the other students in the class." "Tease" is the correct answer.
2. A. If one were to "prevent" something from happening, one might "deter," or discourage someone from doing that thing.
3. D. "Instill" comes from a root meaning "drip" or "drop," and so literally means to add drop by drop. In a broader sense, the word has come to mean to give or impart something gradually. "Our teacher thought it was important to instill in us the value of using good grammar." Thus, "teach" is the correct synonym.
4. B. "Cunning" means "crafty" or "insightful." Something or someone that has cunning can figure out ways to do or accomplish things. Another way to say this is to say that the person or thing is "resourceful."
5. A. "Waver" comes from the same root as "wave." To waver is to vacillate or fluctuate between two choices. In this sense, one might pause or hesitate before committing to something, so "hesitate" is the correct synonym.
6. C. "Havoc" is destruction or confusion. "The shoppers descending on the mall on Black Friday created havoc." Thus, "chaos" is the correct synonym. We can eliminate "privilege" and "certainty," since they have positive connotations.
7. A. A "rival" might be an opponent one is competing against. "The Yankees and Red Sox are the greatest rivals in baseball." Thus, "adversary," which also means "opponent," is the correct synonym. "Adversary" is the only other word with a negative connotation.
8. A. "Imperious" comes from the same root as "emperor." An emperor rules with absolute power, and thus might be commanding or intimidating. We can think of someone who is imperious as being someone who acts like they know better or rule over more than anyone else, or someone who is "arrogant." Someone who is imperious is not necessarily majestic, and someone can be imperious without being royal.
9. C. Something "conspicuous" is clearly visible. "She wore a hot pink coat, which made her conspicuous in a crowd." This is the opposite of "inconspicuous," which means "hard to see." Thus, "obvious" is the correct synonym.
10. C. "Grueling" comes from the root "gruel," which once meant punishment. (These days, "gruel" usually refers to thin oatmeal or a porridge made from ground grains.) Something "grueling" is "punishing," or extremely difficult. "Demanding" is the correct synonym, and shares a similarly negative connotation as "grueling."
11. C. To "ameliorate" is to make better or improve. "The doctor recommended a series of exercises to ameliorate the health of people who sit at a desk all day." In this sense, "enhance" is the best synonym.
12. B. "Egregious" has a negative connotation and generally means to be obviously bad. "The politician's egregious behavior led to his impeachment." Of the choices, only "awful" has a similar connotation and is the correct answer.
13. A. "Aplomb" means "poise." "The actress showed grace and aplomb despite the rowdy crowd of fans." "Composure" is the correct synonym.

14. B. “Tempestuous” comes from the same root as “tempest,” which is another word for a violent storm. Someone who is tempestuous might be said to have stormy moods, or emotions that change the way weather during a storm does. If one is “fickle,” one is not stable or constant, so this is the correct synonym.
15. C. “Concurrent” means “happening at the same time.” The prefix “con-” means “with,” which can help you remember the meaning. “The play and the softball game were happening concurrently, so I had to choose which to attend.” So, “simultaneous” is correct.”
16. A. “Malleable” comes from a root word that means “hammer,” and the original meaning of the word meant literally to be shaped with a hammer, as in metalwork. Today, malleable generally means moldable or shapeable. A person who is malleable might be susceptible to persuasion. Thus, impressionable, or “shapeable,” is the correct synonym.
17. D. “Acumen” comes from the same root as acute. It generally refers to a skill or perceptiveness in practical matters, or a depth of knowledge. “The CEO had great business acumen, and had a knack for choosing smart investments.” Thus, “wisdom” is the correct synonym.
18. D. To be “facetious” is to be funny or humorous. “He is always making facetious remarks, so we rarely take him seriously.” Thus, “silly” is the best synonym.
19. D. “Succumb” has a negative connotation and is a verb meaning “to yield” or “to come to an end.” “After a long argument, she succumbed to pressure and gave up the fight.”
20. A. This sentence shows a contrast. If Carl cares deeply about his grades “in reality,” he must pretend otherwise with his friends. “Apathetic,” which means “showing no emotion,” is the correct word for the blank.
21. D. “Withdrawing” is the clue in this sentence, since the comma after “ocean” shows that the following clause will be logically related. The correct word for the blank is “recedes,” which means to move back or away.
22. B. The pirates raid and pillage despite the law, so they likely do not have much respect for authority. In other words, they have nothing but “contempt,” or a lack of respect, for authority.
23. D. The printing process made the process of copying texts simpler, so the task must have been difficult previously. The clues in the sentence are “before” and “simplified.” Thus, if something had not been as simple before, it was complicated. The word most close to this is “tedious,” which means “tiresome.”
24. A. A part of the movie that did not advance the plot or develop the characters is likely unnecessary, or “frivolous,” which means lacking importance.
25. A. If it would continue raining for months, and since the sentence describes a negative (“no signs of...”), we can infer the word in the blank is a synonym for “stopping.” “Abating” means to decrease in intensity, and is the correct synonym for the blank.
26. C. The accountant is “thorough and precise,” according to the sentence, so the word in the blank will be a synonym of those. “Meticulous,” means “showing extreme care and attention to detail,” so it is the correct word for the blank.
27. B. The phrase “otherwise stagnant” signals that the word in the blank will be an antonym of “stagnant,” which means “stale” or “not advancing.” “Engender” means “to cause to develop,” which is the correct word for the blank.
28. A. In the context of the sentence, the word in the blank will be an antonym for “tiny,” since the sentence shows a contrast. (“Compared with” is the clue.) The word “behemoth” describes something huge, so this is the correct word for the blank.
29. D. If the moon is directly overhead, it has reached the highest point in the sky. This is the opposite of the nadir, which is the low point. The “zenith,” however, is a position directly above the observer.
30. D. The structure of the sentence implies a contrast, so the stock market causes the opposite of prosperity. The best word for the blank should indicate a change or lack of fortune. “Vicissitude” means “change,” and is the best fit for the sentence. If we don’t know the meaning of “vicissitude,” we can arrive at this answer by ruling out the meaning of the other words. A “bias” (preference) or

- “catalyst” (cause) of fortune doesn’t make sense, neither does a “paragon” (example) of fortune (since “after many years of prosperity” implies that the crash led to less or no prosperity).
31. A. “Otherwise” indicates that the word in the blank will contradict “far-fetched.” “Incredulity” implies something is unbelievable, which is an synonym for “far-fetched” and can be ruled out. “Credence” means “believability,” which is a good match for the blank.
 32. B. If Jeremy lost his stuffed animals, he likely remained sad (since the sentence describes the event as “his loss,” which is negative). This is the opposite of pacified and content, so those choices can be eliminated. The word “despite” implies that Jeremy’s mother tried to make him feel better, or “console” him. So “console...indignant” is the correct choice.
 33. C. If the politician was very successful, he must have had positive qualities, so we can infer the words in the blank will have positive connotations. In addition, the clause after the semicolon describes how people found him “charming” and were “captivated by his speeches.” Thus, “eloquent...charismatic” are the correct words for the blank.
 34. C. “Otherwise” implies a contrast so the author used humor when the play was otherwise said, meaning “depressing” and “melancholy” are possibilities for the blank. Since the writer used humor “at just the right times,” he told jokes at “opportune” moments.
 35. D. The word “sensitivity” gives us a clue that we’re looking for words with positive connotations for the blanks. Thus, “tact...discretion” are the correct words for the blanks.
 36. D. If the people would no longer tolerate harsh rule, they likely want their leader to step down. The leader could likely be described by a word with a negative connotation. Thus, “tyrant” is the best word for the first blank, and “abdicate,” which means to give up the throne, is the best word for the second blank.
 37. A. The rebels are likely trying to support or add to their arguments in order to be more persuasive. Thus “augment,” which means to increase, and “garner,” which means to gather, are the correct words for the blanks.
 38. B. In the context of the sentence, the words in the blank are antonyms for “clear.” This is true of both “cryptic” and “ambiguous.”
 39. C. If the gangsters are unable to influence the judge, she must be impartial or have strong values, and the words in the blanks should reflect that. “Scrupulous” means “having moral integrity” and the “in-” in “incorruptible” makes the word mean “unable to be corrupted.”
 40. B. In the context of the sentence, “foolish” and “pointless” are clues about the missing words, which will be synonyms. Thus, “inane...incessant” are the correct words for the blanks; the words mean “lacking significance” and “continuing without interruption.”

Quantitative Reasoning

1. *Distributing*. C. Apply FOIL to the left part of the equation to yield $x^2 + 2x + ax + 2a$. So, $2a = 10$, and $a = 5$. We can rewrite the expression as $x^2 + 2x + 5x + 10$, or $x^2 + 7x + 10$. Therefore, $m = 7$.
2. *Creating Expressions & Equations*. D. In dollars, a quarter is equal to 0.25 and a dime is equal to 0.10. If there are 7 more quarters than dimes, and x is equal to the number of dimes, we can write this equation as $0.10x + 0.25(x + 7) = 4.2$.
3. *Probability*. A. For a six-sided die, the probability of rolling any given number is 1 in 6. So out of 60 rolls, the expected number of 4’s is equal to $60 \div 6 = 10$.
4. *Function Notation*. A. $f(1) = 1$, but $f(-1) = -1$. So, we know that $f(-1) < f(1)$, which allows us to rule out the third and fourth choices. Since $g(0.1) = \sqrt{0.1}$, and we know that $\sqrt{0.1}$ must be positive but less than 1, we know that the second choice must be false. Therefore, only the first choice is true.
5. *Proportions*. B. We can set the denominators equal to one another, just as we can set the numerators equal to one another. $43 = 50 - 7$, so this equation is only true of $x = 50$.
6. *Speed*. C. Speed is equal to distance over time. We know the distance (both ships travel 1,000 miles), but not the time it takes the Astound to reach its destination. Because we know the average speed of the Opulence, we can calculate how long it took the Opulence to travel 1,000 miles. So, if the

Opulence reach its destination 10 hours before the Astound, we can calculate how many hours the Astound took to reach its destination and thus calculate the speed.

7. *Averages.* C. The average score will be equal to the sum of the scores of all tests divided by the number of scores. This question can be represented by the expression $\frac{89 + 95 + 80 + x + x}{5}$, where x represents Rick's score on the last exam. If his mean score is 90, then $90 = \frac{89 + 95 + 80 + 2x}{5}$, and $450 = 264 + 2x$. Simplified, $2x = 186$ and $x = 93$. So, in order to have a mean score of no less than 90, Rick will need to score of 93 or greater on his last exam.
8. *Percents.* D. Let x equal the original price of the phone case. If it was marked down 30%, then the value after the Black Friday sale can be expressed as $0.7x$. If it was marked up 20% afterwards, then we can express this as $0.7x \times 1.2 = 0.84x$. Therefore, compared with the original price of x , the percentage decrease from the original price is $x - 0.84x = 0.16x$.
9. *Arithmetic.* D. When dividing exponential terms that have the same base, we subtract those exponents. In this case, $4^3 \div 4^2 = 4^1$, or simply, 4. Next, simplify inside the parentheses to arrive at $\frac{4 + 8}{8 + 16} = \frac{12}{24} = \frac{1}{2}$. $0.5 \times 4 = 2$.
10. *Exponents.* B. $49 = 7^2$. So, to find the value of 7^8 in terms of 49, we can divide the exponent by 2. $7^8 = 49^4$.
11. *Proportions.* D. 3.5 feet is equal to $3.5 \times 12 = 42$ inches. If each inch represents 4 yards, then 42 inches represents $42 \times 4 = 168$ yards. This is equal to $168 \times 3 = 504$ feet.
12. *Function Notation.* B. Since the asterisk does not mean anything mathematically, this is an imaginary operation. We can substitute the numbers into the equation. Based on the equation $10 * 5 = 3(5) + \frac{10}{5}$. This simplifies to $15 + 2 = 17$.
13. *Histograms.* C. If the minimum height is 54 inches, then the number of people allowed to ride is equal to the sum of the number of people the last three bars represent: $5 + 8 + 11 = 24$.
14. *Consecutive Integers.* C. Let x be the first integer. Three consecutive even integers will be x , $x + 2$, and $x + 4$. The product can be represented as $x(x + 2)(x + 4) = 960$. Since 960 is a multiple of 10, one of factors must be ten. $8 \times 10 \times 12 = 960$. The sum of the integers is $8 + 10 + 12 = 30$.
15. *Fractional Exponents.* D. A fractional exponent indicates that you should take the root as determined by the denominator of the exponent. In this case, the denominator is a 2, so we should take the square root of the base. The negative exponent indicates that the base should be flipped over the fraction line. So, we end up looking for the square root of 25, which is 5.
16. *Slope.* D. In an equation with the format $y = mx + b$, m is the slope of the line the equation represents. If the slope of the line shown is -4 , then the slope of line $y = 2mx + b$ is equal to $(-4)(2) = -8$.
17. *Imaginary Numbers.* C. If $i^2 = -1$, $i^5 = (-1)^2(\sqrt{-1}) = \sqrt{-1}$
18. *Probability – Conditional.* A. Probability is equal to the number of ways something can happen over the total number of outcomes. There are 25 pens total to start. If a blue pen is removed, there are now 24 pens. Since there are 4 black pens, the probability the next pen will be black is equal to $\frac{4}{24} = \frac{1}{6}$.
19. *Nets.* D. Imagine what the net will look like folded into a cube. The correct net will show either adjacent triangles or adjacent arrows pointing in difference directions. This is true of net D.
20. *Algebraic Relationships.* B. If $x + 1 = \sqrt{5}$, then $(x + 1)^2 = (\sqrt{5})^2$. The square cancels out the square root, leaving $(x + 1)^2 = 5$.
21. *Appropriate Units.* B. Since we're measuring the volume of gasoline, we need a unit of volume. Feet and inches are units of length and milliliters are a unit of capacity. Gallons is reasonable because it measures the volume of a something with a high capacity, like a gas tank.

22. *Negative Exponents*. B. Remember that a negative exponent indicates the number should be flipped over the fraction line. So, $\left(\frac{2}{5}\right)^{-3} = \left(\frac{5}{2}\right)^3$. We don't need to simplify both expressions to know that Column A is smaller than Column B. This is because a positive number less than 1 multiplied by itself will result in smaller and smaller numbers. A positive number greater than 1 multiplied by itself will result in bigger and bigger numbers.
23. *Geometry with Variables*. C. The area is equal to the width times the height. So, the area of Rectangle J is $2y4x$ and the area of Rectangle K is $2x4y$. Simplifying both expressions, we see that the areas of both rectangles are equal to $8xy$. So, the values are equal.
24. *Algebraic Relationships*. C. We can rewrite this expression as $x - 11 = y$. Add to both sides, then subtract 11 from both sides.) Thus, the value of Columns A and B are equal.
25. *Area & Perimeter*. D. The perimeter is the sum of the length of all sides of a polygon. Since we don't know the individual lengths of the sides of the rectangle, we cannot determine the perimeter. Therefore, we do not have enough information to determine which value is greater.
26. *Percents*. A. If the stock decreased by 10%, then the price on Tuesday was equal to 90% of the original. $500 \times 0.9 = 450$. So, the price of the stock on Tuesday was \$450. If the stock was 10% less than Tuesday's price on Wednesday, then the price on Wednesday is equal to $450 \times 0.9 = \$405$.
27. *Vocabulary*. B. If the sum of all numbers from 50 to 200 is a , then the sum of all numbers 50 to 199 = $a - 200$, since 200 is no longer included.
28. *Probability*. A. The probability of rolling any given number with the 6-sided cube is $\frac{1}{6}$. The probability of rolling any given number on the 4-sided die is $\frac{1}{4}$. So, the probability of rolling a 2 or a 4 on the 4-sided die is equal to $\frac{2}{4} = \frac{1}{2}$. The probability of rolling a 2 or a 4 on the six-sided cube is $\frac{2}{6} = \frac{1}{3}$. Therefore, the value in Column A is greater.
29. *Formulas*. A. If $\pi = \pi r^2$, then the only value that makes the equation true is $r = 1$. This is greater than the value of Column B.
30. *Arithmetic*. C. Follow order of operations (think of the acronym PEMDAS). In the expression $30 - 5 \times 2^2 + 5$, find the value of the exponential expression, then perform the multiplication, then the addition, then the subtraction. So, $30 - 5 \times 4 + 5$ simplifies to $30 - 20 + 5 = 15$. So, the values of both columns are equal.
31. *Frequency Charts & Graphs*. B. If Bea scored a total of 114 points this season, we can find the value of the missing bar. She scored 2 points in 2 games (4 points), 6 points in 6 games (36 points), 8 points in 5 games (40 points), and 10 points in 3 games (30 points). $114 - 4 - 36 - 40 - 30 = 4$, so Bea scored 4 points in 1 game. $1 < 4$, so the value in Column B is greater.
32. *Algebraic Relationships*. B. If $3x + 2y = x$, then $2x + 2y = 0$ (subtract x from both sides). From this, we can conclude that $x = -y$, so any multiple of x plus the same multiple of y will be equal to 0. Therefore, $12x + 12y = 0$. Thus, the value of Column B is greater.
33. *Averages*. D. The mode is the value that occurs most often. Although we know that 4 students scored 85, we don't know if this was the most frequently occurring score, so we don't have enough information to determine which value is greater.
34. *Interpreting Graphs*. B. The graph shows that Jon's speed increased and decrease, but he never stopped (or traveled at 0 miles per hour), so the value of Column A is 0. Thus, the value of Column B is greater.

35. *Formulas*. D. Since the height is also a variable, we don't have enough information to determine the radius of either cone. Therefore, we do not have enough information to determine which value is greater.
36. *Probability – Conditional*. B. Since there are 4 odd numbers and 4 even numbers on the spinner, the probability of spinning to an odd number is $\frac{1}{2}$, which is the same probability as spinning to an even number. The probability of spinning to an odd then an even number is therefore $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$. Since whether the first spin was odd or even does not affect the second spin, the probability of the second spin landing on an even number is still $\frac{1}{2}$. So, the value in Column B is greater.
37. *Proportions*. A. Convert the units so that they can be directly compared. 1 m = 100 cm. So, $\frac{3}{500} = \frac{100}{x}$. Cross multiply to solve: $3x = 50,000$. $x \approx 16,666.67$, which is greater than 1,500, so the value in Column A is greater.

Reading Comprehension

1. D. *Main Idea*. The passage is mostly about the water cycle and the role it plays in the environment. This is a natural process, so we can conclude that the purpose of the passage is to explain the significance of natural processes to the environment. The passage doesn't delve into why water is good, or explain the reasons for certain natural processes, only telling us what they are.
2. B. *Inference*. In line 10, the author states that sun powers the water cycle. From this we can infer that, if the sun were to disappear, the water cycle would cease to exist. The fact that water is trapped in a frozen state is described as being a pause in the water cycle.
3. C. *Organization/Logic*. These two paragraphs show what happens to water in the order the events occur, or chronological order. Thus, this shows a process in chronological order. There are no recommendations made, nor a particular point of view presented. The author doesn't compare the water cycle with other phenomenon.
4. A. *Supporting Idea*. In lines 48–51, the author explains that glaciers create valleys and define mountains. In other words, ice frozen for a long period of time is responsible for geological features.
5. C. *Vocabulary*. In the larger context of the paragraph, water and ice are responsible for the formation of geological features like valleys and mountains. Rain, likewise, "gouges" rivers and canyons into the land, from which we can infer that "gouges" means "dig."
6. B. *Tone/Language/Style*. The passage is a straightforward explanation of a process and does not include arguments or opinions, so the tone can be described as "objective," or unbiased.
7. C. *Main Idea*. The passage is mostly about the Works Progress Administration, which was a New Deal policy aimed at helping people during the Great Depression. The passage uses evidence to support the argument that the WPA was a smart and effective program. The larger argument is that government intervention helped people during a national crisis. The other choices are mentioned as details or supporting ideas to this broader theme.
8. A. *Inference*. In lines 30–33, the author explains that President Roosevelt intended to invest in employment to help break the depression. The goal was to disrupt the cycle so that the economy could recover. The word "depression" can also be a mental or emotional condition, but in this context, it is used to refer to an economic depression.
9. B. *Supporting Idea*. Although the reason for creating the WPA is detailed in previous paragraphs of the passage, it is not described in lines 41–61.
10. D. *Vocabulary*. The phrase "already high levels of unemployment" indicates that businesses closing and firing employees made a bad situation worse. In other words, "exacerbated" means "worsened."

11. D. *Tone/Language/Style*. The Roosevelt quote restates the larger point of the passage, which is that the WPA had wide-reaching benefits. Thus, we can conclude that the author included the quote to reiterate the point about the WPA helping both people and the country.
12. D. *Inference*. The passage is making an argument in support of the WPA, pointing out that it had a number of benefits, including bolstering arts programs and helping people get jobs in a bad economy. Thus, we can conclude that the author cares about the fact that the WPA was a good program with both short and long-term benefits.
13. C. *Main Idea*. The passage describes a thunderstorm, but it is primarily a description from one person's perspective. So, this is a passage about one person's personal experience with a thunderstorm rather than a scientific explanation of a thunderstorm, or even the steps needed to prep for a thunderstorm.
14. D. *Supporting Idea*. In lines 13–15, the author states that he had witnessed events like these before a thunderstorm in the past, so he knew that a thunderstorm was coming.
15. A. *Inference*. If the author is on “auto-pilot,” that means he is acting without thinking much. We can infer he's gone through this routine a number of times before, so he knows what to check without thinking about it.
16. D. *Vocabulary*. In context, a “few drops gave way to a deafening deluge,” so we can infer that “deluge” is many drops of rain, or a torrent, since a few drops changed into something else.
17. B. *Tone/Language/Style*. A giant's war drums are likely very loud, so this refers to sound. The first part of the sentence refers to the thunder, so we know that this phrase refers to the loudness of the thunder.
18. A. *Organization/Logic*. The last sentence emphasizes that the power is out, so all the author can really do is sit and wait for the storm to pass and the power to come back. This implies a certain amount of helplessness.
19. A. *Main Idea*. The passage is a somewhat humorous account of the author's experience dissecting a frog in biology class. She was excited about this project, but her partner was not. The passage does not give us enough context to support the other statements made.
20. A. *Vocabulary*. In context, the author is excited about dissecting the frog, but Charlie does not share her “zeal.” We can infer that this means Charlie does not have the same enthusiasm for the project.
21. B. *Supporting Idea*. The first thing that happens in the sequence is that “Charlie's face was shrouded in panic and fear” (lines 49–41), so the author can tell Charlie has genuine anxiety about dissecting the frog. The passage contradicts the other statements either in timing or in fact.
22. D. *Inference*. Earlier in the passage, the author explains that they argued sometimes and weren't really friends (paragraph 3). Therefore, we can infer that Charlie is surprised that the author is helping because they otherwise aren't very friendly.
23. C. *Tone/Language/Style*. The author is trying to show irony in the situation. The description is vivid and helps the reader visualize how Charlie looked and what the situation was like.
24. C. *Organization/Logic*. The author does show her relationship with Charlie, tell how Charlie reacts to dissecting the frog, and describe the first few steps in dissecting a frog. However, we are never told why Charlie dislikes dissecting the frog.
25. A. *Main Idea*. The passage is mostly about ancient plumbing systems, like those of the Greeks and Romans. The passage does not draw a clear line between ancient and modern plumbing, except to say in the first paragraph that modern plumbing benefits humanity now.
26. B. *Inference*. If the population is growing but people are crowding into smaller areas, we can infer that the author is referring to people crowding into cities and towns, rather than spreading out in rural or coastal areas, specifically. The author does not specify the type of dwelling.
27. C. *Tone/Language/Style*. In context, “merely” means “simply,” since people simply threw waste away. This shows a lack of sophistication, since it's a simple process, and nothing was dealt with otherwise.
28. A. *Vocabulary*. The sentence draws a parallel with the Romans being “clearly advanced” by saying that other civilizations were “just as savvy.” This context indicates “savvy” means “advanced.”

29. *C. Inference.* The innovation of public baths and wastewater being removed efficiently (lines 29–32) imply that the Roman plumbing system provided better hygiene than earlier systems.
30. *B. Organization/Logic.* The last paragraph describes plumbing systems in Lothal and Crete that were “just as savvy” as the Roman system. These are examples of other plumbing ingenuity and do not undermine the point that the ancient systems were sophisticated.
31. *B. Main Idea.* The passage describes a number of diamonds and why each one is valuable. The author gives the traits that make each diamond prized and points out that different diamonds are valued for different reasons: size, physical attributes, and unique histories. Thus, the purpose of the passage is to explain in a broader way that objects can be valuable for different reasons.
32. *D. Supporting Idea.* The *Great Star of Africa* is prized for its size, and the *Millennium Star* is prized for its color and clarity. Thus, these diamonds both possess extraordinary physical properties.
33. *C. Vocabulary.* In context, the diamonds are not merely famous but instead become legendary. “Transcend” means to go past or “exceed.”
34. *B. Organization/Logic.* The passage describes the path one particular diamond took as it exchanged hands, in chronological order. So, the organization is a chronological order of events.
35. *C. Inference.* The long history of the *Hope Diamond* illustrates that some diamonds are prized for their rich histories (lines 71–72) and not just their physical attributes.
36. *B. Tone/Language/Style.* The proper names of the diamonds show that each one is important. Many of these diamonds have been set in Crown Jewels or are now displayed in museums, or they have unique physical characteristics or histories, meaning they are significant diamonds.

Mathematics Achievement

1. *Sequences.* A. To solve, find a rule for the sequence. Although each term is 6 more than the last, $n + 6$ will yield a series of consecutive numbers. However, if you plug 1, 2, 3, 4, and 5 into $6n - 16$, you get the pattern $(6(1) - 16 = -10; 6(2) - 16 = -4, 6(3) - 16 = 2, \text{ etc.})$. If we plug 50 into this expression, we get $6(50) - 16 = 300 - 16 = 284$.
2. *Factoring.* C. When applying FOIL to $(x + a)(x - a)$, the outer and inner terms will cancel each other out. We can find the factors of $x^2 - 100$ by finding terms for which this is true. Thus, the equivalent expression is $(x + 10)(x - 10)$.
3. *Box-and-Whisker Plots.* D. In a box-and-whisker plot, the third quartile is the median of the larger half of the values. On the plot, this is represented by the right end of the box, which is above 900. So, the third quartile is 900.
4. *Stem-and-Leaf Plots.* D. The range is equal to the largest number in the data set subtracted by the smallest number. Remember that in a stem and leaf plot, the stem represents the tens value and the leaves represent the ones values. The smallest value in this chart is 111 and the largest is 169. $169 - 111 = 58$.
5. *Geometry with Variables.* C. The area of the prism can be represented as $A = lwh$, so the area is equal to $x \times 2x \times y = 2x^2y$.
6. *Algebraic Relationships.* D. Solve for x then substitute into the expression. $9x - 15 = 30$ simplifies to $9x = 45$, and $x = 5$. Therefore, $27(5) - 40 = 95$.
7. *Creating Expressions & Equations.* B. Let n represent the number of towels Nathan folded. This can be represented at $n + 4n = 40$, or $5n = 40$, so $n = 8$. The number of towels the robot folded is $40 - 8 = 32$.
8. *Circle Graphs.* C. There are 360° in a circle, so 30° is equal to $\frac{30}{360} = \frac{1}{12}$. So, the total number of people who took the survey is equal to the number of people who chose giraffe multiplied by 12: $40 \times 12 = 480$. If 40 people chose giraffe, then $480 - 40 = 440$ people chose another animal.
9. *Formulas.* A. Since we know the volume, we can put that in the equation: $36\pi = \frac{4}{3}\pi r^3$. Multiply both sides by 3 then divide by 4π to arrive at $27 = r^3$. We must find the cube root of 27, or a number that multiplies by itself 3 times to arrive at 27. This leaves us with $r = 3$.

10. *Slope*. A. The slope of a line is equal to $\frac{y_2 - y_1}{x_2 - x_1}$, or the difference between the y -coordinates divided by the difference between the x -coordinates. In this case, $\frac{y_2 - y_1}{x_2 - x_1} = \frac{-5 - 10}{10 - 1}$, which simplifies to $-\frac{15}{9} = -\frac{5}{3}$.
11. *Probability*. B. The probability of picking a jack (or a king) is $\frac{4}{52} = \frac{1}{13}$ (the number of ways something can happen over the total number of outcomes). If the jack is picked and then put back, the probability of choosing a king remains unchanged. So, the probability of picking a jack then picking a king is $\frac{1}{13} \times \frac{1}{13}$.
12. *Averages*. C. If a set of 5 numbers has a mean of 10, then they have a sum of $10 \times 5 = 50$. The mean is the sum of the terms over the number of terms, so we can represent the new mean of 15 as $15 = \frac{50 + x}{6}$, where x is the unknown additional term. Multiply both sides by 6 to yield $90 = 50 + x$, so $x = 40$.
13. *Proportions*. B. We can set up this question as a proportion. $\frac{5}{9} = \frac{x}{45}$, where x is the height of the flagpole. Cross multiply to solve. $9x = 45 \times 5$, so $9x = 225$, and $x = 25$ feet.
14. *Distributing*. A. Apply FOIL (multiply the first, outer, inner, and last terms, in that order) to yield $x^2 - bx + ax - ab$. (Be careful with the signs.)
15. *Matrices*. D. To add matrices, add the corresponding values. Here, we'd add $x + x$, $1 + -5$, $3 + 9$, and $2y + 3y$. So, the sum is $\begin{bmatrix} 2x & -4 \\ 12 & 5y \end{bmatrix}$.
16. *Distance on a Coordinate Grid*. C. We can use the Pythagorean theorem to solve. If we take the absolute value of the x -coordinates and add them together, and do the same for the y -coordinates, we find that the distance along the x -axis is 8, and the distance along the y -axis is 6. These are the two legs of a right triangle that are 8 units long and 6 units long. The Pythagorean theorem tells us that $a^2 + b^2 = c^2$, where c is the length of the line connecting the two points (or the hypotenuse of the triangle). $6^2 + 8^2$ simplifies to $36 + 64$, which equals 100. Since $100 = c^2$, take the square root of both sides to find that $c = 10$. Therefore, the length of the line connecting the two points is equal to 10 units.
17. *Absolute Value Inequalities*. C. Isolate q to solve. Since this is an absolute value, $-5q - 5 > 5$ or $-5q - 5 < -5$. Simplify both equations: $-5q - 5 > 5$ simplifies to $-5q > 10$ and then $q < -2$ (remember to switch the direction of the inequality when dividing by a negative number!). $-5q - 5 < -5$ yields $q > 0$. So, the graph should show an open point at -2 and an arrow pointing left, and an open point at 0 and an arrow pointing right.
18. *Angle Sums*. D. The sum of angles in a hexagon is 720° (the formula for finding the total degree measurement of angles in a polygon with n sides is $180^\circ(n - 2)$). Thus, let the missing angle equal x : $44 + 138 + 82 + 68 + 241 + x = 720$. Solving for x yields $x = 147$.
19. *Trigonometry*. C. Remember SOHCAHTOA: In this case, we know the angle A . The tangent is the opposite over the adjacent angle, so $\tan 27^\circ = \frac{8}{x}$, where x is the length of the unknown side. Isolate x to find $x = \frac{8}{\tan 27^\circ}$.

20. *Polygons on a Coordinate Grid*. C. A square has 4 equal sides. The left-most point is 3 units down and 3 units left of the top-most point. The bottom-most point is a similar distance from the left-most point. This means that the missing point must be a similar distance from both the top-most and bottom-most points.
21. *Simplifying Expressions*. B. We can combine like terms, but this means that we must pay attention to the exponent assigned to each variable base (we cannot add or subtract c^2 with c^3 , for example). The equivalent expression is $3c^3d^2 - 5c^2d^3$.
22. *Permutations & Combinations*. C. We can find this answer by multiplying. If 1 person fills a position, there is 1 fewer person to fill the next one, so the number of possibilities is equal to $6 \times 5 \times 4 \times 3 = 360$.
23. *Equation of a Line*. D. This line has points at (0, 6) and (2, 0), so it has a slope of $\frac{y_2 - y_1}{x_2 - x_1} = \frac{0 - 6}{2 - 0} = -3$. The equation for a line takes the form $y = mx + b$ where m is the slope and b is the y -intercept. So, this equation has an equation $y = -3x + 6$.
24. *Solving for Zero*. B. The value of the expression will be equal if either of the terms in the numerator are equal to 0. 2 or -4 would make the expression equal to 0. (The denominator cannot be equal to 0, because that creates an invalid fraction.)
25. *Survey Samples*. C. The best sample will be broad and relevant. If the mosquito may be causing disease in humans in the northeast, testing mosquitos in New York and Maine will give the broadest sample (rather than just in Vermont). California and sub-Saharan Africa are outside of the area of study.
26. *Frequency Charts & Graphs*. A. The range of the data is the largest value subtracted by the smallest. In this data set, $4 - 0 = 4$. So, the range is 4.
27. *Roots of Variables*. B. To find the square root of an exponent, divide the exponent by 2. Thus, the value of the expression is $4x^{72}$ (since the square root of 16 is 4).
28. *Inequalities*. A. Subtract 3 from all 3 parts of the inequality, then divide by -2 to isolate j . Remember to switch the sign because you're dividing by a negative. So, $-7 \leq -2j < 8$ and $\frac{7}{2} \geq j > -4$. The correct graph will have a closed point at $3\frac{1}{2}$ and an open point at -4 with a line connecting the two points.
29. *Appropriate Units*. A. The weight of a cruise ship should be determined in units of mass, which eliminated liters and kilometers. Since a cruise ship is huge, it is more reasonable to measure in kilograms than grams.
30. *Imaginary Numbers*. D. If $x^2 = -400$, then we could express the equation as $x^2 = (-1)(400)$. Taking the square root of both sides, we would arrive at $x^2 = (\sqrt{-1})(\sqrt{400})$. This simplifies to $x^2 = 20i$ (since the square root of -1 is imaginary, and $20 \times 20 = 400$). Since the square of a negative number will be positive, this solution set is $\pm 20i$.
31. *Multiples*. A. The least common multiple is the smallest number that is a multiple of all three numbers. a and b^2 must be factors. 2, 4, and 6 are all factors of 12. Thus, the least common multiple is $12ab^2$.
32. *Roots of Numbers*. B. Subtract first: $100 - 64 = 36$. Then find the square root of 36, which is 6 ($6 \times 6 = 36$).
33. *Scientific Notation*. C. When dividing exponential expressions with the same base (in this case, 10), we can simply subtract the two exponents: $10 - (-5) = 15$. This leaves us with 10^{15} . $7.2 \div 8 = 0.9$. Thus, we are left with 0.9×10^{15} . Notice that this is not an answer choice. Instead, all of the choices begin with 9.0. To convert from 0.9 to 9.0, we must reduce 10^{15} to 10^{14} .

34. *Area & Perimeter*. A. If the circle were not missing a section, the area would be equal to $\pi(4)^2 = 16\pi$.
If the circle were missing a quarter, however, the area of the circle would be $\frac{3}{4}(16\pi) = 12\pi$.
However, there is a right triangle with legs measuring 4 units long. Since we know the area of a triangle is $\frac{1}{2}lw$, we know that the area of the triangle is $\frac{1}{2}(4)(4) = 8$. So, the area of the shaded region is $12\pi + 8$.
35. *Converting Units*. A. We must convert both distance and time. Multiply 80 by 60 to get the number of meters traveled in an hour, then divide by the number of meters in a foot and multiply that by the number of feet in a yard.
36. *Probability – Conditional*. B. If Adam picks a queen and does not return it, then 3 queens and 51 card remain, so the probability that he will pick another queen is $\frac{3}{51}$.
37. *Area & Perimeter*. D. There are 16 shaded squares. If the total area is 64 square meters, then each square has an area of $64 \div 16 = 4$ square meters. This means that each square has a length of $\sqrt{4} = 2$ meters. The perimeter is the sum of the length of each side. There are 18 sides/units, so the perimeter is equal to $2 \times 18 = 36$ meters.
38. *Vocabulary*. A. A real number is any number that is not imaginary. A natural number is a positive integer (there is no positive integer that, when multiplied by itself, gives us 5). A rational number is any that can be written as a fraction (for example, $\frac{5}{1}$).
39. *Formulas*. A. If the diameter is half the height, then the diameter is 6 and the radius is 3. So, the volume is equal to $\frac{1}{3}\pi(3)^2(12) = 36\pi \text{ cm}^3$.
40. *Algebraic Relationships*. D. The only number that can be divided by any number and still equal itself is 0. Therefore, in this equation, x must be equal to 0.
41. *Averages*. D. If the three picked an average of 85 peaches, then the sum of the apples they picked must be $85 \times 3 = 255$. If Alice picked 105 peaches, then the remaining peaches (150) were picked by Bob and Carol. The average is the total peaches divided by the number of people who picked them, so the average number of peaches Bob and Carol picked is $150 \div 2 = 75$.
42. *Percents*. C. 100% more is 2 times the original number (or the whole plus the whole). So, if Michael had 4 hours of homework last night, he has 8 hours of homework tonight.
43. *Area & Perimeter*. B. There are 11 shaded squares. If each has an area of 4 in^2 , then the area is equal to $4 \times 11 = 44 \text{ in}^2$.
44. *Angle Sums*. C. The sum of the interior angles of a quadrilateral is 360° . So, the unknown angle in this quadrilateral (let it equal y) can be found in the equation $100 + 80 + 110 + y = 360$, and $y = 70$. Since x is the supplementary angle to y , the sum of x and y equals 180. Therefore, $x + 70 = 180$, and $x = 110$.
45. *Percent Change*. B. Calculate the change in price: $\$750 - \$600 = \$150$. Then, divide by the original price to find the percentage change: $\$150 \div \$750 = 0.2$, or 20%.
46. *Averages*. D. The last score counts twice and so must be added twice to the total. The question can be represented by the equation $92 = \frac{94 + 100 + 88 + 80 + x + x}{6}$. Note that though there is only one additional test, it is counted twice, which means it must be added twice in the set. This gives us $552 = 362 + 2x$, and $190 = 2x$. So, $x = 95$, which is the score she must get on her final in order to get a 92 average.

47. *Absolute Value Inequalities.* C. This graph shows $-12 < x < 2$. Rewrite some of the answer choices to see what they represent. $|x + 5| < 7$ can be rewritten as $-7 < x + 5 < 7$. Subtracting 7 from all sides also yields $-12 < x < 2$.

